Mech. Engineer
Bridgman's article

The irreversible adiabatic
is Constant heat Content (Some times)
The reversible adiabatic is
Constant entropy

Ink
paper
Cover

A. Beaty

Monday afternoon 1:30

Delaval is a single pressu stage turbine

Turbines.

$$Hp = \frac{W\left(V_{enter}^2 - V_{leave}^2\right)}{550 \times 2g}$$

Where v is in '/sec

Reaction $= \frac{wv}{g} \times 2$

when jet is turned thru 180°

Only $\frac{W}{g} v$ when jet is not turned thru 180°

H. Secoy

ELEMENTS OF
HEAT-POWER ENGINEERING

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ELEMENTS OF
HEAT-POWER ENGINEERING

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Part I
THERMODYNAMICS AND PRIME MOVERS

THIRD EDITION
OF FIRST PART OF
Hirshfeld and Barnard's
Elements of Heat-Power Engineering

COMPLETELY REWRITTEN
By F. O. ELLENWOOD and W. N. BARNARD

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PREFACE TO PART I

In preparing this new edition of the portions of the former book that relate primarily to thermodynamics and the elementary principles of prime movers, it has been found necessary to rewrite them substantially in their entirety in order to incorporate the new methods of treatment that have been evolved while teaching the subject since the former editions were issued, and to include important new material. Because of the notable advances that have taken place recently in the field covered, it is believed that much of the subject matter will be of interest not only to teachers but also to practicing engineers.

Special attention is directed to the following features: the presentation of Reversibility and Entropy in strict accord with modern theory, but at the same time with a frank omission of the rigorous mathematical details; the derivation of the General Energy Equation early in the text so that it may be used as the basis of much of the subsequent discussion; the treatment of the properties, processes and cycles of gases having specific heats that are variable; the attempt to present satisfactory standards, including the "Real Mixture Standard," with which to compare the performance of actual internal combustion engines; the discussion of the dual combustion and composite cycles; the distinction that has been made between the cycles of prime movers using vapors, *i.e.*, the *cycle of the machine* itself, and the *vapor cycle*, or the one through which the working substance passes while traversing the *whole system*, including the boiler, prime mover, condenser and pumps,—a differentiation which has become of increased value because of the use of higher pressures.

An important addition that is believed to be new to the literature of the subject is the development, for engines and plants using vapors, of ideal cycles involving reheating and regeneration with the same number of reheating and bleeding points that are used in the actual plant. It is believed that they will constitute satisfactory standards on which to base the efficiencies of actual units embodying these features.

To as great an extent as seemed feasible, the notation, definitions, and standards given in the Power Test Codes of the American Society of Mechanical Engineers have been followed. The book is so planned that, if desired, many of the sections devoted to the more advanced discussions may be omitted, as indicated by footnotes, without interfering with the continuity of the treatment.

In addition to the main notation, which is tabulated where needed in the various chapters, the index gives the pages on which the symbols are defined. References to sources of much of the information presented, and suggestions of books and articles for further study, are included.

Briefly, Part II will cover, in a separate volume, the discussion of fuels and combustion; combustion equipment; heat-transmission, and apparatus using it; steam boilers, prime movers, auxiliaries and other plant equipment; the economics of the selection of equipment and its operation; and similar subjects.

The authors desire to extend thanks to Mr. C. Harold Berry, now Associate Editor of *Power* and formerly a member of the faculty of Cornell University, for his very great assistance in preparing the important chapters on Reversibility and Entropy, to Professor George B. Upton for his helpful suggestions, especially those regarding the treatment of the ideal cycles of internal combustion motors, to Assistant Professor Frederic C. Evans for valuable aid, and for devising and solving many of the problems, and to Mr. Wilbur S. Cooper for his help in preparing some of the problems and charts.

Ithaca, N. Y.,
Sept. 7, 1926.

NOTE TO INSTRUCTORS

While this volume is intended primarily as a text for a full year of instruction in a college course, it may also be used in short courses by omitting the more advanced portions.

For the ideal Otto and Diesel Cycles, complete numerical solutions are given for the Real Mixture Standards, and although these may be omitted, it is believed worth while to have the student examine them carefully even though he does not carry through the solutions of similar cases. This will assist him not only in obtaining an idea of the general principles involved but also in selecting a reasonable value of the exponent to use in the " air standards " of these important cycles.

As a thorough mastery of the principles covered is best obtained by making application of them, forty pages of problems, with answers, have been added at the end of the book.

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ERRATA

ELEMENTS OF HEAT POWER ENGINEERING, PART I

BY

BARNARD, ELLENWOOD AND HIRSHFELD

PAGE

77, 4th line, eliminate the word "to."

238, 9th line, change "as" to "at."

249, Table VIII, bottom line, change (i) to (b).

286, 4th line, change "of" to "a."

432, Problem 13, 3rd line, insert "at constant pressure" after the word "cool."

434, Problem 8, 1st line, change "if" to "it."

441, Problem 6, interchange the answers to part (a) and also those to part (b).

451, Problem 16, answers should be (b) 111,000 ft. lb. and (d) –33 B.t.u.

453, Problem 35, 2nd line, change 45° to 50° and in the 3rd line change 35 per cent to 85 per cent.

468, Table C, the entropy of saturated vapor at 109 deg. should be 1.9578.

ELEMENTS OF HEAT-POWER ENGINEERING

CHAPTER I

FUNDAMENTALS REGARDING ENERGY AND ITS TRANSFORMATION

1. Heat-Power Engineering. (a) The advancement of the human race has been largely due to the fact that man has greater ability than his fellow creatures to utilize nature's resources. At first he was driven by his own weakness to seek nature's aid for protection, and he thus became familiar with her simpler laws. This knowledge grew steadily and after a time was recorded. Now the accumulated information is too great to be grasped by any individual or group and it has become necessary to specialize. One group of specialists, the scientists, continue to delve after nature's secrets in order to add to the store of human knowledge; another group, the engineers, work to make application of discovered laws to meet the needs of humanity.

The engineer must know these laws and must be familiar with their application in order that he may be able to aid the race in development and improvement of its life. One of the most important of his problems results from the fact that man's body cannot supply the power required to carry out the conceptions of his mind. To solve this problem the engineer draws on nature's store of energy.

In general, the energy of nature's store is not directly available for human uses; it must be changed in kind or quality, transmitted through space, and made available at times of demand. The engineer must provide means for effecting these results.

One of the best examples of such changes is furnished by the conversion of heat energy into the mechanical form by means of **heat engines**. This conversion is of great importance, because the world demands enormous supplies of mechanical energy and because immense stores of easily transportable fuel are distributed over the earth near its surface. This fuel has potential heat energy, which may easily be made available by combustion. It is then the duty of the heat engine to convert as large a part as possible of this energy into mechanical work, at such rates as may be needed, or, in other words, to supply **power** as required.

(b) The term **Thermodynamics** is often applied broadly to mean the science which deals with the relation between thermal and the many other forms of energy. Since these other forms may be mechanical, electrical, chemical, or biological, the subject is a most extensive and fundamental one. As limited to engineering, and as used in this text, thermodynamics is that branch of science which deals with processes whose only significant aspects are thermal and mechanical ones. In connection with it the properties of gases and vapors are investigated for the purpose of determining the energy transformations involved in heat engines, refrigerating machines and gas compressors.

(c) Thermodynamics is based on the study of the behavior of real bodies, in terms of characteristics which can be seen or felt,—such as pressure, volume, temperature and velocity. From the observed relations of these characteristics are established by definition new quantities, such as heat, work, energy and others that are to follow, which are not as obvious to the senses. Often a mental picture of just what happens during a thermodynamic transformation may be developed from accepted views of the constitution of matter, the action of molecules, etc., but it must be remembered that this is done only to assist the memory and to satisfy natural curiosity. In no way do the findings of thermodynamics depend on such speculations; its entire structure rests upon observed relations between measurable quantities that are directly determinable by experiments on actual bodies.

(d) **Heat-Power Engineering** designates that branch of engineering which deals with combustion, heat transfer, thermodynamics, heat-power apparatus, power plants, their economics, and related subjects.

2. Energy.—Whenever a body is capable of doing work by changing its state or condition it is said to possess *energy*. The velocity, pressure, specific volume and temperature of a body commonly determine its *state* and it is the changes in such characteristics that release some energy and makes it available for doing work. The characteristics mentioned are not the only ones that exist; others that the engineer uses because of their convenience will be introduced later.

3. Heat a Form of Energy.—It has been shown experimentally that heat can be produced by the expenditure of other forms of energy, and also that other forms of energy can be produced by the expenditure of heat. Therefore it is justifiable to conclude that heat is a form of energy.

All bodies that man knows possess some heat energy; and whatever the material or state of a body may be, it is usually possible to obtain part of the heat energy from it. It is not known exactly how this energy is stored in matter, but it is certainly possible, and it seems probable, that it is in some way associated with the motions and relative positions of the constituent particles. Beyond this it is not necessary to generalize here in the present state of knowledge.

4. Solar Heat.—Heat for human use probably all comes, directly or indirectly, from the sun. This heat is applied directly to produce a sufficiently high temperature on portions of the earth's surface to render plant growth and animal life possible.

Heat engines have been built which convert into mechanical energy heat directly from the sun; but, because their bulk is great in proportion to the energy transformed, and because the sun's rays are not always available when needed, and also because of their relatively large cost, such engines have not as yet been commercially successful.

The energy of the sun's rays is applied indirectly through the agency of plant growth and geologic processes to produce stores of fuel in the earth's crust. Heat energy, indirectly from the sun, may be evolved for human use from this fuel.

Also the sun's rays falling upon water surfaces cause evaporation whereby heat is converted into mechanical energy. This energy lifts the water vapor, which is again condensed and falls upon the earth's surface as rain or snow. The resulting water flowing to its original level prepares the soil for plant growth; it irrigates plants and turns water wheels to supply mechanical energy.

Heat may be derived from mechanical energy, electrical energy, or from the chemical combination of certain elements. In most cases the ultimate source of the energy is probably the sun.

5. Heat from Mechanical Energy.—Primitive man generated heat to kindle fire by rubbing two sticks together. In such processes the mechanical energy due to the muscular effort that moves the sticks reappears as heat. This heat is derived indirectly from the sun, since the sun's energy makes possible animal life and therefore muscular effort.

The engineer is familiar with the production of heat by machine friction. This conversion is undesirable, since mechanical energy, which should be available for useful purposes, becomes useless heat. The same change occurs when a machine is retarded or stopped by a friction brake. This is a useful change, however, since mechanical energy, which cannot be used and which may become dangerous, is dissipated as heat and rendered harmless.

In general, heat for human use is not derived from mechanical energy because it may be obtained in other ways more conveniently and at much less cost.

6. Heat from Electrical Energy.—The conversion of electrical energy into heat is illustrated by every electric conductor that carries a current; for, though the reason may be unknown, heat results whenever an electric current flows. The heat is always a loss if the delivery of maximum electrical energy is the object of the flow; but it would not be a loss if heat were the object, as in electric furnaces and stoves.

7. Heat from Chemical Reaction.—(**a**) When some substances react chemically heat is liberated, and in other cases it is absorbed. According to the physical-chemist, every chemical system has a certain total energy content, part of which is in the form of heat energy, and the balance may be called chemical potential energy which is not available until some reaction occurs. During a reaction there is an interchange between these two forms of energy, and this results in a redistribution, but the aggregate total energy remains constant. To secure a supply of heat from chemicals it is necessary to utilize systems that form new compounds by reactions in which there is an increase of heat at the expense of the chemical potential energy.

(**b**) To illustrate, consider the production of heat by the combination of carbon and oxygen to form carbon dioxide. The total energy of the system of carbon and oxygen is exactly the same as that of the resulting carbon dioxide, but the distribution is different. Initially, with carbon and oxygen merely mixed together at ordinary temperatures, there is comparatively little heat energy and a rather large measure of chemical potential energy. After the reaction the chemical potential energy with respect to this particular case has been reduced to zero, since carbon dioxide is an inert gas, and all of this energy has been transformed into heat energy which is available for removal as heat if the carbon dioxide be cooled to the initial temperature, or this heat energy is partially available for the development of work if the combustion has taken place within an engine cylinder, in which the carbon dioxide can be allowed to expand.

The usual source of heat in engineering is from the combustion of fuel whose constituents react in various ways with oxygen to form new compounds. These reactions and amount of heat resulting therefrom will be considered in detail in the chapter on Combustion.

8. Units of Heat.—The unit of measurement of energy is usually based upon some effect produced by the kind of energy involved. Under certain conditions a rise in temperature of a body is one of the most obvious phenomena connected with the addition of heat to it, and this effect is used as the basis for defining the unit of heat energy.

In the United States the unit of heat energy commonly used by engineers is known as the **British thermal unit** (B.t.u.) and is defined as follows:

One British thermal unit is $\frac{1}{180}$ *of the amount of heat required to raise the temperature of a pound mass of pure water from* 32 *to* 212 *deg. fahr., the pressure being kept constant at* 14.70 *lb. per sq. in. abs.*[1] (212 $-32=180$.)

The gram-calorie, the kilogram-calorie, and the pound-calorie are units of energy commonly used in the measurement of heat by engineers in other countries.

9. Units of Work, Power and Torque.—(a) **Work** is the action of a force in displacing a resistance through a distance, and is measured by the product of the force multiplied by the distance moved in the direction of the force. One unit of work is the **foot-pound**, which is the amount of work done by a force of one pound acting through a space of one foot. This unit is too small for convenient use in many instances, and therefore much larger units are in common use, especially in power-plant studies. Since work and heat are each forms of energy, the **B.t.u.** may be employed as a unit of work. The relation between these two units is

$$1 \text{ B.t.u.} = 778 \text{ ft-lb.}^2$$

It is customary to use the symbol J to represent the quantity 778, and A for its reciprocal, namely $\frac{1}{778}$.

(b) **Power** is the *rate* at which energy is expended or consumed. This definition does not require that energy be expended as mechanical work; it can be expended in other forms. For example, it is proper to speak of the power of an electric light, or of an electric toaster, or of other forms of energy-consuming apparatus. The two most common units of power used by engineers are the **horsepower** (hp.) and the **kilowatt** (kw.). It is convenient to remember that

$$1 \text{ hp.} = 33{,}000 \text{ ft-lb. per min.} = 2545 \text{ B.t.u. per hour.}$$

and

$$1 \text{ kw.} = 1.341 \text{ hp.} = 3413 \text{ B.t.u. per hour.}$$

(c) Since power is the *rate* at which energy is expended, the product of power multiplied by the time during which the power is used represents a definite quantity of energy. For large composite units of energy engineers commonly use the **horsepower-hour** (hp-hr.) and the **kilowatt-hour** (kw-hr.), which have the following respective values:

---

[1] Where extreme accuracy is not important the British Thermal Unit may be taken as the heat required to raise the temperature of 1 lb. of water 1° F. With this assumption, however, there may be a small error as the amount of heat needed for the one degree rise varies slightly with the temperature and depends on the purity of the water.

[2] The experimental difficulties encountered in the determination of this relation are so very great that there is some uncertainty regarding the value of the third significant figure.

$$1 \text{ hp-hr.} = 1{,}980{,}000 \text{ ft-lb.} = 2545 \text{ B.t.u.}$$

$$1 \text{ kw-hr.} = 1.341 \text{ hp-hr.} = 3413 \text{ B.t.u.}$$

The relation between the various units is given in Table I.

TABLE I

UNITS OF POWER AND ENERGY

(*a*) Units of Power

	Foot-pounds		B.t.u.		Hp.	Watts	Kilowatts	Joules per sec.
	per sec.	per min.	per sec.	per min.				
Horsepower (hp.)	550	33,000	0.707	42.42	1	745.7	0.7457	745.7
Kilowatt (kw.)	737.55	44,253	0.948	56.88	1.341	1000	1	1000

(*b*) Units of Energy

	Foot-pounds	B.t.u.	Hp-hr.	Watt-hr.	Kw-hr.	Joules
Horsepower-hour (hp-hr.)	1,980,000	2545	1	745.7	0.7457	2,684,000
Kilowatt-hour (kw-hr.)	2,655,000	3413	1.341	1000	1	3,600,000

1 B.t.u. = 778 ft-lbs. (Symbol J = 778.) 1 ft-lb. = $\frac{1}{778}$ B.t.u. (Symbol A = $\frac{1}{J}$.)

NOTE. In all heat-power computations engineers generally use the standards that are given in the "Code of Definitions and Values" of the Am. Soc. of Mech. Engineers. Table I conforms to this code.

(d) The **torque** of an engine shaft, or other rotating element, is its turning moment, which is measured by the product of the force and the perpendicular distance from the axis of rotation to the line of action of the force. The customary units of torque are the pound-foot, or the pound-inch.

If a shaft with a torque of T_q lb-ft. is rotating at N revolutions per minute, the horsepower transmitted by it will be

$$\text{hp.} = \frac{2\pi N T_q}{33{,}000} = \frac{N T_q}{5252} = 0.000{,}190{,}4\, N T_q, \quad . \quad . \quad . \quad (1)$$

and if the torque is in pound-inches,

$$\text{hp.} = \frac{N T_{q''}}{63{,}024} = 0.000{,}015{,}87\, N T_{q''}. \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

10. The Working Substance.—(a) One of the important factors relating to a heat engine is the so-called *working substance* which is made to flow through the power plant, absorbing heat-energy at some point and then transforming a portion of this energy into work as it passes through the engine. In the steam power plant the working substance is the steam; and in internal-combustion engines it is the air and other gases that pass through the cylinders. The same kind of working substance may be used in different types of heat engines,—for example, steam is used in reciprocating engines and also in turbines. Furthermore, in a given type of heat engine various kinds of working substances can be used,—thus, there are steam turbines, mercury turbines and gas turbines.

(b) In the common type of *refrigerating* plant a working substance is used for the purpose of absorbing heat from bodies that are to be cooled. It is then compressed, in a compressor or "heat pump," by means of energy supplied from an outside source, until its temperature has become high enough to permit the transfer of a portion of its heat to some other convenient body,—usually water at about atmospheric temperature,—which carries it away.

(c) The choice of a suitable working substance for any given case is a matter of great importance since it affects the size and operating characteristics of the plant to a marked degree, and in making such selection much information is required concerning the properties of various substances over the wide ranges of pressure, volume, and temperature which are found in power plant apparatus.

11. Conservation of Energy and the First Law of Thermodynamics.—(a) It seems to be one of nature's great universal laws that energy cannot be created or destroyed. Experience and experiment have tended to establish this law, and now there is no reason to doubt that it holds throughout the universe. This **Law of Conservation of Energy** may be stated as follows: *Energy cannot be created or destroyed: but all forms of energy are mutually interconvertible.*

(b) The Law of Conservation of Energy, when limited to heat and mechanical energy, is called the **First Law of Thermodynamics,** and it may be stated thus: *Heat and mechanical energy are interconvertible and can neither be created nor destroyed*

From this law it follows that: *A definite amount of heat disappears for each unit of mechanical energy produced from heat; and, conversely, this same quantity of heat is produced from each unit of mechanical energy expended in any way in producing heat.* It should be noted, however, that this does not imply that *all* the energy present in one particular form must be always converted completely into the other form in each instance.

(c) In any heat engine only *part* of the energy supplied as heat appears as mechanical energy. As will be seen more clearly later, this is not because energy is destroyed, but because part escapes conversion and leaves the engine still in the form of heat. However, *each B.t.u. that is converted* is transformed into a definite amount of work, namely 778 ft-lb.

(d) In order to do mechanical work there must be motion, and in all real cases the motion meets with resistance of some form. Anything that resists motion takes away energy; thus, friction might take away heat; a belt might take away mechanical energy; a metallic circuit might take away electrical energy; and if the motion produces sound, energy is taken away by sound waves in the air. If any energy whatever were thus taken away by resistance, and the machine continued in motion without receiving a continual supply of energy, it would have to give out energy that it did not receive. All experience, however, shows that it is impossible to build a machine which will run continuously without some supply of energy. Energy cannot be created and perpetual motion cannot exist. The actual machine would slow down and ultimately come to rest.

12. The Second Law of Thermodynamics.—(a) It is a matter of common observation that in a steam engine, for instance, the steam exhausted after doing work still contains a considerable quantity of heat, and that its temperature is lower than that of the steam supplied to the engine.

This phenomenon of receiving heat from a working substance at a high temperature and rejecting some of the heat while the substance is at a lower temperature, with failure to transform into work all of the heat supplied, is characteristic of every real engine, no matter how perfect, and will be shown later to be true also of every ideal engine. The operation of all such engines is pictured graphically in Fig. 1. Heat energy flows from reservoir I at the high temperature, T_1, into the engine. There part of it is converted into mechanical energy, as shown by the stream Wk flowing out to the right, while the rest of the energy is absorbed by the receiver II at the lower temperature T_2.

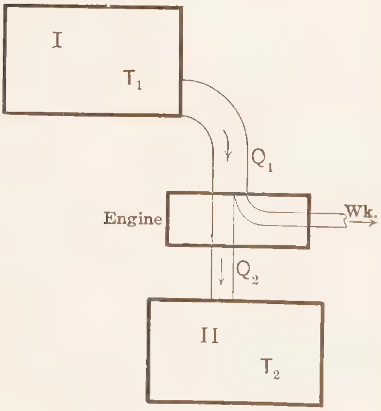

FIG. 1.—Diagrammatic Representation of a Heat Engine.

(**b**) Calling the quantity of heat supplied in a given time Q_1, the mechanical energy leaving Wk, and the quantity of heat leaving Q_2, it follows from the conservation of energy that

$$Wk + Q_2 = Q_1.$$

This rearranged gives

$$Wk = Q_1 - Q_2, \qquad (2)$$

from which it immediately appears that the smaller Q_2 is, the greater will be the work resulting from the use of a given quantity of heat, Q_1. But all observation indicates that Q_2 can never be made zero.

(**c**) Experience has shown in every case, even if friction could be entirely eliminated, that a machine can convert into mechanical energy only part of the heat supplied to it, that it must reject the remainder, Q_2, and that this rejection must be by means of a working substance *at a lower temperature* than that at which this substance received heat. This is summed up in the so-called **Second Law of Thermodynamics** as follows:

No machine, actual or ideal, can both completely and continuously transform heat into mechanical energy.[3]

(**d**) If such *complete* transformation could be effected continuously it would lead to a form of perpetual motion. But as long as heat must be exhausted with the working substance at a lower temperature than it had during the absorption of heat, the possibility of obtaining mechanical energy from heat discontinues as soon as the temperature of all material in the universe has been dropped to the lowest attainable value.

(**e**) *Continuous* and perpetual conversion of heat into mechanical energy would be possible if the necessity of exhausting heat to a body at lower temperature than the source of supply were removed; for then it would still be possible to continue the conversion after all means of obtaining temperature difference had been used up,—that is, after all substances had been reduced to the lowest attainable temperature. As all mechanical energy eventually passes back into heat energy, generally to bodies at low temperature, through friction and allied phenomena, there would be no danger of the supply of heat from these low temperature bodies giving out, hence, the cycle would be an endless one, consisting of the transformation of heat into mechanical energy, the retrogression of this form of energy to heat, the conversion to the mechanical form again, and so on *ad infinitum*. But such form of perpetual motion is impossible even in imagination. Thus, as both the complete and continuous conversion of heat into mechanical energy are impossible, the conclusion is that the second law as stated must be true.

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[3] There are almost as many statements of the Second Law as there are authors of books on Thermodynamics. It is believed that the statement here given is the most satisfactory one for the purpose of this book. The statement by Clausius is: *It is impossible for a self-acting machine unaided by an external agency to convert heat from one body to another at higher temperature.* As given by Kelvin: *It is impossible by means of inanimate material agency to derive mechanical effect from any portion of matter by cooling it below the temperature of surrounding objects.*

13. The State of a Substance.—(a) A working substance may be in any one of the four[4] "states of aggregation" or **phases,**—solid, liquid, vaporous or gaseous; it may change from one phase to another, it may have a mixture of phases, and it may have various temperatures and pressures, which may be constant or changing. The mere statement of phase is not sufficient information for the engineer to determine what he calls the exact "**state,**" or *condition*, of the working substance at any time during a process which he desires to study. To specify completely the states of a known mass of working substance, sufficient additional information must be given to determine its condition in detail. The properties which the engineer most commonly uses to specify the state of a substance are: pressure, volume, temperature, kinetic energy, internal energy, "heat content" and entropy. Usually the state is fully determined by knowing any two of these properties. The meanings of the special terms "internal energy," "heat content" and "entropy" will be given later; the other terms are defined in this section.

(b) The **pressure** of the working substance means the force exerted by this substance on each unit of area of the surface of the container. "*Absolute pressure*" is used in all calculations in thermodynamics, but "gage pressure" is the value read from the ordinary pressure gage. Absolute pressures that are greater than atmospheric are equal to the gage pressures plus the atmospheric pressure, all, of course, being expressed in the same unit.

Atmospheric pressure often varies widely from day to day and place to place, hence its value should always be determined at the place and time needed, a mercury barometer being generally used for the purpose. The *standard atmospheric pressure* adopted for international use is that which supports a column of pure mercury 760 mm. high (29.9212 in.) at 32 deg. fahr., and at a location that has the standard acceleration due to gravity. The corresponding pressure in pounds per sq. in. is 14.693. The value 14.7 lb. per sq. in. is often used for the atmospheric pressure in determining relatively high absolute pressures, such as those in steam boilers, but this value is not accurate enough for determining very small absolute pressures, such as those in steam condensers.

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[4] The physicist usually considers but three states of aggregation,—solid, liquid and gaseous, the vapor phase being included in the latter; but as the engineer has great use for both vapors and gases they are considered distinct in this text.

(c) Pressures less than atmospheric are often used in steam power plant work. The instruments used for the determination of such pressures are called *vacuum gages*, and values obtained from them are subtracted from the atmospheric pressure in order to obtain the absolute pressure. Usually vacuum gages are graduated to read pressures in inches of a column of mercury which is at some definite temperature. At 32 deg. fahr. one cubic inch of mercury weighs 0.4912 lb.

(d) The **volume** of the working substance means the total space filled by the particular amount of substance under consideration. The **specific volume** is the volume of a *unit weight* of the working substance existing under the conditions specified.

(e) The **temperature** of any substance is its thermal condition which determines the tendency of this substance to give or receive heat from other bodies; or it may be said to be a measure of the molecular activity of the substance. It is measured ordinarily by thermometers. The zeros of the temperature scales in common use are arbitrarily chosen ones. The absolute zeros on the Fahrenheit and Centigrade scales are defined in Sect. 65.

14. Kinetic Energy.—A body is said to have kinetic energy when it has capacity for doing work through change in its velocity. If a body have its velocity reduced it will do work on other objects, and if its velocity be increased, work must be done upon it from an outside source. It can be shown that if the velocity of a known mass having a weight w, changes from v_a to v_b, then the amount of work done by the body is

$$Wk = \frac{1}{2}\left(\frac{w}{g}\right) \times (v_a^2 - v_b^2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If v_b is greater than v_a this equation gives a negative result, which means that the work has been done *upon* the body. The work will be in foot-pounds if the velocities are measured in feet per second, w in pounds, and g in feet per second per second.[5]

The kinetic energy of a body whose velocity is v, is given by the equation

$$KE = \frac{w}{2g}v^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

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[5] The standard value of g is 32.174 ft. sec. per sec., and is very closely equal to its value at sea level and at latitude 45°. For ordinary computations 32.2 is frequently used.

15. Transferring, Transforming, and Storing Energy.—In many calculations relating to heat engines, it is very essential to have the beginner realize as early as possible the distinction between the three following quantities: (1) the heat *transferred* as *heat* to or from a working substance during a given process; (2) the heat that may be *transformed* into another form of energy, such as the work done during the process; and (3) the energy that is *stored* within the working substance at the beginning or end of the process. The determination of the quantity of heat that is transferred from one body to another under many different conditions is not always a simple matter and the best method of accomplishing it will depend upon many factors which will be discussed in later chapters.

16. Internal Energy.—(**a**) As a result of experiments it is known that energy may be expended upon a substance, either by transferring heat to it or by doing work upon it, and that this expenditure may, or may not, increase the amount of energy stored *within* the substance. To distinguish between this *stored* energy and the other energy quantities, the former will be called the *internal energy*. As used by engineers, **internal energy**, or **intrinsic energy**, is *the energy possessed by a substance due to the velocity of its molecules and also due to its molecular arrangement* which may, in general, be such that the substance may exist either as a solid, liquid, vapor or gas.[6]

(**b**) Since the internal energy is believed to consist of the kinetic and potential energy of the rapidly moving molecules of which the body is composed, and since the pressure and volume of any known weight of substance determine the arrangement and velocity of its molecules, and hence its state, it follows that the *internal energy of a substance at any time depends only on its state, and is therefore independent of the method or process by which that particular state is reached*. In this regard, internal energy is similar to pressure, volume and temperature; but it is different from each of them with respect to its measurement. Pressure, volume and temperature may each be determined directly by physical measurements. On the other hand, the absolute value of the internal energy of a substance cannot be measured directly by any instrument, nor can this energy ever be reduced to zero; but *differences* in this quantity can readily be determined.

(**c**) The engineer is usually concerned only with the *difference* of internal energy of a substance in two distinct states, just as he is ordinarily interested in the difference in altitudes rather than the absolute elevations of two bodies which he desires to compare. Hence, the internal energy can be referred to any conveniently chosen state in which the internal energy is arbitrarily considered zero. The selection of the particular state to represent the zero of internal energy is important only in so far as it relates to convenience and uniformity; but it should be noted that temperature alone does not always suffice in specifying it. For example, the internal energy of a given weight of water at 32 deg. fahr. is different from the internal energy of the same mass in the form of steam at this temperature. The lack of agreement is due to the fact that the steam at 32° has a large amount of internal energy stored within it due to the energy expended in evaporation.

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[6] The physicist uses the term "internal energy" more broadly as applying also to the structure of the atoms of which the molecules are composed, but the narrower application is sufficient for the engineer's needs at present.

(d) The symbol I will be used to denote the internal energy of a substance in any state designated by the subscript attached to this symbol, and measured above its arbitrarily chosen zero. ΔI will designate the gain in internal energy between any two states under consideration, as indicated by the subscript; thus, $\Delta I_{ab} = I_b - I_a$. This equation is read as follows: the gain in internal energy between the states a and b is equal to the internal energy at b minus the internal energy at a, each being measured above the same arbitrarily chosen zero.

17. The Heat Content.[7]—**(a)** In modern engineering the "heat content" of a substance is one of its most important properties, especially if the substance is a vapor. By *definition* of the term, the *heat content*, H_a, of any substance in any given state a is equal to the internal energy, I_a, of the substance plus the "APV-product or quantity," AP_aV_a, taken for the given state a, the energy quantities being expressed in the same units. Then, according to this definition,[8]

$$H_a = I_a + AP_aV_a, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In this country each term of the above expression is usually given in B.t.u., P_a is the pressure in lb. per sq. ft., V_a is the volume in cu. ft. and the symbol A is equal to $\frac{1}{778}$.

(b) The term AP_aV_a in Eq. (5) is equivalent to the energy that would be expended if the substance having the volume V_a, should be forced into, or out of, a vessel against the constant pressure P_a, corresponding to that at the given state. Since the internal energy is measured above an arbitrarily chosen zero the numerical value of the heat content of a substance in any particular state will also depend upon this same datum. So long as the same zero of internal energy is used in all calculations, the results will be just as useful as though an absolute zero of internal energy had been used, since it is the differences of the heat contents of the same substance in distinct states which are desired.

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[7] The term "heat content" will be used very frequently later when dealing with vapors and this section should be reviewed when starting that subject,—or possibly, it may be omitted until that time. It is introduced here to emphasize its significance and because it may be conveniently applied to all gases as well as to vapors. There are many calculations, such as those relating to the steady flow of gases through a centrifugal compressor, which may be simplified by its use.

[8] This quantity, H_a, is often called the "total heat" and occasionally other names are used for it. It is important to distinguish the modern definition here given, and which is now being generally adopted throughout the world, from the old definition of "total heat" as given by Regnault in 1847.

(c) Referring to Eq. (5), it is evident that the heat content of a body can be changed by varying either one, or both, of the terms I and APV. To illustrate, consider a cubic foot of water at 32 deg. fahr. confined in a vertical cylinder, under a perfectly tight but frictionless piston which is so loaded as to cause the absolute pressure of the water to be, for example, 100 lb. per square inch. The heat content of the entire weight of water in this state will have a definite value. First, with the pressure maintained constant by the frictionless piston, let heat be added to the water, but not enough to cause the formation of vapor. The temperature of the water will rise and this increases the internal energy,—and consequently the heat content. The term APV will increase a very slight amount due to the small expansion of the water, but this is relatively so little for this case that the change in the heat content of the water is due almost entirely to the increase in its *internal energy.*

(d) On the other hand, consider an entirely different process,—one in which no heat is added, but one having a large increase in pressure. Starting, as before, with 1 cu. ft. of water in the cylinder with its initial pressure 100 lb. per square inch and its temperature at 32 deg. fahr., the APV-quantity is $100 \times 144 \times \frac{1}{778} = 18.52$ B.t.u. Now, if the load on the piston is increased until the absolute water pressure becomes 1100 lb. per square inch the APV-quantity becomes $1100 \times 144 \times \frac{1}{778} = 203.7$ B.t.u. Hence the heat content of this water has been increased 185.2 B.t.u., not by adding heat, but merely by increasing the *pressure.*[9]

(e) This last illustration emphasizes the fact that, while the name of this quantity, "heat content," seems to imply that it is always directly related to heat addition or subtraction, such is by no means the case.

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[9] Actually, when the pressure is raised the volume of the water will be decreased slightly (about 3 parts in 1000) and accordingly a small quantity of work will be expended on the water which will slightly increase the internal energy and consequently the heat content. But the relative amount of energy involved in this instance is so small that it has been considered negligible in this illustration.

In many instances, as will appear in the following pages, the heat content of a body undergoes very great changes in value in processes in which no heat is transferred to or from that body. For this reason, this term is far from satisfactory for this very important property, but until a better name is adopted by engineers it will continue in use. When applying it the beginner should always think of the definition of the term rather than the strict significance of the words composing it.

(f) It should be noted very carefully that since the internal energy and APV are each a fixed quantity as soon as the state of a substance has been determined, the same is also true of the sum of these two; hence, *the heat content must be a fixed quantity for a given state of a substance, and this is so regardless of the process by which that particular state is reached.*

18. Notation.—(a) Whether the working substance be in the liquid, vapor, or gaseous phase, there are certain common properties which must be considered and therefore a common set of symbols will be used for them, as follows:

$A = \frac{1}{778}$;
P = absolute pressure, in pounds per square foot;
p = absolute pressure, in pounds per square inch;
p'' = absolute pressure, in inches of mercury;[10]
t = temperature in degrees Fahrenheit;
T = absolute temperature in degrees Fahrenheit, $= t + 460$;
w = weight of the working substance, in pounds;
I = internal energy of w pounds, in B.t.u.;
i = specific internal energy in B.t.u. per pound, $= I/w$;
V = volume of w pounds, in cubic feet;
$\overline{V}$ = specific volume, in cubic feet per pound, $= V/w$;
v = velocity, in feet per second;
H = heat content of w pounds, in B.t.u.;
h = specific heat content, in B.t.u. per pound, $= H/w$;
S or Φ = entropy of w pounds;
s or ϕ = specific entropy, $= S/w$ or Φ/w;
Q_{ab} = The quantity of heat added to the working substance during the process ab;
Wk_{ab} = The work done by the working substance during the process ab.

(b) It should be noted that the use of the letters i, h $\overline{V}$, ϕ or n, for specific values and the corresponding capitals for the entire amount of working substance under consideration becomes of importance only in cases in which the weight is different from unity and in which specific values may be determined directly from tables or charts.

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[10] The temperature of the mercury is assumed to be 32° F.

(c) For those symbols that are used to represent the state of a substance (P, V, v, T, I, Φ, and H), appropriate subscripts are to be attached to indicate the particular states under consideration. Also the Greek letter Δ will be used with these symbols to designate the *gains*, or *losses*, in the quantities between the two states indicated by the two subscripts; thus:

For the gains	For the losses
$\Delta H_{ab} = H_b - H_a$	$\Delta H_{ab} = H_a - H_b$
$\Delta I_{ab} = I_b - I_a$	$\Delta I_{ab} = I_a - I_b$
$\Delta \phi_{ab} = \phi_b - \phi_a$	$\Delta \phi_{ab} = \phi_a - \phi_b$

Similarly for pressures, volumes, temperatures and velocities.

Other symbols will be introduced throughout the text as needed.

CHAPTER II

HISTORICAL OUTLINE

The following history of some of the developments that have taken place in the application of heat to the production of power is necessarily very brief, and only a few outstanding names and inventions are given. It is hoped that the discussion given here will be augmented by supplementary reading. (See references at the end of the chapter.)

19. Dawn in the Use of Heat for Developing Power.—(a) The earliest device known for the production of power from heat is the steam engine described by **Hero** of Alexandria (about **120 B.C.**), which is illustrated in Fig. 2. This was really a " **turbine** " of the " reaction " type, in which the issuing jets of steam caused the hollow sphere to rotate on the pipe-like trunnions through which the steam was supplied from the boiler below. Hero also described the use of heat to cause air in chambers to expand and displace fluids, and mysteriously to cause libations to flow onto altars, temple doors to close, images on a merry-go-round to move and weird noises to occur.

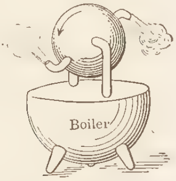

FIG. 2.—Hero's Reaction Turbine (120 B.C.)

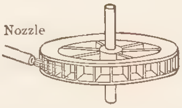

FIG. 3.—Branca's Impulse Turbine (1629).

(b) History is then silent until the beginning of the 17th century when a few attempts were made to use steam for forcing water from chambers to higher levels. About this time (**1629**) **Branca** described an " impulse " type of turbine, Fig. 3, in which steam flowing from a nozzle impinged on the vanes on a wheel which, through suitable shafts and cog wheels, could drive a stamp mill for pulverizing ore. This invention was probably never used in practice.

20. Early Applications.—At the end of the 17th century miners encountered great difficulty in removing large accumulations of water from deep shafts. In 1663 the Marquis of **Worcester** described a steam-pumping apparatus, which was later improved by **Savery** (about **1700**) and widely applied to mines, often several being used in series for pumping from great depths. The device consisted of a boiler, the necesssary piping, hand operated cocks, and two cylinders which were used alternately. With the proper manipulation of the cocks, it functioned as follows: Starting with one cylinder filled with water, steam under pressure was admitted to it to discharge the fluid to the desired height. This steam was then condensed by applying water to the exterior of the cylinder, and the vacuum thus formed therein permitted the vessel to be again filled with water ready for a repetition of the process. While one cylinder was discharging water the other was being filled. The boilers were $2\frac{1}{2}$ ft. in diameter, with soldered joints and had no safety valves, although gage pressures as high as 45 lb. per square inch were common, and sometimes 120 and even 150 lb. were used. In some mines these pumps replaced several hundred horses that were used for water removal. The pumps were also applied to supply water for houses and for driving waterwheels. The Worcester and Savery pumps were "pressure engines," the steam pressures used being relatively high. About this time a steam "digester," or pressure cooker, was used with pressures as great as 1500 lb. per square inch. This device had a safety valve.

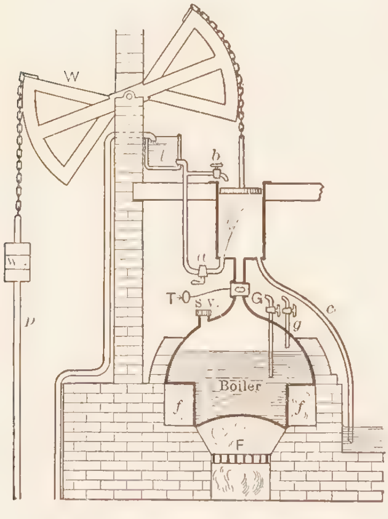

Fig. 4.—Newcomen's Beam Engine (1700)

21. Beam Engines.—In 1690 a water-packed piston was invented, and about **1700 Newcomen** used a vertical cylinder with such a piston to drive one end of a centrally pivoted walking beam, see Fig. 4, the other end of which operated a reciprocating pump by means of a rod which was sufficiently heavy to return the piston to its upper position after the completion of its down, or working, stroke. With the piston at its highest position, steam at atmospheric pressure was admitted to the cylinder, after which a quantity of water was injected to condense the enclosed vapor and thus form a partial vacuum. The atmospheric pressure on top of the piston then caused it to descend, thus operating the pump. After the proper manipulation of cocks the heavy pump rod returned the mechanism to its original position, the cylinder was again filled with steam and the process repeated. The cocks to regulate the steam and condensing water were originally operated by hand. Later it was discovered that the control levers could be moved at the proper times by suitable tappet rods connected to the beam, and this constituted the first self-acting "**valve-gear.**" The boiler had a sheet copper bottom and its upper part was of sheet lead. It was provided with **gage cocks** and a **weighted safety valve.** Subsequently others improved the design. They inserted rope, or leather, in grooves to pack the piston, and they lubricated it with tallow. The condensate was used as feedwater, the boilers were constructed of $\frac{1}{4}$ in. iron plates and engine cylinders as large as 6 ft. in diameter were used. These were the first engines to have trains of mechanism. They were naturally called "*vacuum* engines," the boiler pressures being about atmospheric. They were "single acting," as they did work only during the downward movement of the piston, and they made from 10 to 16 strokes per minute. Such engines were widely used in collieries and for other purposes for about three-quarters of a century.

22. The Father of the Steam Engine.—The mathematical instrument maker, **James Watt** (1736–1819), while repairing a small model of the Newcomen engine (1763) was impressed with the great waste of steam caused by the alternate chilling and heating of the cylinder walls, and saw that those walls should be kept as hot as the entering steam. Being scientifically inclined, and becoming interested in the possibilities of the steam engine, he tested the performance of the model, invented many improvements and experimented with the properties of steam. To reduce the condensation of the entering steam he used a **condenser** that was *separate* from the cylinder and that had an independent **air pump** to remove from it the accumulations of water and air. This change saved three-quarters of the steam and coal previously needed. Later he lagged the cylinder with nonconducting materials and surrounded it with a **steam jacket.** He discovered that if steam was admitted until one-quarter of the stroke was completed and then was cut off and allowed to drive the piston by its **expansion** through the remainder of the stroke, that about 2.4 times as much work could be done per pound of steam (or of coal) as before. He applied this principle of expansion to his engines and also made them **double-acting.** See Fig. 5. The walking beam was retained and to guide the upper end of the piston rod he invented the well-known "*Watt straight-line motion.*" To obtain rotary motion to operate machinery, which was previously driven by windmills or waterwheels, a **connecting rod** attached to the other end of the beam was used to rotate a shaft. As someone else had patented the **crank,** he adopted temporarily the "sun and planet" gear in its place; and to steady the rotative effort he used a **flywheel** on the shaft. He opposed using high steam pressures and usually employed gage pressures of about 7 lb. per square inch. The engines had *poppet valves,* and he invented the first engine **governor** (the "Watt flyball governor") which controlled the *throttle valve* and maintained the rotation at a substantially constant speed. Among his other inventions are the **glass water-gage,** for showing the water level in a boiler, and the steam engine "**indicator,**" for determining graphically how the pressure within the cylinder varies throughout the forward and return strokes. He established the value of the "**horsepower**" which is still in use. Thus to Watt, between the years **1764** and **1786,** can be attributed most of the elements needed in the development of the modern steam engine. The "compound engine" he overlooked, and this was invented by Hornblower in 1781.

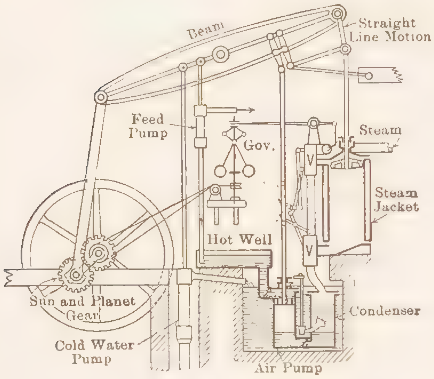

Fig. 5.—Elements of Watt's Engine.

23. Steam Carriages and Locomotives.—Many of Watt's contemporaries attempted to apply steam to the propulsion of road carriages. In **1802 Trevithick,** with some success, used for such purpose a high-pressure noncondensing engine, the exhaust from which was discharged into the boiler stack to increase the draft, an important feature of locomotives to-day; and in 1804 he constructed a locomotive (Fig. 6) that operated on wooden rails.

In 1814 **George Stephenson,** "the father of railroads", began experimenting with locomotives, and in 1829 he built the famous "Rocket," Fig. 7, which at one time attained a speed of 29 miles per hour. His son, **Robert Stephenson,** in **1833,** built a locomotive containing most of the elements that are retained to-day, and invented the Stephenson link reversing valve gear. This locomotive is shown in Fig. 8.

24. Developments in the United States.—(a) In this country some of the earliest uses of steam were for navigation. After experimenting with propulsion by means of paddles and by screw propellers, **John Fitch,** in 1790, built a small passenger boat which had a speed of seven miles per hour. In 1804 **Oliver Evans** constructed his "Oruktor Amphibolis," a flat-bottom boat, on wheels, which propelled itself through the streets of Philadelphia and proceeded down the Schuylkill River driven by a paddle wheel. Evan's engines were noncondensing and he used relatively high steam pressures. He built many engines and boilers to furnish power for mills. In **1807 Robert Fulton,** the "father of the steam ship," constructed the *side wheeler* "**Clermont**" which was driven by an engine purchased from Watt. This ship had the distinction of being the first to be commercially successful. It made regular trips between New York and Albany.

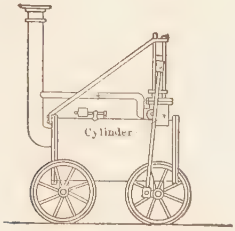

Fig. 6.—Trevithick's Locomotive (1804).

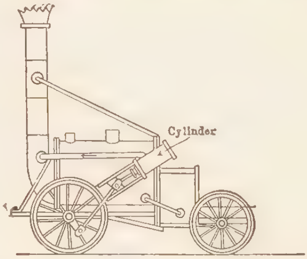

Fig. 7.—Stephenson's "Rocket" (1829).

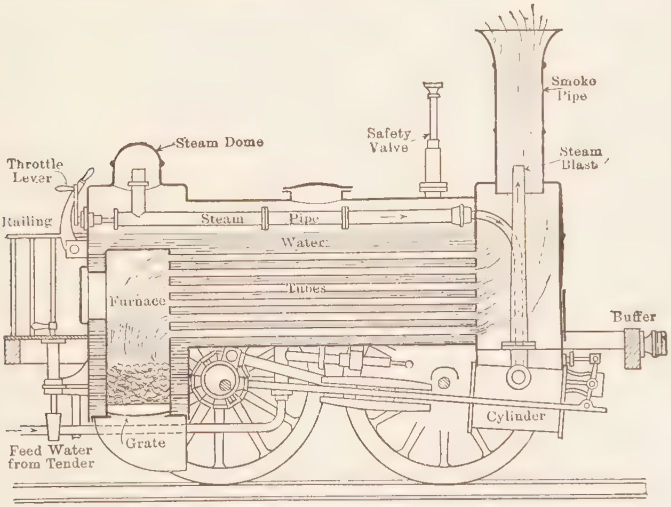

Fig. 8.—Robert Stephenson's Locomotive (1833).

(b) In 1804 **John Stevens** constructed a *screw propelled* vessel which was driven by a high-pressure engine which received steam from a "*water-tube boiler*" of the "sectional" or "safety" type. After he and Fulton had proved the practicality of making steam navigation a success, it was rapidly introduced on both sides of the Atlantic. He and his sons built many of the most successful of the early Hudson River boats. His son, **Robert L. Stevens,** was largely instrumental in developing the type now recognized as the "American river-boat." He conducted extensive experiments on ship propulsion, and constructed many side-wheel vessels which were propelled by vertical engines actuating walking beams overhead. Among his numerous inventions still widely used are the Stevens Valve Gear, having wiper cams to open and close the poppet valves on the engine; the "return tubular boiler"; "feathering" paddle wheels; and the "hog frame," for strengthening shallow ships. In 1842 he started to build the first well-planned ironclad battleship. He died in 1856.

(c) After building his wheeled boat, Oliver Evans asserted that **carriages** propelled by steam would soon be in common use, and should excel in speed the swiftest horse. In 1812 John Stevens set forth the advantages of **steam railways** over canal navigation. He proposed iron rails as a substitute for the wooden ones used around mines, and suggested that the wheels be of cast iron and have flanges. He saw nothing to hinder moving at 100 miles per hour. But it was not until 1829 that the first locomotive was regularly put to work in this country. At that time **Horatio Allen** imported from England a locomotive for the Delaware and Hudson Coal Company's road. In 1831 the famous DeWitt Clinton locomotive was built for the Mohawk and Hudson Railroad.

25. Stationary Steam Engines.—(a) The early "*stationary*" steam engines had slow moving pistons, and low rotative speeds. In the middle of the 19th century several types of "trip cutoff" valve gears were invented, the most successful of which was the "*Corliss gear*," invented by **Geo. H. Corliss,** in **1849,** which greatly improved the efficiency of engines, and is widely used to-day in modified form. In 1861 **Charles T. Porter** invented the "loaded" type of flyball governor and applied it to control the Allen valve gear. Later, he introduced the Porter-Allen "*high speed*" type of engine, with fast-moving piston and high rotative speed. Also he planned a commercial "*line*" of engines, with graded sizes, from which selection could be made to meet the usual needs, and with parts interchangeable as far as was consistent. This established the practice now generally followed by manufacturers of power apparatus. In the 70's Professor **John E. Sweet** developed the "Sweet straight-line," high-speed engine with many new features that have been commonly adopted, and he designed and used a "shaft type of automatic governor," many forms of which were invented about this time. The widely used Rites " inertia " shaft governor was patented in 1894.

(b) **Multistage engines** were developed as time progressed, and these effected a considerable economy in the amount of steam and fuel consumed. Compound engines were quite commonly used, and triple and quadruple expansion engines became standard in the marine and pumping applications. **Steam jacketing,** and the **reheating** of steam between the cylinders, were introduced to improve the economy still further; and in 1908, **Stumpf** showed the possibilities of what he termed the " **uniflow** " engine, which, under certain conditions, with a *single* cylinder, approaches the thermal economy of the ordinary multi-cylinder type.

26. The Steam Turbine.—The turbine principles of Hero and Branca seem to have been forgotten until about 1820 when turbine patents began to appear. Then followed a period of experimentation and development without satisfactory outcome until **Charles A. Parsons** in **1884** showed the possibilities of the multi-stage **reaction** type and **Gustaf deLaval** in **1889** produced a commercially successful **impulse** turbine. The real development of these prime movers, however, began with the present century. Now turbines are built from the smallest sizes to those that are several times as powerful as any steam engine that was ever built, and some are being designed for steam pressures and temperatures which until recently were considered prohibitive, and for operating cycles previously unknown or believed not feasible.

27. The Internal Combustion Engine.—Engines in which the combustion process takes place within the working cylinder are called " internal combustion engines." This type of engine is believed to have originated about 1690 when gun powder was experimentally used to drive a piston in a cylinder. The first gas engine patent was issued in England in 1794. Knowledge of details was gradually acquired and the difficulties were progressively overcome until in 1860 **Lenoir** produced the first practicable engine. In **1876 Otto** developed a design, suggested by **Beau de Rochas,** which established the gas engine on a firm commercial basis. The use of *oil* as a fuel in engines met with increasing success and now has generally replaced gas except in special cases where large quantities of cheap gas are available. In **1892 Dr. Rudolf Diesel** invented the very important oil engine that bears his name, and these engines and many modifications of them are being rapidly applied to generate power on land and sea. With the advent of the automobile, early in this century, the *gasoline engine* for its propulsion began its marvelous growth, and the subsequent development of the airplane has had a marked influence on the further improvement of these engines.

28. Progress in Theory and Performance.—(a) It will be noted that the development of the steam engine by Watt occurred at about the time of the American Revolution and that its modernization began at about the centennial of that event; also, that most of the development of the steam turbine and of the internal combustion engine has occurred in the present century.

(b) The early inventors had little theory or data to guide them. In **1824 Sadi Carnot** presented his law for the ideal engine, which was then little appreciated. It was not until after **Joule's** experiments on the equivalency of heat and mechanical energy, published in 1843, that the theory of heat engines started vigorous growth. After **1849** the science began to be developed rapidly, notably by **Clausius, Rankine** and **William Thomson** (Lord Kelvin), and it has been continued by many others since then. Although engineers now have logically presented and well established scientific laws and data to guide them, there is much work of development in this field still remaining to be done.

(c) The following table presents some data which illustrates in an approximate way the progress that has occurred in steam engineering:

TABLE

Year	Prime Movers	Steam Pressures. Lb. per Sq. In., Gage.	B.t.u. per Hp-hr.	Lb. of Coal per Hp-hr.
1700	Savery's engine. Steam in contact with the water pumped	45–150	420,000	30
1720	Newcomen's beam engine, with piston.	0	280,000	20
1782	Watt's double acting, expansion engine.	7	112,000	8
1880	Modern type	80	56,000	4
1900	Multiple expansion engine	150	21,000	1.5
1920	Steam turbine power plant (large)	275	14,000	1
1925	Steam turbine power plant (large)	300–1200	10,000	.75

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SMITH, Life of John E. Sweet.

CHAPTER III

ILLUSTRATIONS OF MODERN POWER PLANTS

29. Introductory.—In power plants that depend on heat as the source of energy, there must be provided some form of fuel, proper means and conditions for burning it efficiently, a working substance, apparatus for transferring the heat to the latter and for conveyance to the point of utilization, a prime mover for converting some of the heat into mechanical energy, and means for delivering the power where needed. It is often convenient to consider a power plant as made up of a number of **systems,** each of which is composed of a closely related group of parts that function together. A few illustrations of power plants, accompanied by rather brief descriptions, will be presented in this chapter for the purpose of giving a very general understanding of what power plants may consist.

30. The Automobile Power Plant. (a) The power plant in the usual type of automobile is one of the simplest and most generally understood. Such a plant is shown in Fig. 9. It functions as follows: The fuel is metered in the carbureter where it is supplied with oxygen-conveying air; then the mixture is introduced into the cylinders where ignition and combustion take place. The heat generated is added to the products of combustion which become the **working substance.** These gases act on the pistons, from which the power is transmitted through suitable chains of mechanism to the rear wheels where it is usefully expended.

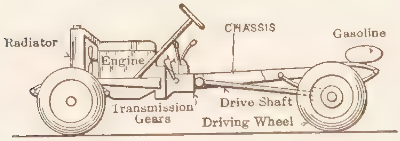


FIG. 9.—Automobile Power Plant.

One arrangement of the engine, or prime mover, is illustrated in Fig. 10, in which the principal parts are labeled. After leaving the carbureter, the fuel mixture passes through an intake manifold to the respective cylinders, thence through inlet valves into the cylinders where it is ignited by the spark plugs. After doing work on the pistons the gases are discharged through the exhaust valves, exhaust pipe and muffler. The overheating of the cylinder walls is avoided by a water jacket through which is circulated water whose temperature is kept within limits by the radiator. Provision is made for oiling the bearings and other rubbing surfaces. The engine is governed by hand.

(b) The plant may be considered as including the following systems: (1) Fuel and Combustion; (2) Ignition; (3) Energy Conversion; (4) Power Transmission; (5) Water Cooling; (6) Lubrication; and (7) Control. These systems will be studied later.

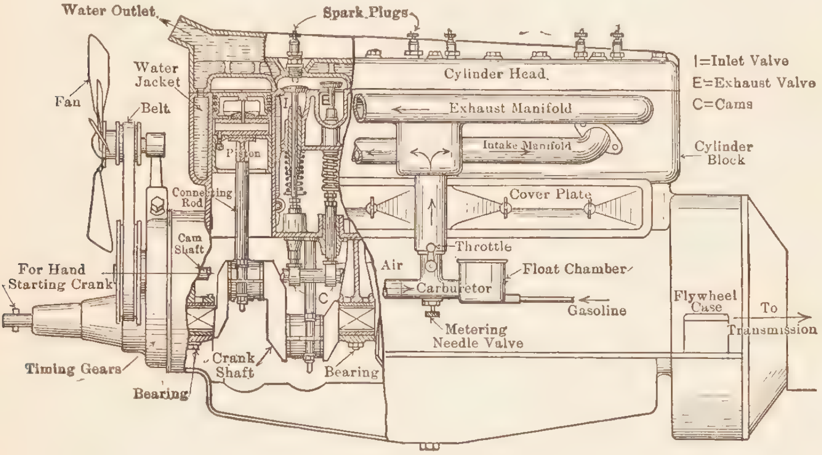


FIG. 10.—Elements of a Six-cylinder Automobile Engine.

31. Oil Engine Power Plant.—A plant using an oil engine of the Diesel type is illustrated in Fig. 11. Oil is pumped to the fuel valve which opens at the proper time and allows air under high pressure to spray the liquid into the cylinder in which the piston has compressed a charge of air to such an extent that its temperature is sufficient to cause ignition. The heat resulting from combustion is added to the gases formed and these, functioning as the working substance, expand and drive the piston during its working stroke. The power is transmitted to the shaft, which may deliver it by belt or other flexible connector, or which may be " direct connected " to an electric generator or other device for utilizing the energy. The engine is provided with an inlet valve, for admitting the main charge of air that is needed for combustion, and an exhaust valve for discharging the spent gases. The cylinder is always water jacketed. An auxiliary piece of apparatus, the multi-stage air compressor, is used for forcing air under high pressure into storage tanks from which it is withdrawn for injecting the fuel into the cylinder, or for starting the engine. The rotative speed of the engine is controlled by an automatic governor.

The Diesel plant is thus seen to be made up of the same systems as the preceding type, except for the addition of the compressed air system and the elimination of the ignition apparatus.

There are many other types of oil engines but they need not be considered here. They will be discussed in a later chapter.

32. Gas Engine Plant.—(a) Artificial gas is usually too expensive to employ for power production. Only when there are available cheap natural or by-product gases in adequate quantities, do gas engines become commercial possibilities. When they are used, the power plant consists almost entirely of the engine itself.

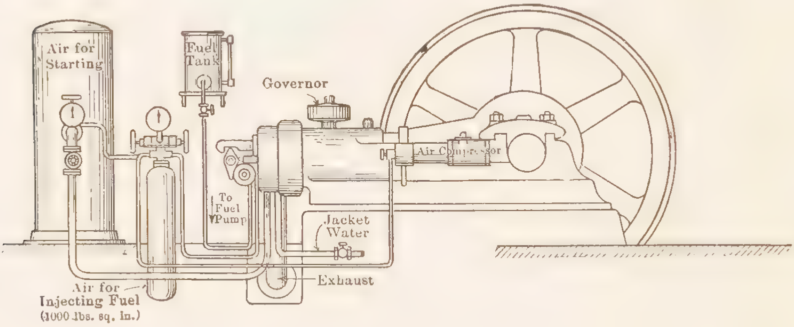


Fig. 11.—Elements of a Diesel Engine Plant.

(b) Outside of the natural gas regions, the most extensive applications of these engines are in connection with blast furnace plants in which large quantities of combustible gases are produced as a by-product of the main process, and in which power is needed to operate blowing engines, for furnishing air to the furnaces, and to supply electricity for the plant and neighboring communities. Usually much more gas is produced in such plants than is needed in generating the amount of power that can be utilized. The remainder is used in " stoves " for heating the furnace air, burnt under boilers, used in other ways, or even wasted. Before it is suitable for engines, the gas must be thoroughly cleaned. Fig. 12 shows diagrammatically one arrangement of a blast-furnace plant. Blast-furnace engines are usually large, with sizes ranging from 2000 hp. to 4000 hp. or more per unit.

33. The Elementary Steam Power Plant.—(a) In addition to the **boiler** and the prime mover, or **main unit,** in a steam-power plant, there are many pieces of auxiliary apparatus, commonly called **auxiliaries,** the necessary **piping,** and certain **accessories,** or secondary devices on the main apparatus, **trimmings** and **fittings.**

(b) The simple elements of a plant are shown diagrammatically in Fig. 13 in which the principal parts are labeled. Assuming that coal is used, this fuel is placed on the **grate** through which and above which air is passing in proper amount to supply oxygen to support the combustion, which occurs partly in the fuel bed and partly in the **combustion space** in the **furnace.** The hot gases on their way to the **stack**, or **chimney**, pass over the boiler tubes, or **heating surface**, on the other side of which circulates water which absorbs some of the heat, the steam thus generated being the **working substance.** This steam will have a temperature of a definite value which depends on the pressure carried.

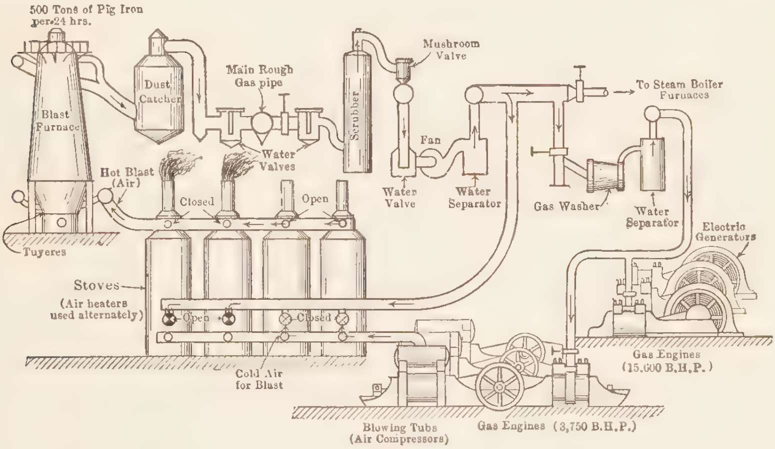


FIG. 12.—Diagrammatic Arrangement of a Blast Furnace Plant.

It is often desired, however, to use steam at a higher temperature than this, in which case the vapor is passed through a **superheater** where it absorbs additional heat from the hot gases. After passing through the **boiler stop valve,** the **pipe line** and the **engine throttle valve,** the steam is allowed to do work in the engine (**prime mover**) from which it is exhausted to the atmosphere, or to a **condenser.** In the former case the discharge is at about atmospheric pressure and the engine is said to be operating **non-condensing.** Even if the engine normally operates condensing, provision must be made to permit **atmospheric exhaust** to be used in case of accident to the condenser. This relief is accomplished automatically by the opening of an **atmospheric relief valve** in a pipe which discharges outside the building, through an **exhaust head** which prevents the spraying of oil and water over the neighborhood. A **condensing water pump** is used to circulate the cooling medium through

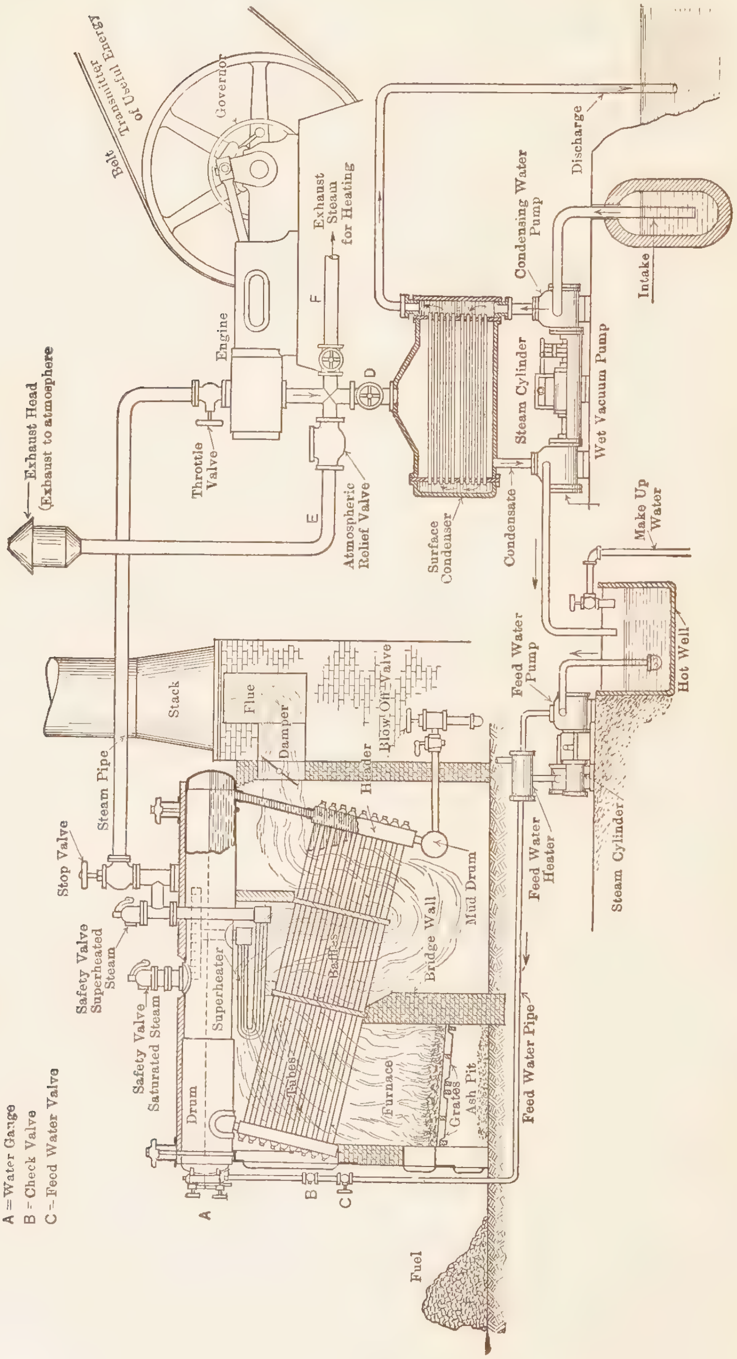


FIG. 13.—Elements of a Simple Steam Power Plant.

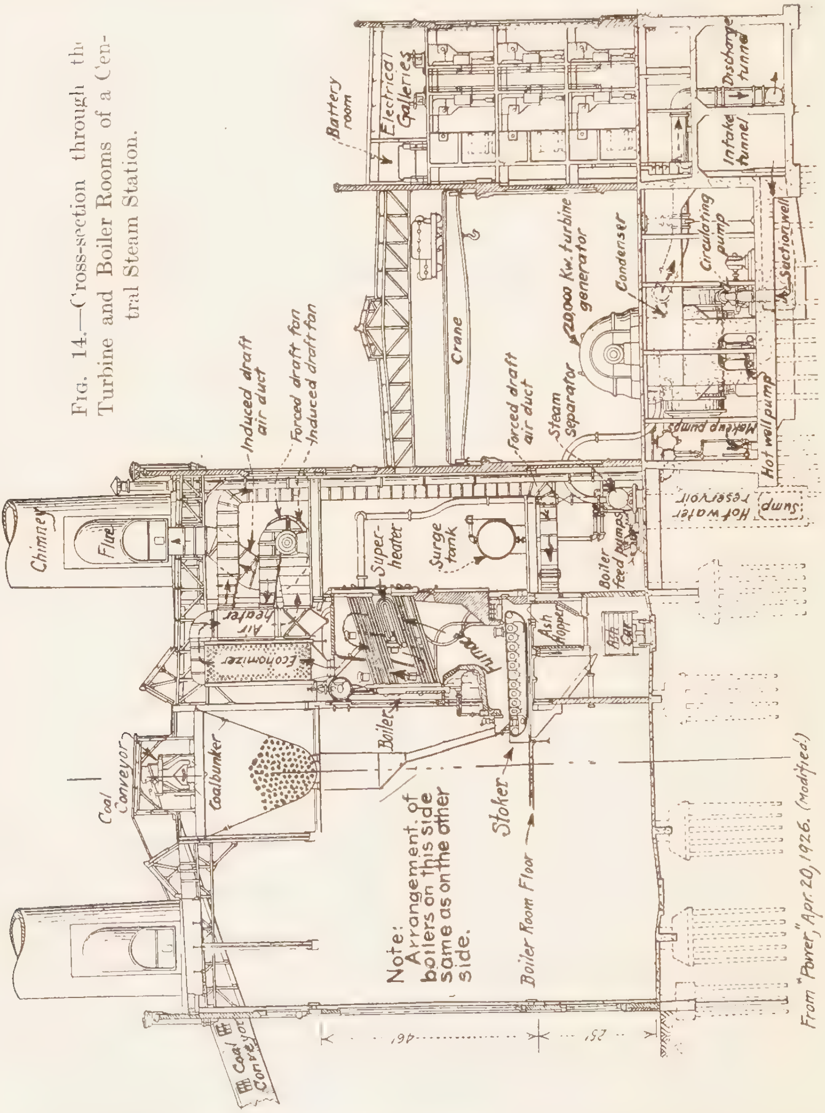
the condenser in order to abstract sufficient heat from the exhaust steam to reduce it to the liquid form and thus produce a partial vacuum. The resulting **condensate** is at a pressure corresponding to the " vacuum " which is maintained in the condenser. A **wet vacuum pump** is used to raise the pressure of this water, and of any air that may be present, up to atmospheric pressure, and to deliver it to the **hotwell.** The condensate is distilled water and is therefore free from impurities which might be detrimental to the boiler. Also it is hotter than the supply of raw water and consequently requires less heat, and, therefore, less fuel, to convert it into steam than is the case if cold water were used; hence the condensate is normally used as boiler **feedwater.** The **feed pump** delivers the water from the hotwell to the boiler, through the **feedwater pipe** line which is provided with control and check valves. The exhaust steam from the feed pump is passed through a **feedwater heater.**

(c) The **boiler** is a heat-absorbing container for the water and steam, and it is so arranged that the water circulates rapidly over the heating surfaces which are swept on the other side by the hot furnace gases. Above the water surface is a **steam storage space.** The boiler must always be provided with a **pressure gage,** a **glass water gage** and **try cocks** for determining the water level, and also with **safety valves** for relieving the pressure if it exceeds that for which the boiler was designed. It has a **mud drum** and **blow-off valves** for removing sedimentary deposits, and access to its interior is provided by **manholes** and **handholes** for the purposes of inspection and for cleaning the surfaces.

(d) The **furnace** and boiler are surrounded by a brick **setting,** lined with firebrick, and the gases are guided by **baffles** so as to pass over the full extent of the heating surface available. The chimney produces the draft that is needed for bringing air into the furnace and for disposing of the products of combustion. Means for controlling the air supply and draft are provided.

(e) In case steam is needed for **heating** buildings, or for industrial processes, it is more economical to use for this purpose exhaust steam, which has done some work, than to utilize " live " steam taken directly from the boiler. At *F* in Fig. 13, is shown a pipe for **exhaust heating.** In this pipe line a **pressure regulating valve** might be placed in order to maintain an approximately constant pressure in the heating system.

34. Central and Isolated Steam Stations.—(a) A power plant which supplies energy to a single building, such as a hotel, or an office building, or to a factory having one or more buildings, is commonly termed an **isolated station, or plant.** Such plants are usually relatively small, but are not necessarily so. When a plant serves a large community in which there are many users of power, it is referred to as a **central station,** and is relatively large. When several large communities are supplied by a single station, or by a group of interconnected plants, the terms **super-power plant,** or, **super-power system** are sometimes used. A large steam plant, or central station, is illustrated in Fig. 14.



From "Power," Apr. 20, 1926. (Modified.)

FIG. 14.—Cross-section through the Turbine and Boiler Rooms of a Central Steam Station.

(b) The kinds of equipment and methods of operation used in a plant depend on the magnitude of the demand for power, on the character of the service to be rendered, on local conditions and on many other factors which need not be discussed at this point. A steam plant is most easily analyzed by considering the systems of equipment of which it is composed, and there may be a great many of these systems, especially if the plant is a large one. The following brief description applies more particularly to a large plant using coal as the fuel, and it is necessarily limited to a consideration of only the most important of the systems.

(c) **Coal and Ash Handling System.**—Assuming that the fuel is delivered in barges or railroad cars, some special labor-saving, **mechanical unloading equipment** may be employed for its removal. The coal then may be weighed, sampled and tested, and conveyed to a storage pile in the yard. Before using, it may be crushed, pulverized or otherwise **prepared** by special equipment. It may then be transported to **overhead bunkers** in the boiler room, by means of a belt-, bucket-, or other **mechanical conveying system.** From the bunker, the fuel may gravitate through **down spouts,** to the **hoppers** of the **mechanical stokers,** which automatically feed the coal to the furnace. The **ashes** may also be disposed of by some mechanical system. By the use of such expedients the employment of manual labor can be reduced to a minimum.

(d) **Draft System.**—Supplementing the **chimney** for providing the draft needed for supplying the air for combustion and for removing the gases formed, there may be used fans, or other equipment, which constitute a **mechanical draft system.** The flues are provided with dampers, and the air supply and draft are controlled by a hand-operated, or automatic, regulating system.

(e) **Condensing System.**—The condensers require large quantities of cooling water, which may be supplied through **intake tunnels** and be carried away by **discharge tunnels.** This water may have to be passed through **intake screens** to remove debris; and if reused, as in case of a limited supply, **cooling towers** or **cooling ponds** may be needed for lowering its temperature before its return. In connection with the condensing system there may also be one or more **condensing water pumps, condensate pumps,** and air removal pumps, commonly called **air pumps.**

(f) **Feedwater System.**—In addition to the **feed pump, hotwell** and **piping,** the feedwater system may also have provision for **water purification.** The "raw," or impure, make-up water may be distilled in an **evaporator,** or it may undergo **chemical treatment;** and in addition the feedwater may be passed through a **de-aerator.**

(g) **Heat Balancing System.**—In order to reduce the amount of fuel consumed, and thereby minimize the cost of operation, heat that would be wasted, or be less efficiently utilized elsewhere, may be added to the feedwater. Often **exhaust steam** from auxiliary engines and turbines, or **extraction steam** which is bled from between stages of the prime mover, is used in **heaters** for this purpose. Also the hot flue gases which have left the boiler may be passed through **economizers** for heating the feedwater, or through **preheaters** for heating the air that is used for combustion.

(h) **Piping System.**—The main and auxiliary piping that connect the various pieces of apparatus may be quite extensive, complicated and costly. Besides the pipe, there are many **valves** of various kinds; moisture and oil eliminators, called **separators**; also **drains, traps,** etc. In addition, provision must be made for pipe line **expansion,** by introducing expansion joints, loops or similar expedients; also for **drainage,** and for decreasing **heat losses** from the piping.

(i) To the foregoing may be added the **auxiliary drive system,** including the electric or steam operated motors for driving auxiliary equipment, the house turbines, etc.; the **control systems,** with the necessary **instruments** and **gages**; and many other systems which will not be considered here. From this brief outline of the equipment that composes the modern steam station it is evident that considerable study of the individual systems will be necessary in the following chapters.

35. The Locomotive.—The railway locomotive is merely a complete power plant which is so arranged on wheels that the power expended is utilized in moving itself and in usefully drawing the train of cars. It contains the elements already discussed in connection with other steam-power plants. Fig. 15 shows one of the most usual arrangements of a passenger locomotive, and on it many of the more important parts are labeled. Locomotives will be considered in some detail later.

36. The Marine Power Plant.—The power equipment of vessels may be quite complicated, for, in addition to the main units needed for propulsion, and their auxiliaries, there may be provided prime movers for furnishing electric light and power, hoisting engines, air compressors, refrigerating machine, draft apparatus, ventilating and heating equipment, bilge pumps, sanitary pumps, fire pumps and much else. The prime movers may be internal combustion engines or steam engines or turbines. Fig. 16 shows diagrammatically the power equipment for a relatively small and simple plant. In order to provide as much cargo-carrying capacity as possible, such plants, although quite complex, are made very compact and of light weight. The details of marine plants are highly specialized and are beyond the scope of this text.

37. Other Plants.—The power plant for a hotel, or office building, may be as diversified and compact as is the marine plant. To the more common types of plants already mentioned may be added many special ones, such as refrigerating and ice-making plants, air and gas compressor plants, municipal and other water pumping plants, contractors' portable or semi-portable plants, farm plants, and many others.

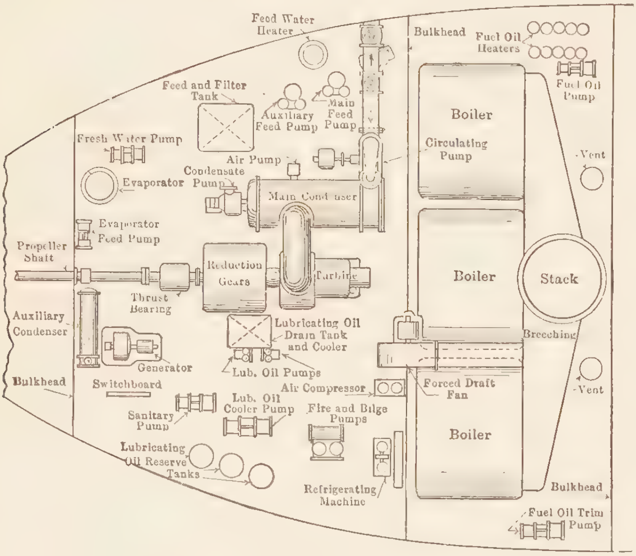


FIG. 16.—Small Marine Power Plant.

38. Engineering of Power Plants.—The relative simplicity and high thermal efficiency of the internal combustion power plant has its appeal, but in many cases the interest and other charges on a possible greater first cost of the plant, and a possible relatively large expense for costly fuel, may militate against its selection. The steam plant, although more complicated, often can furnish power at less expense than can oil-engine plants, and in many instances can also outrival hydraulic power plants. To determine the proper kind of plant to install under given conditions entails a careful engineering study, which means an analysis of the **economic, financial,** and **technical** considerations involved.

It is the constant aim of power-plant engineers to improve the design, arrangement, operation and the economic performance of power-plant equipment; thus heat-power engineering constitutes an unlimited field of endeavor.

CHAPTER IV

ENERGY EQUATIONS

39. Introductory.—In Heat Power Engineering it is frequently necessary to determine quantitatively the energy involved in the various processes encountered, and it is the purpose of this chapter to develop the four most important equations needed. The first of these equations deals with the work done by a given amount of working substance while expanding according to some known law; and it will be called "the equation for work done by an expanding substance." The second is used for the determination of the heat added to any working substance under simple conditions, and will, therefore, be referred to as the "simple energy equation." The third, "the specific heat equation," shows how the specific heat may be used to determine the amount of heat transferred to any working substance during any known process. The fourth is applicable to the more general cases in which all forms of energy, except electrical and chemical, are involved, and it will, therefore, be spoken of as the "general energy equation."

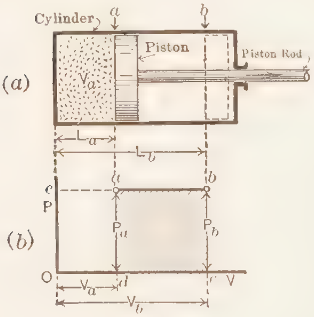


FIG. 17.

40. The Work Done by an Expanding Substance.—(a) Whenever a working substance expands it does work upon those bodies which it moves by virtue of this expansion. This statement can be explained in connection with Fig. 17(*a*), which shows the elements of an engine with the piston initially in position *a*. Imagine the space V_a to be filled with some suitable working substance which is at the pressure P_a, and then let more of this material be forced into the cylinder at just the right rate so that as the piston moves to the right, towards its extreme position *b*, the pressure will be maintained constant at its original value. If a record be plotted, with pressures as ordinates and volumes as abscissas, of the simultaneous values of P and V for the constantly increasing amount of working substance within the cylinder during the movement of the piston, it will result in a horizontal line, such as *ab* in Fig. 17(*b*), which is the graph of this *constant pressure process*. In this diagram *a* represents the initial state and *b* the final state of the working substance that has entered the cylinder, and the line *ab* shows the pressure volume relationship throughout the process.

(**b**) A similar horizontal graph might have been obtained in the following manner: As before, imagine any kind of working substance in the cylinder in sufficient quantity to fill the space V_a at the pressure P_a, and then, instead of admitting any more working substance, let heat be supplied to the fluid already present so that the pressure is maintained constant while the piston moves from *a* to *b*. Here again the line *ab* in Fig. 17 (*b*) will represent on the *PV*-diagram the process occurring. In both cases the pressure is maintained constant, but the means by which this is accomplished in the two instances are clearly very different.

(**c**) The work done on the piston during each of the above processes is the same, and equal to the total force acting on the piston multiplied by the distance through which this force acts. Taking the pressure in pounds per square foot to be P_a and the area of the piston in square feet to be F, then the total force in pounds acting on the piston is P_aF. The distance in feet, through which this force acts is represented by $L_b - L_a$, Fig. 17(*a*); hence, for the movement of the piston corresponding to this distance, the work done becomes $P_aF(L_b - L_a)$. But FL_b and FL_a represent respectively the volumes V_b and V_a, corresponding to the final and initial states; hence the work done in foot-pounds during this constant pressure process is

$$Wk_{ab} = P_bV_b - P_aV_a = 144p(V_b - V_a), \quad . \quad . \quad . \quad . \quad (6)$$

in which p represents the constant pressure in pounds per square inch, absolute. If it is desired to determine this work in B.t.u., the expression then becomes

$$Wk_{ab} = \frac{(P_bV_b - P_aV_a)}{778} = AP_bV_b - AP_aV_a. \quad . \quad . \quad . \quad (7)$$

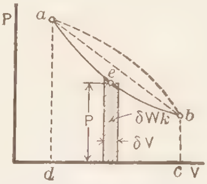


Fig. 18.

(**d**) The more common case, however, is the one in which work is done by a substance that is expanding under *variable* pressure. Consider the solid curve *ab*, in Fig. 18, to represent the pressure-volume history of a working substance expanding in a cylinder under such conditions; then, at some particular instant the *PV*-condition is shown by the coordinates of the corresponding point, such as *e*, on the curve, the instantaneous value of the pressure being P. If, at this pressure, considered constant, there is a small change in volume, equal to δV, the corresponding small amount of work done will be $\delta Wk = P\delta V$, which is shown by the sectioned area in the figure. Similarly, for a very small expansion at this instant along the line *ab*, with the same small change in volume, approximately the same amount of work would be done, and $\delta Wk = P\delta V$ approx., which at the limit $(\delta V \doteq 0)$ takes the form

$$dWk = PdV.$$

Then, if the relation between the pressure and volume during the entire expansion from *a* to *b* is known, the calculus enables one to determine the work done during the whole process *ab*. Thus, expressed in its most general form,

$$Wk_{ab} = \int_a^b PdV, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

= the area *abcd*, to scale, in Fig. 18.

(e) Eq. (8), which may be termed the "equation for work done *by* a substance," is perfectly general as it applies to *any* substance expanding or being compressed in any manner such that the pressure-volume relationship is known throughout the entire process from the initial state *a* to the final state *b*. The yielding of a negative quantity would express the fact that the process *ab* is a compression.

(f) It is the particular functional relationship existing between *P* and *V* that is the determining factor in locating the path, or curve, to represent the process *ab* on the *PV*-diagram, rather than the *kind* of substance used. From an inspection of Fig. 18 it will be seen that any number of paths may be drawn connecting the two states *a* and *b*, as shown by the dotted curves; and it will be clear that the areas under these curves, or in other words the values of $\int_a^b PdV$, will not be the same since each is dependent on its specific path. Obviously the integration cannot be performed until the pressure-volume history of the particular process involved is known. Hence, to speak of the work done between any two states, without specifying the connecting process involved, is to use an indefinite expression. The various kinds of processes will be discussed in a later chapter.

41. The Simple Energy Equation.—(a) When heat is added to working substances it may produce important effects other than a change of temperature. It may increase the internal energy of the working substance, it may cause the substance to expand, thus doing work on other bodies, it may increase the velocity of flow of the stream of the working substance itself, it may produce a change in elevation of a substance, it may cause chemical or electrical changes, and some of the energy may be lost by radiation or in other ways. Hence, the determination of the quantity of heat which is transferred to or from a working substance can be made only by considering all the resultant effects. To include all the possible forms of energy in one equation makes it much longer than is necessary for most cases, hence it is customary to simplify such equation by eliminating those energy terms whose values do not change during the process considered, and those changing by negligibly small quantities in comparison with the terms which are retained.

(**b**) Consider any known process *ab* resulting from the addition of a quantity of heat to a working substance inside the cylinder of an engine or compressor, and take the case in which such addition changes the internal energy of the substance and causes work to be done, no other energy transformations or transfers occurring. Then, by the law of conservation of energy the following important relation must hold for this simple case:

$$\left\{\begin{array}{l}\text{The heat added } \textit{to}\\ \text{the working sub-}\\ \text{stance during the}\\ \textit{given process ab.}\end{array}\right\} = \left\{\begin{array}{l}\text{the } \textit{gain} \text{ in internal}\\ \text{energy of the work-}\\ \text{ing substance be-}\\ \text{tween the states } a\\ \text{and } b.\end{array}\right\} + \left\{\begin{array}{l}\text{the work done } \textit{by}\\ \text{the working sub-}\\ \text{stance during } \textit{the}\\ \textit{process ab.}\end{array}\right\} \quad (9)$$

Or, expressed more concisely by symbols, the "*simple energy equation*" is

$$Q_{ab} = \Delta I_{ab} + Wk_{ab}, \quad \ldots\ldots \quad (9a)$$

or

$$Q_{ab} = I_b - I_a + Wk_{ab}. \quad \ldots\ldots \quad (9b)$$

(**c**) It has been shown in Sect. **40** that the work done by any working substance in passing from state a to b depends upon the process or path connecting these two states. Since the above energy equation shows that the quantity of heat added to a working substance equals the gain in internal energy plus the work done during the process under consideration, it must follow that *the quantity of heat added is also dependent upon the process under consideration.* In other words, in reading the above energy equation, *the process or path must always be specified when giving the terms* Q_{ab} and Wk_{ab}.

(**d**) It should also be noted that the strict use of the preposition *to* in the first term, and *by* in the last term of Eq. (9), combined with the word *gain*, or *increase*, in the middle term, will always furnish the key to the correct interpretation of a negative sign which may be encountered for any of these three terms.

This simple energy equation is important as it will be used frequently in dealing with reciprocating heat engines, air compressors, and refrigerating machines, whether the working substance is in the form of a liquid, vapor, or gas.

42. The Energy Equation Involving Specific Heat.—(*a*) It has been shown in Sect. **41** that the quantity of heat transferred to or from any working substance in order to change its state from a to b, depends upon the process ab connecting these two states. It therefore follows that *the amount of heat required to change the temperature of a unit weight of any substance through one degree* will also depend upon the process involved. This amount of heat is called the **specific heat**[1] of the substance *for the given process.* Since the engineer has to deal with many different processes he designates the corresponding specific heats by adding a subscript to the general symbol for this term. Thus, c_p is the general symbol for specific heat at constant pressure, c_v is for constant volume, c_i is for constant internal energy, and so on for any other process involving a *constant characteristic.*

(**b**) If the heat added with each degree change in temperature, over the range $\triangle T_{ab}$, remains unchanged, then *c is constant* and the quantity of heat added to the weight w is

$$Q_{ab} = wc\triangle T_{ab} = wc(T_b - T_a), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

or

$$c = \frac{1}{w}\,\frac{Q_{ab}}{\triangle T_{ab}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

In engineering work in this country c is commonly used in B.t.u. per pound per degree Fahrenheit. When the energy is expressed in foot-pounds the symbol K is used for the specific heat. Obviously

$$K = 778\ c, \quad K_p = 778\ c_p, \quad K_v = 778\ c_v.$$

(**c**) For most substances, however, the specific heats are *variable* during certain processes. In such cases the above equations may sometimes be employed by substituting for c its average value over the prevailing temperature range. This *average specific heat* will be represented by $\bar{c}$. As its value depends on the temperature range, the latter can be given as a further subscript,—thus $\bar{c}_{p_{75-180}}$ represents the average specific heat at constant pressure within the temperature limits of 75 and 180 degrees.

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[1] This definition of specific heat involving the *unit weight* is the usual one in most engineering calculations. However, in dealing with gases, it is sometimes more convenient to use what is called the *volumetric specific heat,* which is based upon the *unit volume instead of* the unit weight.

(**d**) Not only does the specific heat depend on how the heat is added, i.e., on the process, and on the kind of substance used, but it is also often very materially affected by the magnitude of the temperature. For example, the specific heat of air at constant pressure is appreciably larger at very high temperatures than for ordinary conditions. With this in view, the relations between the specific heat and all the other factors on which it depends is best given by the general expression

$$c = \frac{1}{w}\frac{dQ}{dt}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

from which

$$dQ = wcdt, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

or

$$Q_{ab} = w\int_{t_a}^{t_b} cdt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

This last equation reduces to $wc(t_b - t_a)$ only in case the specific heat during the process *ab* is independent of the temperature, it being assumed that the weight *w* does not change during this process.

(**e**) The two most important specific heats for all working substances are those at constant pressure and at constant volume, and their ratio is frequently used. It is convenient, especially when dealing with gases, to employ a symbol for this ratio, the Greek letter gamma being commonly used for this purpose; thus

$$\gamma = \frac{c_p}{c_v} = \frac{K_p}{K_v}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

(**f**) It is essential to note the distinction between the term *specific heat* and *specific heat-content*, or *specific total heat*. Each of the latter terms designates the *heat content*, *h*, or the *total heat* per unit weight of substance, which is very different from the specific heat.

43. Conditions for Steady Flow.—(*a*) There are many important pieces of apparatus through which the working substance passes under conditions which are described by the term "steady flow." Using 1 to indicate the section through which the working substance enters the apparatus, and 2, the point of exit, and using these numbers as subscripts to designate the state of the working substance passing these respective sections, the term "steady flow," as applied to energy equations, means that the following three conditions must be fulfilled:

> (*A*) The weight of working substance flowing per unit of time, or rate of flow, past section 1 is constant and equal to the rate of flow past section 2.

(*B*) The pressure, temperature, velocity and specific volume of the working substance remain constant at section 1; and similarly they are constant at section 2. (Note, however, that the quantities at 2 do not necessarily have the same values as the corresponding ones at 1, for, in general, some will have different values, and all may be different.)

(*C*) The apparatus has been operating for a sufficient length of time to have reached a stable condition, which will be maintained throughout the period in which all observations are made.

(b) As an illustration of steady flow, consider the steam turbine. In this apparatus, section 1 would commonly be taken at the turbine throttle valve and section 2 would be at the exhaust opening. The total weight flowing past sections 1 and 2 per unit of time would be the same, assuming no leakage or addition or extraction of steam for any purpose, but the pressure, temperature and specific volume of the steam would be very different at section 2 from their values at section 1, and the velocities of flow at these two points might be quite different.

(c) Other common illustrations are the centrifugal air compressor, the centrifugal boiler feed pump, the steam condenser, and the steam boiler. In each of these cases a working substance is flowing through the apparatus which is either adding energy to, or abstracting it from, this fluid; or the working substance may receive energy at some places within the apparatus and give up energy in other parts of it. The energy received by the apparatus, or given up by it, may be in the form of heat, or it may be mechanical, or it may be a combination of both.

(d) In some cases the fluid is made to flow through the apparatus by reason of the energy supplied to drive the machine, as in the case of a centrifugal air compressor. In other cases, such as prime movers, the fluid flows through the apparatus by reason of the pressure drop permitted therein, and mechanical energy may, therefore, be delivered to other machines. *The condition of the fluid need be determined only at the entrance and exit sections, irrespective of what happens within the apparatus between these two points.* The only specification imposed upon the apparatus itself is that it is *not to act as a reservoir* in which the amount of energy stored varies over an appreciable interval of time. However, the apparatus may pass through cyclic changes, as in the case of reciprocating machines, so that the kinetic or thermal energy stored within such apparatus also undergoes corresponding cyclic changes; but so long as the pulsations set up in the working substance do not violate the provisions of (*A*) and (*B*), the equation of steady flow may properly be applied.

(e) A case in which the steady flow equation does not apply is that of an engine, turbine or similar apparatus, initially at room temperature and at rest, being warmed up and brought to running speed. In a few simple cases of this kind the equation of steady flow may be slightly modified by the addition of terms to cover the variable energy stored within the apparatus, but the general equation as developed in the next section does not apply to such processes.

44. General Energy Equation for Steady Flow.—(a) Having an understanding of what is meant by steady flow, it is now desired to develop an equation which will contain all the forms of energy involved in such a passage of a fluid through any kind of apparatus. The law of the conservation of energy states that energy cannot be created or destroyed; hence, the problem resolves itself into the determination of the amount of energy, in all its various forms, that is flowing past sections 1 and 2, and that is being supplied to the apparatus, or abstracted from it, between these sections.

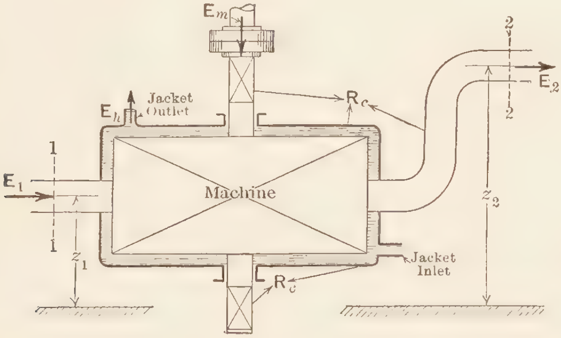

FIG. 19.—Diagram to Illustrate the Factors Involved in the General Energy Equation for Steady Flow.

Consider the diagrammatic sketch in Fig. 19 in which, for any unit of time,

E_1 = the amount of all the forms of energy flowing across Section 1;
E_2 = ditto, for Section 2.

Also, between Sections 1 and 2:

E_m = the amount of mechanical energy supplied to the machine;
E_h = the heat energy abstracted from the machine, by some suitable form of apparatus to transfer heat, such as the water jacket of an air compressor;
E_g = the work done in lifting the working substance against the force of gravity from elevation z_1 to elevation z_2, and

R_c = the energy lost by radiation, conduction and convection of heat from all external parts of the machine.

It should be noted that this last term includes the energy lost by friction in the bearings of the machine, and also that the friction and turbulence of the working substance within the machine cause an increase in the values of E_2 and R_c.

(**b**) In case the machine is *delivering* mechanical energy, as does a steam turbine, or other prime mover, the term E_m is negative; and similarly when the apparatus is supplying heat *to* the working substance, as does a steam boiler, the value of E_h is likewise negative. Even R_c is negative in the case of ordinary refrigerating machines.

(**c**) Assuming each of the terms to be positive and therefore in the directions of the energy streams as indicated by the arrows in Fig. 19, then, from the law of conservation of energy it follows that the total energy supplied to the apparatus equals the total amount leaving it. Hence

$$E_1 + E_m = E_2 + E_h + E_g + R_c, \quad . \quad . \quad . \quad . \quad (16)$$

or, the mechanical energy supplied to the machine is

$$E_m = E_2 - E_1 + E_h + E_g + R_c. \quad . \quad . \quad (17)$$

It remains to determine the values of the various quantities in these equations.

(**d**) The term E_1 is made up of the kinetic energy of the stream flowing past section 1, plus the internal energy of the fluid, plus the energy expended in forcing the volume of the fluid across this section against the constant pressure maintained there. Similarly the term E_2 is composed of the corresponding forms of energy for section 2.

Expressing the rate at which energy is flowing past any section in B.t.u. per second, it follows that for section 1, the *kinetic energy* due to the velocity, v_1, with w lb. flowing per sec., is

$$KE = wv_1^2 \div (778 \times 2 \times 32.17) = \frac{wv_1^2}{50,000}{}^{*}.$$

The *internal energy* due to the thermal state of w pounds of the substance is wi_1. The energy expended, or the *work done*, in forcing each pound of working substance, having the specific volume $\overline{V}_1$, across section 1, against the constant pressure P_1 is $AP_1\overline{V}_1$; and for the entire weight w, the corresponding value is $wAP_1\overline{V}_1$ or AP_1V_1. The two terms, i and $AP\overline{V}$, may be added together giving the specific heat-

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[*] Inasmuch as the quantity 778 is somewhat uncertain in the third significant figure, the round number, 50,000 may be used.

content, h. Corresponding expressions may be written for section 2. Then the values of E_1 and E_2 may be expressed as follows:

$$E_1 = \frac{wv_1^2}{50{,}000} + wi_1 + wAP_1\overline{V}_1 = w\left(\frac{v_1^2}{50{,}000} + h_1\right). \quad . \quad (18)$$

$$E_2 = \frac{wv_2^2}{50{,}000} + wi_2 + wAP_2\overline{V}_2 = w\left(\frac{v_2^2}{50{,}000} + h_2\right). \quad . \quad (19)$$

(e) The work done in lifting the working substance, expressed in B.t.u. per second, is,

$$E_g = \frac{w(z_2 - z_1)}{778}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

This item is usually extremely small when dealing with vapors and gases and may often be neglected in such cases.

(f) Then, substituting in Eq. (17) these values of E_2, E_1, and E_g, and remembering that $H = wh$, it follows that

$$E_m = H_2 - H_1 + \frac{w(v_2^2 - v_1^2)}{50{,}000} + E_h + \frac{w(z_2 - z_1)}{778} + R_c. \quad (21)$$

(g) This equation states that the energy *supplied to* an apparatus by mechanical means, in order to operate it, is equal to the increase in the heat content of the fluid forced through this machine, plus its increase in velocity energy, plus the energy delivered to the cooling apparatus, plus the work done in lifting the fluid, plus the energy lost by radiation and conduction. This last item includes all friction in the bearings, gearing and elsewhere in the apparatus, except insofar as parts of such frictional losses may appear in the energy E_2, going out with the fluid. This form of the equation is the one most convenient for use with such machines as centrifugal compressors and pumps.

(h) The term E_h in this equation is determined by various means, depending upon how the heat is transferred from the apparatus. For instance, if a water jacket is used, E_h would be the product of the rise in temperature of the water, in degrees Fahrenheit, the weight flowing, in pounds per second, and the specific heat.

The value of R_c may be determined if all the other terms are known. It is sometimes estimated as a percentage of one of the other terms and often it is relatively so small that it may be neglected altogether.

(i) The mechanical input of a machine is usually expressed in terms of horsepower or the kilowatt. To convert it to B.t.u. per sec., the following may be used:

$$\begin{aligned} E_m &= 0.707 \times \text{(horsepower delivered to drive the machine)} \\ &= 0.948 \times \text{(kw. delivered to drive the machine).} \end{aligned}$$

(j) For any engine or turbine, the mechanical energy is *delivered by* the apparatus, and the working substance leaves with less energy than it had at entrance. Also energy may be *supplied to* the machine in the form of heat, E_h', by some such means as heating coils; and the elevation z_2 is usually less than z_1. For such cases the energy equation may be more convenient if written thus:

$$E_m' = H_1 - H_2 + E_h' + \frac{w(v_1^2 - v_2^2)}{50,000} + \frac{w(z_1 - z_2)}{778} - R_c. \qquad (22)$$

This equation states that, for a machine driven by the flow of working substance through it, the mechanical energy E_m' delivered by the machine is equal to the loss of the heat content of the fluid in passing through the machine, plus the heat energy E_h' supplied to the fluid by any reheating apparatus, plus the energy given up by the decrease in velocity, plus the work done on the fluid by gravity, minus the radiation and conduction losses, all taken between sections 1 and 2.

(**k**) Very often with engines and turbines E_h' is zero, $z_1 = z_2$, and $v_1 = v_2$. Then for such cases

$$E_m' = H_1 - H_2 - R_c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

That is, for any prime mover operating under these conditions the energy delivered by this apparatus in any unit of time is equal to the difference of the heat contents at entrance and exit from the apparatus, for the entire amount of working substance flowing in this unit of time, minus the radiation and conduction losses from the apparatus.

(l) If the engine has much energy lost by friction of its mechanical parts, that loss appears as heat in these parts, as shown by their increased temperature. For example, the bearings may become heated. The hotter these parts are, with other conditions remaining the same, the faster is energy lost from them by radiation and conduction. In other words the value of R_c is increased by machine friction.

If the engine has rough surfaces over which the working substance passes at high velocity, or if the passageways have numerous abrupt turns, there will be much fluid friction and turbulence within the machine. These result in losses in energy which appear as heat in the fluid and in the adjacent parts of the machine. Whatever heat is given to the machine tends to raise its temperature thus increasing the radiation losses from it. The effect of adding to the working substance heat produced by turbulence and fluid friction, is to cause its internal energy and its specific volume to become greater, thus resulting in an increase in its heat content, $H_2(= I_2 + AP_2V_2)$.

The exit velocity of the substance will become larger in proportion to the increase in specific volume, since the areas of the passageways and the rate of flow, or quantity flowing, are considered constant. Thus the loss due to the exit velocity, v_2, will be somewhat increased by turbulence and fluid friction in the machine.

The temperature of the working substance may increase or may remain constant as a result of the turbulence and fluid friction, depending on whether it is a gas or a vapor.

45. Available Energy.—(a) In heat-power engineering **ideal machines** are defined as those having *no mechanical friction*, and through which the working substance flows without *leakage* or *friction*, and without *heat transfer* or *turbulence*, except as they may be intentionally introduced.[2] Therefore, for ideal engines having steady flow with $v_1 = v_2$ and $z_1 = z_2$, it follows from Eq. (22) that the output of mechanical energy is

$$E'_{m(\text{ideal})} = (H_1 - H_2) + E'_h. \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Thus, the general energy equation reduces to this very simple expression for the ideal engine using, under steady flow conditions, *any* working substance which enters and leaves the machine with the same velocity and at the same elevation. Often E'_h is zero, and, consequently, for such cases the right-hand member of Eq. (24) becomes simply the difference in the heat content of the working substance at entrance to and exit from the machine.

(b) Eq. (24) gives the amount of mechanical energy that could be developed if no losses occur, hence this quantity is commonly called the "**available energy** *for steady flow conditions*." No actual engine can be operated without having some loss, but it should approach as nearly as possible the performance of the ideal machine and deliver as much of the available energy as is possible.

(c) To be able to compute the available energy for different kinds of working substances, under various conditions, requires a considerable knowledge of the thermodynamic processes of vapors and gases. These processes and the methods of determining the values of H_1 and H_2, for different cases, will be given in subsequent chapters.

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[2] Extreme turbulence may occur in refrigeration equipment and in a few other special cases which will be discussed later.

CHAPTER V

EQUILIBRIUM, REVERSIBILITY AND AVAILABILITY

46. Introductory. The primary purpose of the study of engineering thermodynamics is to find processes and to devise apparatus for their application and control, whereby the maximum amount of useful mechanical work is developed at the expense of the energy of the working substance, this energy having been derived from some convenient source of heat, usually the combustion of fuel. As will be seen in detailed instances later, it is never possible to convert into useful work the equivalent of *all* of the heat supplied, for even in ideal cases only a portion is available as work. The object of this chapter is to study the conditions that influence the extent of this availability.

47. Equilibrium.—(a) First it is necessary to discuss the meaning of the term *equilibrium*, as used in thermodynamics. Its significance in processes having only mechanical aspects is well known. It designates a condition in which everything involved is in a state of balance. The system, without the application of external force, is by itself incapable of changing. Similarly, in processes having thermal as well as mechanical aspects, *thermodynamic equilibrium designates a state in which the system under consideration is incapable of spontaneous change.*

Systems not in equilibrium always undergo spontaneous change until they ultimately reach some equilibrium state. A few examples will help to make these statements clear. For instance, consider the simple mechanical system consisting of the earth, air, a pendulum, and its supports. With the pendulum displaced from its lowest position and released this system is not in equilibrium; the pendulum will immediately move, and will ultimately come to rest in its lowest position. The process will take some time, but it will surely progress continuously until mechanical equilibrium is attained. Likewise, a block of ice floating in a body of hot water constitutes a system which is not in equilibrium; the ice is very actively melting, and the water is being cooled. Heat is being transferred from the hot water to the ice, and also to the cold water resulting from the melting of the ice. Ultimately the entire mass will attain a uniform temperature throughout and this is the state of final thermodynamic equilibrium for this system. Again, consider two masses of gas confined in a rigid vessel, with a partition wall between them, one mass being at pressure higher than the other. Let a valve in the partition be opened. The system will not be in equilibrium, and gas will flow from one portion of the vessel to the other until the pressure is uniform throughout its entire mass, and equilibrium is thus established.

(b) It should be noted that systems which are not in equilibrium may persist in an unbalanced condition for some time. Such systems are far from impossible,—indeed, they are very common; but they all display one common characteristic—active spontaneous change. This change may be slow or fast, depending upon circumstances, but such systems ultimately reach some state of equilibrium.

(c) A body in thermodynamic equilibrium will be characterized by uniform pressure[1] and temperature throughout the entire mass, uniform mass distribution, that is, uniform density[1] and specific volume, and uniformity of all other properties whose values are determined when the state of the body is known. The words *uniform* and *constant* should not be confused with each other. The temperature of a body may be uniform throughout, that is, at any given instant, it may have the same value at all points throughout the entire mass, and yet it may be slowly changing as a whole, in response to outside influences. If these influences are held constant, the body in equilibrium is incapable of any spontaneous change whatever.

(d) Since any body in equilibrium is characterized by uniform values of all the variables whose magnitude depends upon its state, it is possible to specify these values numerically; but if a body is not in equilibrium this specification cannot be made, because it is not possible to determine the instantaneous values of any property for the different parts of the body. Moreover, if one variable is not uniform throughout the mass of the body, some, at least, of the other variables cannot have a uniform value. For example, in a mass of gas, if the temperature distribution is not uniform, then either density or pressure must be non-uniform, since these are related for each element of mass. The state of a body not in thermodynamic equilibrium cannot be described in simple terms.

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[1] Uniform pressure and uniform density involve the assumption that the effect of gravity is negligible. In a mass of liquid, the pressure at various levels is different, due to the difference in hydrostatic head, but the variation in density is slight for most liquids, because of their relatively low compressibility. Likewise, in a mass of gas extending to an appreciable height, there will be significant variations in both pressure and density. In small volumes of gas, however, the pressure and density are sensibly uniform throughout. These statements apply to one phase states. Two or more co-existant phases will have the same temperature and, except for gravity, the same pressure, but the other properties will differ widely, as will be shown in the study of vapors.

(e) The concept of thermodynamic states of equilibrium is analogous to that of the familiar rigid body of mechanics; such states do not exist in reality, but, for many purposes, they are very closely approximated in practice, hence the study of the simple ideal cases yields conclusions which are sufficiently accurate for practical purposes, and are of great value.

48. Availability of Energy.—Several types of processes will now be discussed to determine the conditions influencing the extent to which the energy of the participating system is made *available as useful work.*

(a) First, consider a simple process in which the thermal aspect is so secondary that it is usually overlooked,—the *falling of a body* from a higher to a lower level. To make the discussion concrete, assume 10 lb. of iron, elevated 10 ft. above the ground. In its upper position, the system consisting of the earth and the iron possesses 100 ft-lb. of potential energy, because that amount of work must have been expended on the system to elevate the iron to that level. Whether 100 ft-lb. of useful work may be derived by allowing the body to fall to its lower position *depends altogether upon the manner in which the fall is controlled.*

(b) Suppose that the body is simply dropped, its fall being entirely uncontrolled. Then the useful work done is zero. Neglecting the energy expended against atmospheric resistance, all of the potential energy, of the system has been converted into kinetic energy by the time the ground is reached, and this energy is in turn all converted into useless heat upon impact. True, some of this energy may be expended in setting up sound waves, but their energy must also ultimately be dissipated as heat. In short, all of the energy of the body is uselessly expended.

(c) Again, suppose the 10-lb. body in its upper position, suspended by a cord which passes over a sheave and has its other end attached to a 5-lb. counterweight on the ground. Upon setting the system free, the 10-lb. body will fall to the ground as before, but more slowly, while the 5-lb. one is elevated 10 ft. The process is somewhat controlled, and 50 ft-lb. of useful work have been developed in lifting the counterweight. Half of the energy of the falling body has been made available, and half has been dissipated as heat.

(d) If the counterweight be increased to successively larger values (say 9 lb., 9.9 lb., etc.), as the process is repeated, the useful work derived will be progressively increased, and the dissipated energy will be correspondingly decreased. Evidently the mechanism is approaching an ideal limiting condition, in which both weights are equal. In such an ideal mechanism all friction, including air resistance, must be absent. Using such an arrangement, let the first weight be in its top position, the second one being on the ground. The application of the very slightest momentary force to the system will set it in motion, and the first body will fall with extreme slowness, while the second is lifted equally slowly to the upper position. The process is entirely controlled, since the smallest imaginable force could stop it at any time during its course. During the process as a whole the useful work done in raising the counterweight is equal to the entire potential energy of the initially elevated body. There is no dissipation. The original energy has been made wholly available in useful form.

49. Reversibility of a Mechanical Process.—(a) The last process being completed, the first body is on the ground, and the second is elevated 10 ft. The application of a slight impulse in the opposite direction will cause the identical process to be reversed, under the same perfect control which characterized the direct process. At the end of the reversed process, everything is just as it was initially—the first weight is at its upper position, the second is on the ground, and no energy has been dissipated. The only thing unaccounted for is the energy expended in applying slight impulses to the perfectly balanced mechanism. However, as the magnitude and duration of these impulses are decreased, the process approaches as closely as desired the limiting imaginary, or ideal, case in which the two bodies are always in exact equilibrium, and may be thought of as moving in the direct or reverse sense with equal facility. Hence this ideal limiting process may be designated a *reversible* process.

(b) At any time during a reversible process the following conditions must hold: (1) *The process may proceed in either direction with indifference;* (2) *All bodies concerned are always in equilibrium throughout the process;* (3) *The dissipation of energy is zero;* and (4) *The availability of energy in the form of useful work is a maximum.* This discussion will not be carried further for purely mechanical processes.

50. Availability of Energy and Reversibility of Thermodynamic Processes.—(a) In a manner exactly analogous to the foregoing, a thermodynamic process, involving both thermal and mechanical aspects, may approach an ideal reversible type, in which it is entirely controlled, or harnessed, so that it is made to develop the maximum possible quantity of useful work. This may be illustrated most simply by a study of the behavior of a mass of gas in a cylinder having a movable piston.

(b) Before proceeding with the study of this process the apparatus to be employed will be described briefly. See Fig. 20 (*a*). Let the cylinder be vertical, and have a long hollow piston which is absolutely leak-proof, yet free to move without friction, and of weight that is relatively small and definitely known. The piston may be loaded at will with various weights and it is assumed that this load comprises a large number of thin plates, which may be transferred to or from the piston in whatevery number desired. Beside the line of travel of the piston imagine an apparatus to which brackets may be attached at any level, for receiving portions of the piston load whenever it is desired to slide them off. It is thus possible to change the load on top of the piston, and in that way to alter the pressure to which the confined gas is subjected. Further, it must be considered possible to lock the piston in any position.

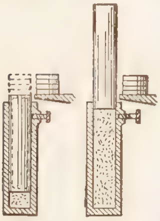

Fig. 20.

It should be noted that *sliding* the load from the top of the piston to a bracket or from a bracket to the top of the piston when both are at the same level, involves no work, assuming friction eliminated from this apparatus. *Lifting* the load, however, does call for the expenditure of work. If the load is lifted by the piston, the work is done by the confined gas beneath the piston, and when the load is lifted while on one of the brackets, the work is derived from some outside source.

Two sorts of processes will be employed in the study of this apparatus, it being understood that useful work means the lifting of a load. The first process is a simple mechanical one of shifting load to or from the piston and noting the resulting compressions and expansions of the confined gas. The second, which is considered in Sect. **52**, involves the transfer of heat to and from the confined gas, with or without changes in the piston load.

(c) Now, let the mass of gas be confined in its cylinder, with the piston loaded so that the pressure is, for example, 100 lb. per square inch, absolute. This position of the load is shown by dotted lines in Fig. 20(a). As a first experiment, lock the piston, and then remove the entire load from it to the position shown by the solid lines. If the piston is now unlocked it will move upward abruptly and without control, since the upward push of the gas is much greater than the downward force due to the weight of the piston. The piston will finally come to rest at the upper end of its travel as indicated by Fig. 20(b) and with the entire piston load left behind at the initial elevation. Such a process will be entirely uncontrolled and no useful work in raising the load will be done. Within the mass of gas itself, during this rapid process, conditions will be very much confused. At the instant of release, the piston will start to rise, and this action will allow the gas immediately adjacent to the piston face to expand. Before this release of pressure begins to be felt near the bottom of the cylinder an appreciable fraction of a second will elapse. This delay results in setting up pressure waves within the mass of gas. During such a period, it is not possible to state a single value for the pressure sustained by the mass of gas as a whole, for the pressure differs from point to point. Since there will be a non-uniform distribution of gas mass, neither the density of the gas nor its reciprocal, the specific volume, is capable of numerical specification and, consequently, the total volume cannot be used in computing thermal results.

(d) With density and pressure distributed erratically throughout the mass, it is probable that the temperature of the mass also differs from point to point. In thermodynamic language, the gas under such conditions is said to undergo a *turbulent process* in which it is never in equilibrium except in the initial and final states. *In general during a turbulent process, the condition of the working substance is not capable of simple quantative description at any stage except the initial and final ones.*

(e) Can this process be reversed? That is, can the gas be made to contract of its own accord, passing through all of the stages which it traversed while the piston was moving up, but in the reverse order? All experience shows that such a result cannot be obtained, hence this process is called an irreversible one. *All turbulent processes are irreversible.*

(f) However, the fact that a process is irreversible does not imply that the working substance can not be returned to its initial condition. In the case just considered it would be relatively easy to compress the gas so that it would return to its initial state. This could be accomplished by elevating the bracket to the upper position, locking the piston and sliding the load on to it. The piston, upon being released, would then return to its original position. Such a compression would involve doing work upon the gas at the expense of energy received from an outside source (used in raising the weight) and the gas would not pass through the same intermediate stages that it traversed during the expansion. For these reasons the process has not been reversed although the gas has been returned to its initial state.[2]

(g) Next, repeat the experiment in a slightly different way. Again assume that the load is on the piston, which is locked in its lowest position. Remove half of the load, Fig. 21(*a*), and then release the piston.

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[2] Temperature changes and heat transfers would be involved in this process, but they are purposely omitted from this discussion in order to simplify the problem for the beginner.

Again the latter will move upward abruptly, but since it now carries half of its original load, it will move more slowly, and will ultimately come to rest, Fig. 21(b), at an elevation lower than in the first experiment, in which there was no load at all. Now lock the piston again, and remove the remainder of the load; then release the piston and allow the second step to complete itself. (Fig. 21(c).)

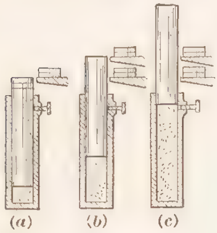

Fig. 21.

The process has thus been conducted with some measure of control, and some useful work has been done, in that half of the load has been lifted to some elevation above that at which it stood initially.

(**h**) As in the first case, the gas can be restored to its original condition by elevating the loads on the brackets, sliding them on to the piston, etc.; but in this second case less work will be done in lifting the loads, since a portion is already elevated, and the return process will be conducted in *two steps*, corresponding to the original ones. The process is not reversed in detail, but one intermediate state of the gas and piston will be reproduced. The process is still irreversible.

In this case, there is one stage intermediate between the initial and final at which the gas is in equilibrium, for it is allowed to stop its expansion midway and come to a uniform condition, so that, in that stage of the process, its state can be completely specified.

(i) Instead of carrying out the process in two steps, let it be done in *ten steps*. At the conclusion of these steps the piston will be in its upper position, with the load distributed in ten portions, at corresponding elevations (Fig. 22). The control of the process is more complete, and more useful work has been done, since, on the average, the load has been lifted higher. To return the apparatus to its initial state, elevate all of these portions of the load approximately one-tenth of the total travel, and then conduct the ten return steps, the piston accumulating an increment of load at each step. The process as a whole is still irreversible, but it is obviously approaching a condition of reversibility, since in the first case no intermediate steps were reproduced, whereas now ten such steps are duplicated.

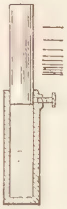

Fig. 22.

(j) Now imagine this series of experiments extended indefinitely, the load on the piston being removed in progressively smaller portions, until the expansion of the gas, and the return compression, have been reduced to a series comprising an indefinitely large number of very small steps. As the number of steps is thus increased, the actual processes evidently approach the case in which the direct and reverse processes correspond, step for step, as closely as desired. In other words, the actual direct process approaches as a limit an ideal reversible one. As this limit is approached, the useful work done will correspondingly increase, and the outside work which must be done preparatory to the return process will decrease. In the limit, for the strictly reversible process, a maximum of useful work will be done, and no outside work will be necessary to cause the reverse process to take place.

51. Reversible Process Defined.—*A reversible thermodynamic process is one in which everything involved can be made to traverse completely and precisely, but in the reverse order, the same steps which it passed through in the direct process.* During such a process, the working substance is constantly in equilibrium,—that is, a *reversible process is a continuous series of equilibrium states* of the working substance. The conditions for reversibility as given in Sect. **49**(**b**), are general and hence apply to this case.

The concept of a reversible process is of great value in thermodynamics for two reasons; first, because, during such a process the history of the working substance is completely known, since it is always in equilibrium; and secondly, because, during such a process the maximum possible useful work is developed.

52. Availability and Reversibility with Heat Transfers.—(a) So far, it has been considered that there has been no transfer of heat to or from the gas within the cylinder. A process involving such transfer will now be examined. For this purpose there will be needed thermal reservoirs which will be designated as "*hot bodies*" or "*cold bodies,*" depending on their relative temperatures. Each of these bodies has the special property of giving forth or receiving large quantities of heat without alteration in its own temperature. Such bodies are not as fanciful as they may seem at first. A cylinder containing a mass of liquid and its vapor, under constant pressure, and at uniform temperature throughout, is just such a body. When the body receives heat, a portion of the liquid will evaporate, and when it gives out heat, a portion of the vapor will condense, but so long as the pressure is constant, the temperature will not vary.

(**b**) Now, imagine that the lower head of the cylinder is made of some material which transmits heat without resistance. During this experiment, the loading of the piston will not be changed nor will the piston be locked. Assume that the mass of gas in the cylinder is initially in equilibrium at a temperature of 50 deg. fahr., with a certain load on the piston. Place the cylinder in contact with a hot body which is constantly at a temperature of 150 deg. Immediately heat will flow into the gas within the cylinder, and the temperature and volume of the gas will increase, moving the piston and its load upward, and ultimately equilibrium will be attained when the gas has reached a uniform temperature of 150 deg. However, during the process, the gas has not been in equilibrium, since receiving heat rapidly through the cylinder head necessarily sets up a temperature gradient in the mass of gas and results in a variable gas density, giving rise to convection currents. Hence the gas has been in a slightly turbulent state. To return the gas to its initial state, employ a cold body constantly at a temperature of 50 deg., which will abstract heat from the gas, and bring it back to the starting point. But this action will not reverse the process which occurred within the gas mass; the process is irreversible. The net result of the direct and return process is that the hot body has lost a certain amount of heat to the cold body.

(c) Again, employ *two hot bodies*, one at 100° and the other at 150°, and conduct the process in two steps. Then return the gas to the initial condition by using *two cold bodies*, one at 100° and the second at 50°. For the former, the " hot body " which is at that temperature may be used. The net result of these two processes, each consisting of two steps, will be that the hot body at 150° has lost heat, the body at 100° has recovered all heat that it formerly gave up, and the cold body at 50° has gained a quantity of heat equal to that lost by the hot body at 150°. But the heat thus transferred will be only half as much as in the first experiment. The process is still irreversible.

(d) Next, just as when changing loads on the piston, increase indefinitely the number of steps. In this case the 100° rise in temperature will be accomplished by using a series of hot bodies, one for each step and each at a different temperature. Then, since the gas will be in equilibrium at every intermediate state, as the number of steps is increased the process as a whole will approach as a limit a continuous series of equilibrium states,—that is, it will approach a reversible process, and the corresponding net heat transfer between the hot and cold bodies will be progressively reduced and approach zero as the limit.

(e) In these thermal processes it is not so obvious that the useful work developed increases, or, conversely that the waste of energy decreases, as the reversible process is approached, but it is nevertheless true. At this point it is difficult to prove this fact, but the argument is briefly as follows: *Whenever heat is transferred from a hot body to a cold one, over a finite range of temperature, it is possible to interpose a heat engine, receiving heat from the hot body, converting a portion of it into useful work, and rejecting the remainder to the cold body. If the heat is transferred without the interposition of such an engine, as a part of the par-*

ticipating system, as in the case just considered, the possibility of developing this work has been sacrificed. But as the difference in temperature between the two bodies is reduced, the capacity for developing work by such an engine is lessened, and as this temperature interval approaches zero, the work developable also approaches zero. Later (in Sect. **121**) a definite proof will be given to show that the reversible process sacrifices no energy that could be utilized and hence develops a maximum of useful work.

53. Reversibility and Irreversibility Combined in a System.– (a) In engineering practice, heat must very often be transferred between a gas or vapor within a cylinder, and hot or cold bodies outside, but rarely are these hot bodies at constant temperature, and never are they at the same temperature as the material within the cylinder,—in fact, seldom do they even approximate such equality. Such heat transfers are entirely turbulent, or irreversible processes, so far as the entire system of participating bodies is concerned, and therefore the maximum possible work development is not realized, and, strictly speaking, little can be done in the way of detailed analyses of the processes as a whole. For example, it is impossible at any instant to give numerical values for the pressure, temperature, etc., of the entire mass of working substance and the material of the other participating bodies. However, since such processes are of such frequent occurrence, they will be examined further.

(b) Suppose the heat transfer process is conducted in a somewhat different manner from that in the preceding section. Again start with the gas at 50°, but employ only one hot body, at 150°. With the cylinder head made of material which conducts heat without resistance, marked temperature gradients are set up within the gas, hence the state of the gas, considered by itself, is turbulent. Of course, the state of the entire system, comprising the hot body, cylinder, and gas, is also turbulent. The process executed by the entire system is irreversible, and so is that for the gas within the cylinder, considered alone.

(c) Now assume that the cylinder head is made of some material which transmits heat very slowly, that is, it is of some substance that is *partly* heat-insulating. Heat will now pass to the gas more slowly, the resulting temperature gradient within the gas mass will be less pronounced, and the expansion of the gas will be slower. By using better and better heat insulation for the cylinder head, the action can be retarded to any desired extent, so that, at any instant, the departure of the gas from equilibrium can be made as little as desired. That is, the temperature gradient within the gas can be reduced to as small a fraction of one degree as may be assigned, and the pressure, density, etc., are also made correspondingly sensibly uniform. *The limiting process is a continuous series of equilibrium states for the mass of gas considered* **by itself,** *and hence it becomes a reversible process for* **that** *body alone.* So far as the gas within the cylinder is concerned, the process is identical with that resulting from the use of an indefinitely large number of hot bodies, each at a different temperature and the useful work developed is the same. It is capable of complete analysis in exactly the same manner.

(**d**) *But the transfer of heat through the cylinder head, from the hot body to the colder gas within the cylinder is irreversible* since the two bodies are at such different temperatures that there is a sacrifice of useful work that might have been developed, had an engine been imposed between the hot body and the colder gas. Hence, *although a part of a system may be performing a reversible process and thus be converting into useful work a maximum of the heat energy it receives, yet the process as a whole, considering the entire participating system, may be irreversible and consequently not developing the greatest amount of useful work possible.* In such cases the state of the system as a whole cannot be specified numerically except initially and in the final state of equilibrium to which it naturally and inevitably progresses. From the foregoing it is obvious that, in general, when a reversible process is said to result in a maximum development of work it must be remembered that all participating bodies in the entire system must be considered as included and that they must all be executing reversible processes.

(**e**) In engineering studies, although actual processes may be irreversible, it is usually proper to think of them as split up into parts, some reversible and others irreversible. The latter are susceptible of analysis only by special methods, if at all, but fortunately they are often merely of secondary interest. The reversible parts, on the contrary, are usually of prime importance and since they may be analyzed completely, without reference to the exact manner in which they may be caused to occur, a knowledge of such processes is of great practical value.

54. Actual Approximations to Reversible Processes.—(**a**) Since, strictly speaking, only reversible processes are capable of complete analysis, it is now pertinent to inquire how closely certain important processes approach irreversibility. For example, if air is compressed in a cylinder, many results are computed on the assumption that the process is subject to complete numerical description, that is, that the process is reversible, so far as the gas within the cylinder is concerned. It is impossible to determine just how much error is thus introduced, but it will usually be negligible. The speed with which the process is conducted evidently has much to do with the condition of the gas.

However, considering that sound travels through the air, by waves, with a velocity of some 1100 ft. per second, and that it will travel faster as the air is compressed to higher densities, it seems evident that no material pressure differences can long be sustained within the cylinder of any machine of feasible size. Large temperature differences can be sustained by stagnant gas, but it would seem that gas being compressed with reasonable rapidity will be somewhat agitated so that only small temperature differences can persist. If pressure and temperature are sensibly uniform throughout the mass of gas, density must be also, and likewise any other variables whose values depend upon the state of the gas. It has been pointed out that the gas mass is probably agitated, but if the variables whose values depend upon its state have sensibly uniform values throughout the mass, any relative motion of one element of mass with respect to others is ordinarily of little thermodynamic significance. Motion of the mass as a whole is important, as when compressed gas escapes into the atmosphere through a nozzle, but mere confused motion within the gas itself does not, taken alone, generally involve any thermodynamic consequences.

(**b**) It is entirely safe to assume that the usual processes in which gases and vapors are compressed or allowed to expand may be considered to be reversible *so far as the gas or vapor alone is concerned.* Except in special cases, the substance may always be thought of as in an equilibrium state, subject to numerical description. It is important to remember, however, that the gas or vapor under consideration is often only a part of the entire system which is participating in the process. There may be adjacent bodies transmitting heat to or from the active working substance, and these heat transfers may be, and usually are, irreversible, so that the process as a whole is irreversible, and hence fails of developing the maximum work.

55. Graphical Representation.—(**a**) The fact that equilibrium states and reversible processes are such that the state of the working substance at every instant can be described fully in numerical terms makes possible interesting and valuable graphical representations of such states and processes. For example, the expansions and compressions of the gas confined within the cylinder described in Sect. **50** (**c**), in which the load was shifted to and from the piston, may be represented graphically by employing pressure and volume coordinates. The initial state of the gas, with full load on the piston, was characterized by high pressure and small volume, and may therefore be represented by point *a* in Fig. 23. The final state, following sudden expansion due to the removal of the entire load, was characterized by low pressure and large volume, and may be represented by point *b* in the same figure. The intermediate states cannot be represented at all, since at any instant, the pressure, density and other properties of the gas, were not uniform, and hence cannot be stated numerically.

(**b**) In the second experiment, in Sect. **50** (**g**), conducted in two steps, the gas is in equilibrium in the two end states, and in one intermediate state, thus the three points, *a*, *b*, and *c* of Fig. 24 represent all that is known of this process.

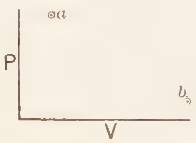

Fig. 23.

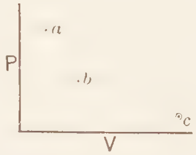

Fig. 24.

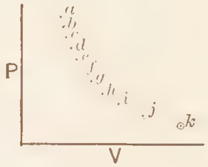

Fig. 25.

(**c**) For the ten-step process, in Sect. **50** (**i**), Fig. 25 represents what took place during that experiment.

(**d**) As the number of steps is increased indefinitely, the points come closer and closer together, hence they approach as a limit a continuous curve, which indicates the mode of variation of pressure with respect to volume throughout the reversible process, as shown in Fig. 26.

(**e**) In the experiments in which heat was added to the gas, with constant load on the piston, Sect. **52** (**b**), the volume and temperature of the gas changed, and all that is known regarding these two variables is shown by Fig. 27. The initial state of the gas was characterized by

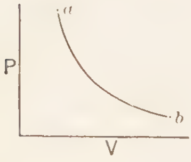

Fig. 26.

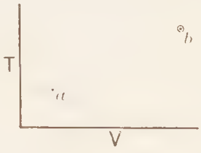

Fig. 27.

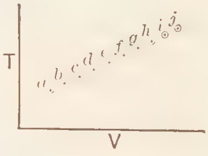

Fig. 28.

low temperature and small volume (point *a*), and in the final state both temperature and volume had increased (point *b*). Intermediate points cannot be plotted, because the intermediate states of the gas were turbulent.

(**f**) In Sect. **52** (**d**) the process of transferring heat was considered to be conducted in progressively smaller steps, for which Fig. 28 is a typical plot. As before, the states intermediate between the equilibrium points which mark the division of the process into steps cannot be plotted. As the steps become progressively smaller the process approaches as a limit a reversible one which may be represented by a continuous curve, as shown in Fig. 29.

(g) Similar curves might be drawn with other coordinates, such as specific volume against specific heat content, pressure against temperature, specific internal energy against specific volume, etc. Since there are several such properties, there are a number of possible pairs. Some of these combinations give diagrams which are of great value in engineering applications, others are of more interest in highly technical studies, and some yield such meager information that they are rarely, if ever, used.

(h) In studying reversible changes of state, it is customary to speak of the entire series of equilibrium states intermediate between the initial state a and the final state b, as the *path* of the change of state ab. Any curve plotted to represent this series of states is also referred to by the same term, or as the path of the process.

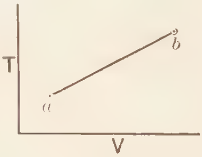

FIG. 29.

56. Summary.—(a) A body is said to be in equilibrium when it is incapable of spontaneous change. Bodies in equilibrium are characterized by uniform[3] (not necessarily constant) values of pressure, density, specific volume, temperature, specific internal energy, specific total heat, etc. Only equilibrium states can be described numerically. The opposite of an equilibrium state is a turbulent one, in which the body is characterized by non-uniform distribution of pressure, density, temperature, etc. Most actual states of working substances, used in engineering applications of thermodynamics, approximate equilibrium conditions sufficiently closely to justify the assumption that they are in equilibrium.

(b) A reversible process is the limit approached by an actual process as it is increasingly controlled, or harnessed, so as to develop progressively more work. A reversible process is a continuous series of equilibrium states of the entire system of bodies participating in the process. The history of the entire system during such a process is completely known in terms of numerical quantities depending upon the state of the system. During a reversible process, the maximum possible useful work is developed. The opposite of a reversible process is an irreversible one, during which the system passes through turbulent states. Such a process cannot be described numerically, since momentary states of the system as a whole cannot be determined.

(c) By themselves, certain parts of a system may be considered to execute reversible processes and their action may be analyzed on that assumption, despite the fact that the system as a whole may be undergoing an irreversible process.

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[3] Except for the effect of gravity, and except for two phase substances, such as a coexistant vapor and its liquid.

(**d**) An equilibrium state of a body may be represented graphically by a point in a field whose coordinates are the values of any two characteristics of the body. A reversible process may be represented in such a field by a curve setting forth the mode of variation of one property as compared with another. A turbulent state, or a turbulent process, cannot be represented in this way, in terms of any variables whatever. The series of equilibrium states which constitute a reversible process, or any curve representing this series, is called the path of the process.

CHAPTER VI

ENTROPY

57. Explanatory.—(a) In many thermodynamic problems a certain property, known as "*Entropy*," is found of great importance. When applied to working substances its use simplifies engineering calculations that would be very complicated without it. When applied to a system of bodies it also enables scientists to draw certain sweeping conclusions with regard to natural phenomena,—conclusions which would otherwise be difficult to formulate and which materially assist in developing the laws governing thermodynamic transformations.

(b) When heat is added to a working substance during any process, the rate of addition may be constant or variable, depending on the conditions of the transfer. But, in any event, the aggregate amount added while passing from the initial state a to the final state b, along some specified path, is represented by the symbol Q_{ab}, which may also be expressed more significantly by $\int_a^b dQ$ in which the integration is along the given path. But, as it stands, this integral cannot be evaluated, because, as noted in Chapter IV, the heat added to any working substance during any process depends upon that process, or path, and is not determined solely by the initial and final states. In other words, it is impossible to find a single quantity X, such that

$$Q_{ab} = X_b - X_a.$$

(c) However, if dQ can be expressed in terms of any properties of the working substance whose values have determined the continuous series of states through which the substance passed in going from a to b, then this integral takes a form that can be evaluated. Thus, if c is the specific heat of a substance for a given process, then $dQ = wcdT$, and $Q_{ab} = w\int_a^b cdT$, which may be integrated if the value of the specific heat for the particular process is known in terms of the temperature for the given case. Expressing dQ thus in terms of specific heat and temperature is often of great value, but it is frequently much more convenient and useful to express it in terms of temperature and "entropy," which will now be considered.

58. Definition of Entropy, ϕ.—(**a**) The absolute values of the entropy of any working substance are not of importance to the engineer since he uses only its changes. Hence, its definition will be given in terms of the latter; thus: *The entropy, ϕ, of a substance is that property whose* **gain** *between any two states a and b is such that*

$$\Delta\phi_{ab} = \int_a^b \frac{dQ}{T}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

this integration being made for any reversible path between a and b. It will be seen later that the result will be the same regardless of what reversible path is used for this integration.

(**b**) In the special case in which the substance remains at constant temperature while receiving heat, the increase in entropy is the total amount of heat thus added divided by the constant absolute temperature prevailing. However, in most cases heat is not transferred at constant temperature and it is therefore usually necessary to use the more general expression given in Eq. (25).

(**c**) Entropy is like the pressure, volume and temperature, in that it assumes a single definite value measured above some convenient datum for any given state of the substance, *no matter how that state is reached.* Hence, whenever the substance is brought from state a to state b, its entropy changes from one *definite* value to another,—that is,

$$\Delta\phi_{ab} = \phi_b - \phi_a, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

and in this respect it differs from Q_{ab}, which is not independent of what occurs between the initial and final conditions, but depends upon the process ab.

(**d**) From Eq. (25) it is evident that, for a reversible process,

$$dQ = Td\phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

and hence

$$Q_{ab} = \int_a^b dQ = \int_a^b Td\phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

which is the general equation for the heat added to the working substance during any reversible process ab, expressed in terms of temperature and entropy.

(**e**) If a curve be plotted with Tϕ-coordinates to represent the reversible process ab, as in Fig. 30, the $\int_a^b Td\phi$ will be shown by the area under the curve, extending down to absolute zero, hence Q_{ab} is shown by the area *abmn* in the figure.

(f) It is ordinarily impossible to integrate a differential expression, from one value to another of the independent variable, until the relation between the variables involved is known. For example, the differential expression xdy cannot be integrated until some equation connecting x and y is given. Thus, if $x = y$, the integral becomes $\frac{y^2}{2} + K$; if $x = \log y$, the integral becomes $y \log y - y + K$; and if $x = y^m$, the integral becomes $\frac{y^{m+1}}{m+1} + K$. In like manner, the differential expression $Td\phi$ cannot be integrated for any change of state until the path of that change is known—that is, until the relation between T and ϕ is determined throughout the process. For reversible processes this relation is definitely known and hence Q_{ab} in Eq. (28) can be determined for such cases, as will be seen later.

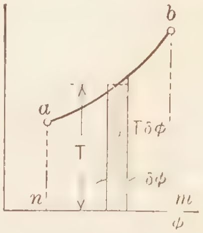

FIG. 30.

(g) For turbulent processes Eq. (28) is obviously indeterminate since equations cannot be written for their paths because the quantities involved do not have uniform values throughout the entire mass and hence cannot be related in any definite manner. On the contrary, the entropy expression $\frac{dQ}{T} = d\phi$ may be integrated between the states a and b, and the same change of entropy is secured whatever may be the path of the process between these states. Hence, to determine the difference in values of the entropy of a substance in any two specified states, it is merely necessary to devise any reversible process whereby the substance in question may be conducted from one of these states to the other, then write the equations defining the path of this process, and integrate the general expression as given by Eq. (25). In later chapters this method will be illustrated for a number of cases.

59. Entropy Units.—(a) Since, in Eq. (25), dQ is expressed in heat units and T in absolute temperature, the change of entropy must be in terms of heat units per unit of temperature. Thus, entropy is expressed in heat units per degree and will, therefore, depend on both of these units, for example it may be stated in B.t.u./deg. fahr., or in cal./deg. cent., which result in different numerical values. It is customary not to name the unit, but simply to give the entropy of any substance in any state, or the change of its entropy during any process, as a number, sometimes adding the phrase "units of entropy."

(b) Since the heat added to a substance to produce any change in its state is proportional to its weight, the corresponding entropy change has the same proportionality, and in this respect is like internal energy, total heat, and volume. For simplicity, when these quantities are referred to in terms of unit weight the word "specific" is usually prefixed to the name.

60. Physical Significance of Entropy.—Much time and effort have been expended in the search for an easily comprehended physical significance of entropy, but the results are more likely to confuse the beginner than to assist him. Until one has studied thermodynamics and the theory of the constitution of matter very much further than this textbook pretends to go, it is usually undesirable to seek for a physical meaning for the entropy of a substance. Let it be regarded as a mathematical concept, whose existence is demonstrable (although this book does not attempt to prove this rather involved mathematical theorem), and whose relations to other things are such that it is of great utility in engineering calculations. Like moment of inertia, internal energy, and heat itself, entropy is of great value and may be used without any direct conception of just what it is in a physical sense. Entropy is not an observable property which can be measured, like pressure and temperature, but it is a mathematical function of quantities which can be so determined. It is real, although it is not appreciable to the senses.

61. Illustrations Concerning Entropy.—(a) The experiments discussed in Chapter V will now be considered further to examine the entropy changes involved.

(b) In the simple mechanism in which a falling weight lifted another mass by means of a cord running over a sheave, Sect. 48 (c) and (d), the thermal aspects of the problem are secondary, hence the study of the entropy changes, while entirely possible, would be of little value.

(c) However, the case of the block of ice floating in hot water is worth examining further. As long as any ice remains as such, it will still be at the temperature 32 deg. fahr., while the water from which it is receiving heat will be at a higher temperature. Assuming no losses, the heat absorbed by the ice is equal to that given up by the water. That is $Q_{ice} = -Q_{water}$. But the heat abstracted from the water is some function of the variable water temperature, T_w, thus $Q_{water} = f(T_w)$. Then it follows that the *gain* in entropy of the *water* is negative and is equal to

$$\Delta\phi_{water} = -\int\frac{dQ_w}{T_w} = -\int\frac{d\cdot[f(T_w)]}{T_w}. \quad . \quad . \quad . \quad . \quad (A)$$

This integral cannot be evaluated until the function $f(T_w)$ is known;

but, from Sect. **58**(b) it follows that, since the absolute temperature of of the melting *ice* remains constant at 492°, the *gain* in its entropy is

$$\Delta\phi_{ice} = \frac{Q_{ice}}{492} = \frac{-Q_{water}}{492} = -\int\frac{d\cdot[f(T_w)]}{492}. \quad . \quad . \quad . \quad (B)$$

Then, since T_w is always greater than 492, it follows from equations (A) and (B), which have the same numerators, that

$$\Delta\phi_{ice} > -\Delta\phi_{water}.$$

This inequality leads to the conclusion which may be startling for a moment, namely, that the entropy of the ice increases more than the entropy of the water decreases. In other words, the aggregate entropy of the entire mass of ice and water increases until they reach the same temperature when the process stops. *This phenomenon is characteristic of all spontaneous processes,—they are always accompanied by an increase in the entropy of the system executing the process.*

(d) A similar result will follow from the study of the process in Sect. **52** (**b**), of heating a gas by means of a hot body at a higher temperature. Assuming no external loss, the rate of heat transfer by the hot body is always equal to the simultaneous rate of heat absorption by the gas, but the temperature of the hot body must be *greater* than that of the gas, hence the rate of decrease of entropy of the hot body is less than the rate of increase of entropy of the gas. Therefore the entropy of the entire system increases, as in the previous case. When the heating is carried out with an increasing number of hot bodies, as described in Sect. **52** (**d**), the temperature difference between the gas and the particular hot body, which is momentarily in contact with it, is progressively decreased. This difference approaches zero as the reversible process is approached, and with it the entropy increment of the entire system approaches zero.

62. Generalizations.—(a) From the foregoing discussion several generalized statements may now be made. Certain processes may proceed by themselves, that is, there are processes during which the working substance passes spontaneously from a state in which it is not in equilibrium to one of equilibrium, without any outside interference. During such a process, the useful work developed is zero, the increase of entropy of the system is the maximum possible, and the internal energy of the substance is constant, since it is cut off from outside influences. As this process is harnessed more and more effectively, an increasing measure of useful work is done, the gain in entropy of the working substance is reduced, and its internal energy is changed due to the work developed. *The completely controlled reversible process develops a maximum of work, and entails no change in the entropy of the entire system.*

(b) *For any given state, the entropy of the working substance is fixed, whatever the path by which that state is reached.* During a process, any energy that is dissipated as heat, which is later returned to the working substance, increases the entropy precisely as though an equivalent quantity of heat had been purposely added from outside sources at similar temperatures. If a relation is known between the rate of this heat addition and the simultaneous temperatures, the corresponding increase in entropy can be computed. *If such a relation is not known it is necessary to obtain the final state of the working substance from other considerations, and then, its state being known, its entropy may be determined.*

(c) In any process, when all participating bodies are considered, an increase of the aggregate entropy of the entire system shows that this process has failed to develop the full quota of useful work which would have been possible were the process more fully controlled. In other words, *the gain in entropy of the entire system is a measure of the unavailability of work, or of the dissipation of energy, during the process.*

(d) Processes may be thought of as divided into parts, some irreversible, others reversible. The *reversible portions may be analyzed completely,* and the changes of entropy may be computed readily for the body executing those parts of the process. These reversible changes of entropy, however, do not make up the change in the aggregate entropy of the entire system, and are therefore not related to the availability of work in any simple manner.

(e) The law of the conservation of energy and the general principles just learned about the entropy of an isolated system are sometimes generalized by stating that *the energy of the universe is constant, while the entropy of the universe tends toward a maximum.* This is a generalization from small, well-known processes, to vast phenomena which are little understood, and the conclusion is possibly too broad. It may be that in the processes whereby nebulae come into being and develop into planetary systems, actions may occur during which entropy decreases, and perhaps energy increases. Such questions are far beyond the scope of technical thermodynamics and must be left for the speculations of the astronomical physicist. However, the generalized statement is true for all processes with which the engineer deals.

(f) In the chapters that follow, various changes of state of gases and vapors will be studied by computing, or obtaining graphically, such results as may be of service in everyday engineering applications.

In connection with these studies a knowledge of entropy will be found of great assistance. The most convenient methods of determining the numerical values of entropy for many different cases will be shown in detail when studying the various thermodynamic processes.

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CHAPTER VII

THE PROPERTIES OF GASES

63. Phases of Matter.—(a) All substances are known to exist, under proper conditions, in various distinct *phases*, some solid, some liquid, and some vaporous or gaseous. Most substances are commonly known in only one or two of these phases. For example, copper is familiar in the solid phase, but it can be melted rather easily, and at high temperatures it can be vaporized. Again, carbon dioxide is ordinarily in the gaseous phase, but if it be subjected to sufficiently low temperature and high pressure, it will condense to the liquid phase, which in turn may freeze to the solid phase. Sulphur is an example of a substance which has five phases,—two solid, two liquid and one gaseous.[1] Similarly water may exist in several solid phases which depend on the pressure to which it is subjected.[2]

(b) This chapter is devoted to the study of those substances which ordinarily appear in the gaseous phase, far removed from the conditions under which condensation can occur. Such substances are often called " permanent " gases, a term which has come down from the time when it was believed that they could not be liquefied. Because of the ease with which they can be handled, these " permanent gases " were studied very early, and about a century and a half ago their behavior was reduced to very simple mathematical relations. More recent researches have shown that these simple statements are subject to small errors, which increase as the temperature is lowered, and as the pressure is raised; but for engineering work the errors are negligibly small, except in problems requiring a high degree of precision, or in processes in which

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[1] The transformations from phase to phase are usually designated as follows:

solid to liquid	— melting
liquid to solid	— freezing or solidifying
liquid to gaseous	— evaporating or vaporizing
gaseous to liquid	— condensing or liquefying
solid to gaseous	— subliming
gaseous to solid	— condensing or solidifying

[2] See Bridgman, "Properties of Matter Under High Pressures," Mechanical Engineering, March, 1925, p. 161.

the substances involved are carried to temperatures or pressures approaching too closely the conditions of condensation.

64. Boyle's Law.—This law may be stated thus: *When the temperature of a given mass of gas is constant, the volume and pressure vary inversely*: that is

$$PV = a\ constant = K_1, \text{ the temperature not changing.} \quad (29)$$

As soon as the simultaneous values of pressure and volume for any given mass of gas are known, the value of the constant K_1 is established, and the volume corresponding to any pressure, or the pressure corresponding to any volume, may then be computed. The value of K_1 depends upon the kind of gas, the weight considered, and the temperature at which it is maintained.

65. Charles' Law and Absolute Temperature.—(a) One form of statement of Charles' law is: *When the volume of a given amount of gas is held constant, the change in the pressure of the gas is proportional to the change in its temperature.* This law may be represented graphically by the straight line abc in Fig. 31 in which the simultaneous values of temperature and absolute pressure are given for the three states a, b, and c, the temperatures being measured above any convenient datum t_0. In order to satisfy this law of Charles the three states a, b, and c, must be so located that

$$\frac{p_b - p_a}{t_b - t_a} = \frac{p_c - p_a}{t_c - t_a} = \frac{\Delta p}{\Delta t} = \text{a constant}, \quad . \quad . \quad . \quad (30)$$

in which the constant represents the slope of this line.

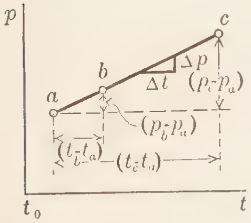

FIG. 31.

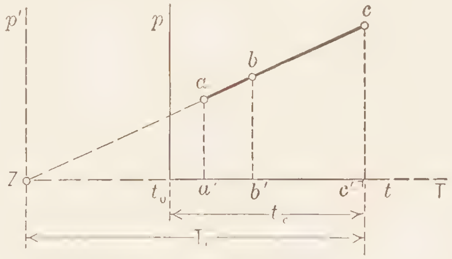

FIG. 32.

(b) If this law should hold perfectly for the gas when cooled indefinitely at constant volume, the absolute pressure would eventually become zero, as is indicated in Fig. 32 by the point z, through which coordinate axes p' and T are drawn. From the figure it is seen that

$$\frac{aa'}{za'} = \frac{bb'}{zb'} = \frac{cc'}{zc'}.$$

But the distances aa', bb' and cc' represent the values of the absolute pressure of the gas in three states a, b, and c, respectively, since they are each measured above the true zero of pressure. Similarly the distances za', zb', and zc', may be said to represent values of the *absolute temperature* of the gas in states a, b, and c, respectively, since they are each measured above a *true* or *absolute zero* of temperature, rather than above an *arbitrarily* chosen one, such as t_o. The particular temperatures that are used as the arbitrarily chosen zeros of our usual thermometer scales are thus found to be a certain number of degrees, measured by such a scale, above the absolute zero of temperature. Hence it follows that the absolute temperature may very properly be expressed in terms of any arbitrarily chosen temperature scale, such as the centigrade or the Fahrenheit. Careful experiments indicate that the absolute zero is very close to **491.6** fahrenheit degrees, or **273.1** centigrade degrees, below the temperature of melting ice. Thus, using T for the absolute temperature and t for ordinary temperature, and retaining only three significant figures, the following relation must hold for the usual scales.

In centigrade degrees, $T = t + 273$.

In fahrenheit degrees, $T = t + 492 - 32 = t + 460$.

Similar relations exist for other thermometer scales.

From this discussion it is apparent that the statement that the absolute temperature of a body is so many degrees, means nothing until it is specified or understood what scale is used for measuring these degrees. Throughout this text, unless otherwise stated, the " absolute " and " ordinary " temperatures will be expressed in fahrenheit degrees.

(**c**) By using absolute temperatures, Charles' Law may now be stated in simpler terms than given in (**a**). Thus, it becomes: *In a given mass of gas the pressure is proportional to the absolute temperature, if the volume is constant;* that is

$$P = K_2 T, \text{ the volume being constant,} \quad . \quad . \quad . \quad (31)$$

in which K_2 is a constant which depends upon the kind of gas, the mass considered and also upon the volume occupied.

(**d**) Charles' Law is also stated for the case in which the *pressure* is kept constant, as follows: *When the pressure of a given mass of gas is constant, the volume is proportional to the absolute temperature;* that is

$$V = K_3 T, \text{ the pressure being constant,} \quad . \quad . \quad . \quad (32)$$

in which the constant K_3 depends upon the kind of gas, the mass considered and also upon the pressure sustained.

66. The Characteristic Equation of Gases.—(a) Let a known weight of a specified gas be changed from state a to any other state b. It is desired to find the equation expressing the relation between pressure, volume and absolute temperature at these two states. Assume c to be some other state such that $T_c = T_a$ and $V_c = V_b$. Then, from Boyle's law, since $T_c = T_a$, it follows that $P_aV_a = P_cV_c$,

or

$$P_c = \frac{P_aV_a}{V_c} = \frac{P_aV_a}{V_b}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

and from Charles' law, since $V_c = V_b$, $P_c/P_b = T_c/T_b$,

or

$$P_c = \frac{P_bT_c}{T_b} = \frac{P_bT_a}{T_b}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

Then from Eqs. (a) and (b),

$$\frac{P_aV_a}{V_b} = \frac{P_bT_a}{T_b},$$

or

$$\frac{P_aV_a}{T_a} = \frac{P_bV_b}{T_b}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

That is, since a and b represent any two different states of the gas, it follows that for *any* state of the gas,

$$\frac{PV}{T} = K_4, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

in which K_4 is a constant depending only upon the weight of gas and upon the nature of that gas.

(b) For any given gas, if the pressure and temperature be maintained constant, the volume is clearly proportional to the weight of gas. Similarly, if the volume and temperature be fixed, the pressure will be proportional to the weight, since to confine twice as much gas within the same volume twice as much pressure is required. Again, if the pressure and volume be constant, the absolute temperature *will be inversely proportional to the weight of gas*. Accordingly, the constant K_4 must be proportional to the weight of gas considered, and it may, therefore, be replaced by the product wR, in which R is a constant depending only upon the nature of the gas considered. Thus

$$\frac{PV}{T} = wR, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

or

$$PV = wRT. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

This equation will be used so often that it should be memorized.

(c) It has already been stated that no *real* gas behaves precisely according to the Laws of Boyle and Charles, and it follows that Eq. (36) is not strictly accurate for any real gas. It is, however, sufficiently exact for all except the most precise engineering computations, for all of the more common "permanent" gases.[3] Strictly speaking, however, this equation defines the behavior of no real gas, but of an imaginary series of "ideal" gases (a series because the constant R may assume different numerical values), and for this reason it is usually designated as the *characteristic equation of ideal gases*, or, more briefly as *the ideal gas law.*

Eq. (36) taken alone does not define the concept of the ideal gas. An **ideal gas** is defined as one that follows Eq. (36) exactly, and also has c_v constant at all temperatures.

(d) From this ideal gas equation, it would appear that if any weight of such a gas were reduced to extremely low absolute temperatures, its pressure-volume product would become very small, approaching zero as the absolute temperature approached zero. That is, at absolute zero temperature, either the pressure of an ideal gas or its volume must be zero. Zero pressure implies the complete absence of molecular activity, which is consistent for absolute zero. Zero volume, on the other hand, is scarcely conceivable even for an ideal gas. If *real* gases be cooled to extremely low temperatures, they will condense to liquids, and freeze to solids, thus departing altogether from the behavior which is described by the ideal gas equation. This aspect of the ideal gas equation, however, is of no significance as applied to real gases, for the equation is merely an empirical expression of relations observed at temperatures well above the absolute zero. It ceases to be even approximately true for real gases when their temperature is lowered far below ordinary atmospheric levels. The failure of this equation at low temperature is an excellent illustration of the impropriety of extending the results of restricted experimental observations far beyond the range of values within which the results were obtained.

67. Units.—(a) In this text, the basic units for the ideal gas equation will be taken thus: pressure in pounds per square foot, volume in cubic feet, weight in pounds, and absolute temperature in degrees fahrenheit. The numerical values of R, given in Table II on page 80, are for these units. For air, for example, its value is 53.3, that is, from Eq. (36)

for air,
$$PV = 53.3wT. \qquad (37)$$

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[3] For a discussion of the deviation of real gases from this ideal gas equation see U. S. Bureau of Mines Technical Papers No. 131 and 158, on the Compressibility of Natural Gas. It is pointed out that under everyday working conditions, natural gas deviates from the ideal gas law by as much as 15 per cent.

Sometimes it is convenient to base computations on the pressure in pounds per square inch, because this is a much more common unit for actual measurement of pressures than is the pound per square foot. Since $P = 144p$, Eq. (36) may be written,

in general,
$$pV = \frac{wRT}{144}, \quad \ldots \ldots \quad (38)$$

where p is measured in pounds per square inch. The value of $R/144$ for air is 0.3701, hence the equation becomes,

for air,
$$pV = 0.3701wT. \quad \ldots \ldots \quad (39)$$

(b) Similarly, pressures are sometimes taken in inches of mercury at 32 deg. fahr. The density of mercury at the ice point being 0.4912 lb. per cubic inch, the pressure in inches of mercury at 32 deg. fahr., p'', multiplied by 0.4912, equals the pressure in pounds per square inch; or $p = 0.4912p''$. This value may be substituted in the foregoing equation, giving,

in general,
$$0.4912p''V = \frac{wRT}{144},$$

or

$$p''V = \frac{wRT}{70.73}, \quad \ldots \ldots \quad (40)$$

where p'' is measured in inches of mercury at 32 deg. fahr. The value of $R/70.73$ for air is 0.754, thus,

for air,
$$p''V = 0.754wT. \quad \ldots \ldots \quad (41)$$

In Table II, on page 80, there are given values of R, $R/144$, and $R/(0.4912 \times 144)$, for a number of gases.

68. The Mol and the Universal Gas Constant.—(a) According to **Avogadro's Law,** *equal volumes of all gases at the same temperature and pressure contain an equal number of molecules.* From this law it follows that the densities of any two gases, say 1 and 2, at the same pressure and temperature, are in the same proportion as their molecular weights, m_1 and m_2. But since the specific volume, $\overline{V}$, is the reciprocal of density, d, it therefore follows for these conditions that, for any two gases indicated by the subscripts 1 and 2,

$$\frac{\overline{V}_1}{\overline{V}_2} = \frac{d_2}{d_1} = \frac{m_2}{m_1}. \quad \ldots \ldots \quad (a)$$

Hence, if the gases be measured by a new unit of weight called the **mol,** which is defined as the weight of m pounds (or grams) of the particular gas, it follows that

$$\frac{\text{Volume of 1 mol of gas}_1}{\text{Volume of 1 mol of gas}_2} = \frac{\text{Volume of } m_1 \text{ pounds of gas}_1}{\text{Volume of } m_2 \text{ pounds of gas}_2} = \frac{m_1\overline{V}_1}{m_2\overline{V}_2}; \qquad (b)$$

or, from (a)

$$= \left(\frac{m_1}{m_2}\right)\left(\frac{m_2}{m_1}\right) = 1. \qquad (c)$$

In other words, from Avogadro's Law, the value of the volume of 1 mol is the same for all gases when at the same pressure and temperature. This *constant volume of the mol* is usually taken for a pressure of one atmosphere and a temperature of 32 deg. fahr., and under these conditions its value is very close to **360** cu. ft.

(**b**) The ideal gas equation for *one pound* of gas, is

$$P\overline{V} = RT, \qquad (d)$$

or

$$\overline{V} = \frac{RT}{P}, \qquad (e)$$

$$\therefore m\overline{V} = \frac{mRT}{P}. \qquad (f)$$

But $m\overline{V}$ is the volume of 1 mol of the gas, and is same for all gases under the same pressure P and temperature T, hence mR must also be constant.

This product, mR, is often designated as the **universal gas constant**, and the symbol R' may be used for it. Since, for any weight of gas

$$PV = wRT,$$

then

$$P(mV) = w(mR)T = wR'T,$$

or

$$PV = \frac{wR'T}{m}. \qquad (42)$$

(**c**) The *value of R'* is close to **1544**, but an inspection of Table II will show that all of the real gases do not yield this value exactly, probably because many depart considerably from an ideal gas.

For approximate purposes it is convenient to remember that the *gas constant R*, for any gas (for pressure in pounds per square foot), is approximately equal to 1544 divided by the molecular weight of that gas. For gas mixtures, like air, there is, strictly speaking, no molecular weight, and the gas constant must be determined by the method indicated in Sect. **75**.

(d) The gas constant R, represents the work done when a unit weight of the gas is heated one degree at constant pressure. Thus, with the system of units used in the text, R represents the foot-pounds of work done by one pound of gas when heated ~~to~~ one degree fahrenheit at constant pressure. Hence it follows that the universal gas constant, mR or R', with this system of units, would be the foot-pounds of work done by m pounds of the gas when heated one degree fahrenheit at constant pressure.

69. Joule's Experiment.—**(a)** About 1844, Joule experimented with the so-called *free expansion* of gases, —that is, with the expansion of a gas in such a way that it develops zero work, and neither receives nor loses heat. Under such conditions, the internal energy of the gas must remain constant, since, from the simple energy equation, Eq. (9*b*).

$$I_b - I_a = Q_{ab} - Wk_{ab} = 0.$$

His experimental apparatus consisted of two large flasks, connected by a short tube with a stop cock, all immersed in a water bath, together with thermometers, etc. One flask was filled with gas at a high pressure, 22 atmospheres, and the other was evacuated as completely as possible. After the entire apparatus had reached a uniform temperature, the stopcock was opened, permitting the gas confined in one flask to expand to fill both vessels. As the volume of the two flasks was unchanged, no external work was done. Since the water bath had the same final temperature, after equilibrium was established, as it had originally, there was no net transfer of heat between the gas and the water. Hence the internal energy of the gas was the same before and after the expansion. The cardinal observation of Joule was that *the temperature of the gas was also the same before and after the expansion.* That is, when the internal energy of an ideal gas is constant, the temperature is also constant, regardless of the changes in pressure and volume,—or, in other words, *the internal energy of an ideal gas depends only upon its temperature.*

(b) It is now desired to find the quantitative value of the change of internal energy of a given amount of ideal gas passing by any process whatever from temperature t_c to t_d. From the definition of specific heat at constant volume it follows that, when an ideal gas is heated from t_c to t_d at constant volume, the heat added is

$$Q_{cd} = wc_v(t_d - t_c).$$

Since no work is done during a constant volume process, the simple energy equation, Eq. (9*b*), becomes, for this case,

$$I_d - I_c = Q_{cd} = wc_v(t_d - t_c).$$

But, since Joule's experiment shows that the internal energy of an ideal gas depends only upon its temperature, it follows that, when there is a change from t_c to t_d, the gain in internal energy must be $wc_v(t_d - t_c)$, whatever the nature of the process; that is, for any two given states of an *ideal* gas, say 1 and 2

$$\Delta I_{12} = I_2 - I_1 = wc_v(t_2 - t_1), \quad . \quad . \quad . \quad . \quad . \quad (43)$$

regardless of how the gas passed from one state to the other. Note that the gain in the internal energy of the gas equals the heat added during a constant volume process *only;* for other processes the heat added is not all used to increase the internal energy.

(c) The case in which c_v is not constant, as with real gases, is discussed later, in Sect. **73.**

70. The Relations between R, K$_v$, K$_p$, and γ.—In making thermodynamic calculations for a given gas it is very convenient to have available the established relations that exist between its specific heats, K_p and K_v, and its constant R. They may be found by the application of the preceding principles, as follows:

If heat be added to an ideal gas during any *constant pressure* process, such as *ab*, the volume must increase, and hence the work done by the gas must be, from Eq. (8),

$$Wk_{ab} = P(V_b - V_a).$$

But, from Eq. (36), $P_bV_b = wRT_b$, and $P_aV_a = wRT_a$, wherefore, since $P_a = P_b$, the work done is

$$Wk_{ab} = wR(T_b - T_a) = wR(t_b - t_a), \text{ (in ft-lb.)} \quad . \quad . \quad . \quad (a)$$

The heat added will be, from Eq. (14),

$$Q_{ab} = wc_p(t_b - t_a), \text{ (in B.t.u.)} = wK_p(t_b - t_a), \text{ (in ft-lb.)} \quad . \quad (b)$$

The gain in internal energy will be, from Eq. (43),

$$I_b - I_a = wc_v(t_b - t_a), \text{ (in B.t.u.)} = wK_v(t_b - t_a), \text{ (in ft-lb.)} \quad . \quad (c)$$

The simple energy equation for this process is, from Eq. (9),

$$Q_{ab} = (I_b - I_a) + Wk_{ab}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (d)$$

The values of the several terms in Eq. (*d*) must of course be in the same units, so that either the work done must be expressed in B.t.u., or else the heat added and the change in internal energy must be expressed in foot-pounds. For the present purpose, the latter unit is more convenient. Substituting in Eq. (*d*) from the three preceding equations gives,

$$wK_p(t_b - t_a) = wK_v(t_b - t_a) + wR(t_b - t_a),$$

and dividing through by the common factor $w(t_b - t_a)$ leaves

$$K_p = K_v + R, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (44)$$

or, changing units,

$$c_p = c_v + \frac{R}{778} = c_v + AR. \quad . \; . \; . \; . \; . \; . \; . \quad (45)$$

By definition,

$$\gamma = \frac{c_p}{c_v} = \frac{K_p}{K_v}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (46)$$

From Eqs. (44) and (46) it follows that

$$K_v = \frac{R}{\gamma - 1}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (47)$$

$$K_p = \gamma \cdot \frac{R}{\gamma - 1}; \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (48)$$

or, changing units,

$$c_v = \frac{R}{778(\gamma - 1)} = \frac{AR}{\gamma - 1}, \quad . \; . \; . \; . \; . \; . \; . \quad (49)$$

$$c_p = \frac{R}{778} \cdot \frac{\gamma}{\gamma - 1} = AR\left(\frac{\gamma}{\gamma - 1}\right). \quad . \; . \; . \; . \; . \quad (50)$$

71.[4] **Variable Specific Heats of Gases.** (a) For real gases, at the wide range of temperatures involved in combustion and in a few other cases, the values of c_v and c_p are far from constant. Many experiments have shown that the specific heats are affected very little by high pressures, but materially by high temperatures. In many cases these experimental determinations are not entirely consistent with each other and it requires much study to decide which values should be used. A very careful investigation of this subject was conducted by Goodenough and Felbeck [5] and their conclusions as to the most reliable equations to use for all gases may be expressed as follows:

$$c_v = M + BT + CT^2. \quad . \; . \; . \; . \; . \; . \; . \quad (51)$$

$$c_p = M' + BT + CT^2. \quad . \; . \; . \; . \; . \; . \; . \quad (52)$$

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[4] For short courses of instruction it may be desirable to omit the remainder of this chapter.

[5] See Bulletin No. 139 Engineering Experiment Station, Univ. of Illinois, 1924, on "An Investigation of the Maximum Temperatures and Pressures Attainable in the Combustion of Gaseous Fuels," by G. A. Goodenough and G. T. Felbeck.

TABLE II

GAS CONSTANTS

For a temperature of 32° F. and a pressure of 14.7 lbs. per sq. in., abs. Except for those cases marked with an asterisk, the values of c_p and c_v have been calculated from the equations of Goodenough and Felbeck, in Bulletin No. 139, Univ. of Illinois.

Name of Gas	Molecular Formula	Atoms per Molecule	Molecular Weight		c_p	c_v	$\gamma = \frac{c_p}{c_v}$	$R = 778(c_p - c_v)$	$\frac{R}{144}$	$\frac{R}{144 \times 0.4912}$	R × mol. wt.	Specific Volume, cu. ft. per lb.	Density, lb. per cu. ft. $= \frac{14.7 \times 144}{492R} = \frac{4.3024}{R}$
			Approximate	International (1923)									
†Air	...	..	(29)	28.95	0.240	0.1715	1.40	53.3	0.3701	0.7535	1547	12.39	0.0807
*Argon	Ar	1	40	39.9	0.1269	0.076	1.67	39.6	0.2750	0.5598	1580	9.21	0.1086
*Helium	He	1	4	4.00	1.251	0.754	1.659	386.6	2.685	5.466	1546	90.0	0.0111
Carbon Monoxide	CO	2	28	28.005	0.2485	0.1775	1.40	55.3	0.3840	0.7815	1550	12.86	0.0778
Hydrogen	H_2	2	2	2.016	3.140	2.160	1.46	762.0	5.292	10.77	1525	177.1	0.00565
*Nitric Oxide	NO	2	30	30.008	0.2310	0.167	1.38	49.5	0.3437	0.6997	1485	11.52	0.0869
Nitrogen	N_2	2	28	28.016	0.2485	0.1775	1.40	55.3	0.3840	0.7815	1550	12.86	0.0778
Oxygen	O_2	2	32	32.00	0.2175	0.1553	1.40	48.2	0.3347	0.6812	1544	11.22	0.0891
Carbon Dioxide	CO_2	3	44	44.005	0.2025	0.1575	1.285	35.0	0.2450	0.4990	1540	8.14	0.1230
*Nitrous Oxide	N_2O	3	44	44.016	0.226	0.174	1.30	40.6	0.2818	0.5737	1786	9.43	0.1060
Acetylene	C_2H_2	4	26	26.026	0.391	0.314	1.243	59.9	0.4160	0.8468	1558	13.92	0.0718
Methane	CH_4	5	16	16.037	0.54	0.416	1.295	96.5	0.6700	1.364	1544	22.40	0.0446
Ethylene	C_2H_4	6	28	28.042	0.357	0.286	1.25	55.3	0.3840	0.7815	1550	12.86	0.0778

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* See Landolt and Bornstein, "Physikalisch-Chemische Tabellen," 1923.

† The values of c_p and c_v for air were calculated as shown in Section **75** (f). The most reliable values determined experimentally for air appear to be, $c_p = 0.239$ and $\gamma = 1.403$.

In Table III the values of the constants, M', M, B and C are given for a number of gases, and have been calculated for the pound as the unit of weight, instead of the mol, as given by these investigators in their Table 15.

TABLE III

Instantaneous Specific Heats of Gases

(In B.t.u. per pound per degree Fahrenheit)

$$C_p = M' + BT + CT^2$$

$$C_v = M \ + BT + CT^2$$

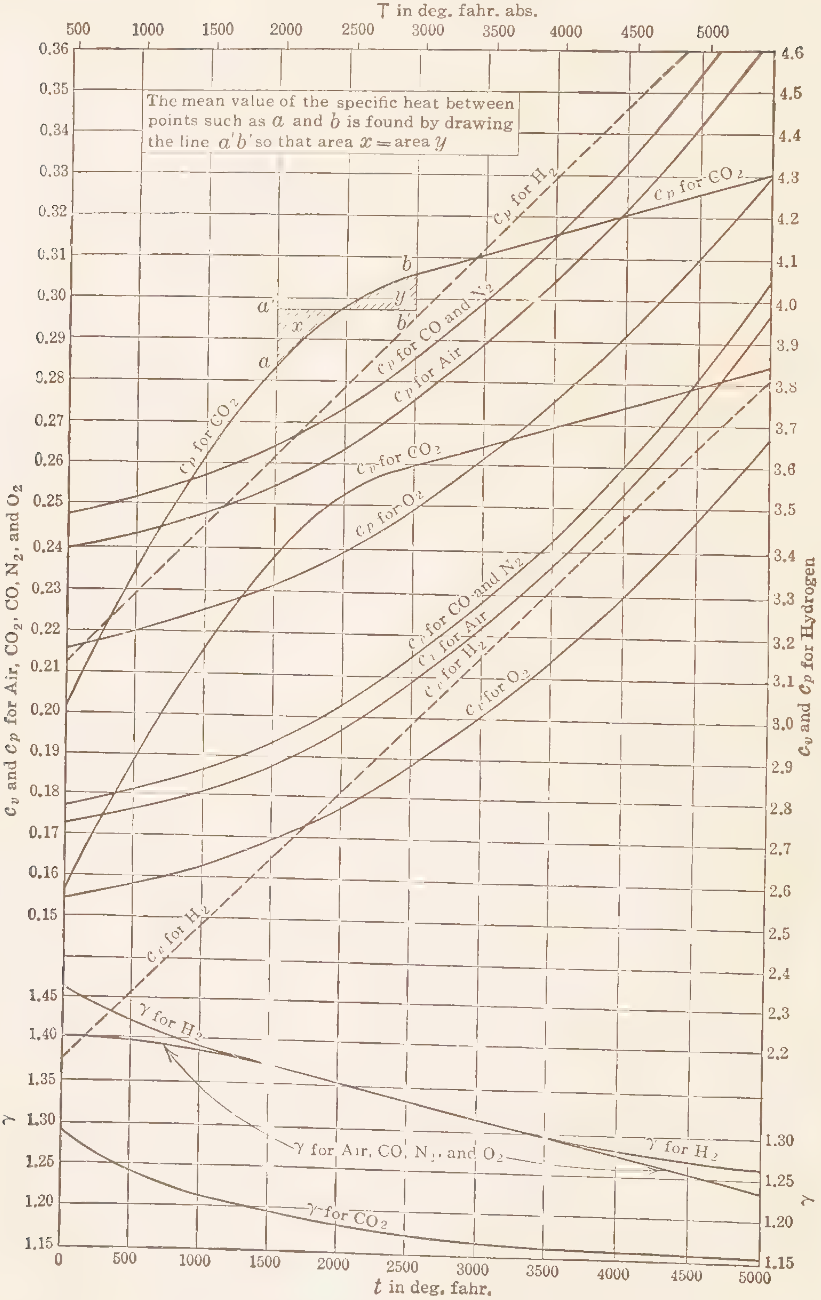
in which T = abs. temp. in deg. fahr., and the constants have the following values:

Gas	M'	M	B	C	For $T=492$		For $T=2000$	
					c_p	c_v	c_p	c_v
CO	0.2475	0.1765	0	$4.28\ (10)^{-9}$	0.2485	0.1775	0.265	0.194
H_2	2.98	2.00	$3.31\ (10)^{-4}$	0	3.140	2.160	3.64	2.66
N_2	0.2475	0.1765	0	$4.28\ (10)^{-9}$	0.2485	0.1775	0.265	0.194
O_2	0.216	0.154	0	$3.75\ (10)^{-9}$	0.217	0.155	0.231	0.169
Air	0.239	0.1705	0	$4.138\ (10)^{-9}$	0.240	0.1715	0.2555	0.187
$CO_2 (T<2900)$	0.1625	0.1175	$8.86\ (10)^{-5}$	$-1.36\ (10)^{-8}$	0.2025	0.1575	0.285	0.240
$CO_2 (T>2900)$	0.277	0.232	$9.55\ (10)^{-6}$	0				
C_2H_2	0.238	0.161	$3.11\ (10)^{-4}$	0	0.391	0.314	0.860	0.783
CH_4	0.216	0.092	$6.59\ (10)^{-4}$	0	0.540	0.416	1.53	1.41
C_2H_4	0.238	0.167	$2.42\ (10)^{-4}$	0	0.357	0.286	0.722	0.651

The values of M', M, B and C for air are those calculated in Sect. **75** (f). For the other gases the values of these constants are based on the ones of Goodenough and Felbeck, as given on page 106, Bulletin No. 139, University of Illinois Experiment Station.

(b) How the instantaneous values of c_p, c_v and γ, for certain of the gases vary with the temperature, is shown in Fig. 33.

72. The Characteristic Equation of Real Gases. Even though the specific heats of real gases may change appreciably, the difference, $(c_p - c_v)$, remains a constant, and will equal $(M' - M)$, if the values of c_v and c_p follow the laws given in Eqs. (51) and (52). Hence, since $R = (c_p - c_v) \times 778$, it remains fixed and has for the various gases the values that are given in Table II. Thus, the equation $PV = wRT$ will hold for the "permanent gases" to a very satisfactory degree of ac-



The mean value of the specific heat between points such as a and b is found by drawing the line $a'b'$ so that area x = area y

FIG. 33.—Chart Showing how the Instantaneous Values of c_p, c_v and γ for Certain Gases Vary with the Temperature.

curacy even when using high pressures and temperatures, such as are found in internal combustion engines. Obviously, $M' = M + AR$.

73. Internal Energy of a Gas with c_v Variable.—For gases in which c_v is a function of temperature Eq. (43) cannot be used for obtaining the internal energy. For such cases

$$dI = wc_v dT, \qquad \qquad (53a)$$

or

$$\Delta I_{1,\,2} = I_2 - I_1 = w\int_{T_1}^{T_2} c_v dT. \qquad \qquad (53b)$$

Or,

$$\Delta I_{1,\,2} = w\bar{c}_v(T_2 - T_1), \qquad \qquad (53c)$$

if the average specific heat $\bar{c}_v$ for the range of temperature is known.

From Eq. (51) and Table III the relation of c_v to T may be found for the gases listed and the integral in Eq. (53b) can then be evaluated between any two values of temperature. With $c_v = M + BT + CT^2$,

$$\begin{aligned}\Delta I_{1,\,2} &= w\int_{T_1}^{T_2}(M + BT + CT^2)dT \\ &= w\left[M(T_2 - T_1) + \frac{B}{2}(T_2^2 - T_1^2) + \frac{C}{3}(T_2^3 - T_1^3)\right]. \qquad (53d)\end{aligned}$$

74. Heat Content of Gases.—(a) It is sometimes useful to find the value of the heat content of a gas in some given state, but usually the *difference* in the heat contents for two known states is desired. Let it be required to find $H_b - H_a$ for w pounds of a gas: Then, since by definition, $H_b = I_b + AP_bV_b$, and $H_a = I_a + AP_aV_a$, it follows that

$$\begin{aligned}H_b - H_a &= I_b - I_a + AP_bV_b - AP_aV_a \\ &= w\int_{T_a}^{T_b} c_v dT + AP_bV_b - AP_aV_a, \qquad (54)\end{aligned}$$

which becomes determinate as soon as the values of P_a, V_a, P_b, V_b, and c_v are known.

(b) For an *ideal gas* c_v is constant; then for w lb. of such gas the difference in heat content is

$$\begin{aligned}H_b - H_a &= wc_v(T_b - T_a) + wAR(T_b - T_a) \\ &= w(c_v + AR)(T_b - T_a) \\ &= wc_p(T_b - T_a); \qquad (55a)\end{aligned}$$

and for *one pound*

$$h_b - h_a = c_p(T_b - T_a). \qquad \qquad (55b)$$

By substituting $\bar{c}_v$ for c_v, Eqs. (55*a*) and (55*b*) may also be used for real gases, if the average values of their specific heats are known for the temperature range involved.

(**c**) If $c_v = M + BT + CT^2$, then

$$H_b - H_a = w\left[M(T_b - T_a) + \frac{B}{2}(T_b^2 - T_a^2) + \frac{C}{3}(T_b^3 - T_a^3)\right] + AP_bV_b - AP_aV_a; \quad . \ . \ . \ . \ . \ . \quad (56a)$$

and, since $AP_bV_b = wART_b$ and $AP_aV_a = wART_a$ and $M' = M + AR$, Eq. (56*a*) reduces to

$$H_b - H_a = w\left[M'(T_b - T_a) + \frac{B}{2}(T_b^2 - T_a^2) + \frac{C}{3}(T_b^3 - T_a^3)\right]. \quad (56b)$$

75. Gas Mixtures.—(**a**) According to **Dalton's Law,** *when several gases, which have no chemical affinity, occupy a vessel in common, each constituent behaves as though the others were not present; hence the mixture exerts on the enclosing walls a pressure which is the sum of the pressures exerted by the constituents.* The pressure which would exist if any one of the gases occupied the vessel alone is termed the *partial pressure* for that gas; then, the total pressure exerted by the mixture is the sum of the partial pressures, each taken for the given temperature.[6]

(**b**) If a mixture be composed of several gases weighing w_1, w_2, $w_3 \ . \ . \ . \ w_n$ lb., respectively, then the weight of the mixture is $w_m = w_1 + w_2 + w_3 \ . \ . \ . + w_n$, and, since the gases are intimately mixed together in the same vessel, it follows that $V_m = V_1 = V_2 = V_3 = \ . \ . \ . = V_n$ and that $T_m = T_1 = T_2 = T_3 . \ . \ . \ . = T_n$. Assuming that each constituent obeys the characteristic law of ideal gases, then, according to Dalton's law,

$$P_1 = \frac{w_1R_1T_m}{V_m}, \quad P_2 = \frac{w_2R_2T_m}{V_m}, \ . \ . \ . \text{ and } \quad P_n = \frac{w_nR_nT_m}{V_m};$$

and for the mixture,

$$P_m = \Sigma P = \frac{(w_1R_1 + w_2R_2 \ . \ . \ . + w_nR_n)T_m}{V_m} = \frac{\Sigma(wR)T_m}{V_m}. \quad (57)$$

This *characteristic law of a mixture of ideal gases* may be rewritten as follows:

$$P_m = \frac{w_mR_mT_m}{V_m}, \quad . \ . \ . \ . \ . \ . \ . \ . \quad (58)$$

in which R_m is the gas constant for the mixture. Obviously,

$$R_m = \Sigma(wR) \div w_m. \quad . \ . \ . \ . \ . \ . \ . \quad (59)$$

---

[6] Later it will be shown that Dalton's law also applies to vapors, and to mixtures of gases and vapors.

(c) The *specific heats of a mixture* of a number of gases may be calculated from the following equations:

$$c_{pm} = \frac{(w_1c_{p_1} + w_2c_{p_2} \ldots + w_nc_{pn})}{w_m} = \frac{\Sigma(wc_p)}{w_m} \quad \ldots \quad (60a)$$

$$c_{vm} = \frac{\Sigma(wc_v)}{w_m}. \quad \ldots \quad (60b)$$

(d) For a mixture of real gas with $c_v = M + BT + CT^2$, it is necessary to determine the value of the constants M, B, and C for the combination. The value of M can be obtained in the following manner: If w_m pounds of a mixture of gases is composed of constituents with respective weights w_1, w_2, w_3 . . . and w_n and with constants M_1, M_2, M_3 . . . and M_n, then $w_mM = (wM)_1 + (wM)_2 + (wM)_3 \ldots + (wM)_n = \Sigma(wM)$. Hence, for the mixture

$$M_m = \frac{\Sigma(wM)}{w_m}. \quad \ldots \quad (61a)$$

Similarly,

$$B_m = \frac{\Sigma(wB)}{w_m}, \quad \ldots \quad (61b)$$

and

$$C_m = \frac{\Sigma(wC)}{w_m}. \quad \ldots \quad (61c)$$

(e) In illustrating the application of Eqs. (59) and (60), let the mixture be **air,** the composition of which by weight, when dry, is approximately as follows:[7] Nitrogen (N), 75.5 per cent; Oxygen (O), 23.2 per cent; and Argon (Ar) 1.3 per cent.

Then, from Eq. (59), the value of R_m *for air* is

$$R_{Air} = \frac{(w_NR_N + w_OR_O + w_{Ar}R_{Ar})}{w_{Air}}$$

$$= \frac{(0.755 \times 55.3 + 0.232 \times 48.4 + 0.013 \times 39.6)}{1} = \mathbf{53.3}.$$

Obtaining the values of c_v and c_p from Table II for the constituents of the mixture, then the *specific heats of air* at 32 deg. fahr. are found from Eqs. (60*a*) and (60*b*) to be as follows:

$$c_{p\text{Air at }32°} = \frac{(0.755 \times 0.2485 + 0.232 \times 0.2175 + 0.013 \times 0.127)}{1} = \mathbf{0.2395}$$

---

[7] These data were calculated from the volume percentages given in the "Smithsonian Physical Tables," published by the Smithsonian Institution, Washington, D. C. These tables also show that air contains 0.00065 per cent H_2 and 0.046 per cent CO_2, but these quantities have been omitted in the illustration as they are negligibly small.

$$c_{v\,\text{Air at }32^\circ} = \frac{(0.755\times0.1775+0.232\times0.1553+0.013\times0.076)}{1} = \mathbf{0.171}.$$

Hence

$$\gamma_{\text{Air at }32^\circ} = \frac{0.2395}{0.171} = \mathbf{1.402}.$$

(f) As a check on the foregoing values, the specific heats of air may be calculated from the general equations of Goodenough and Felbeck. From them the specific heats of air at any absolute temperature, in terms of the *mol*, are as follows:

$$C_{p_{mol}} = 6.93 + 1.2\,(10)^{-7}T^2,$$

$$C_{v_{mol}} = 4.945 + 1.2\,(10)^{-7}T^2.$$

Then, if air be considered to have a molecular weight of 29, the specific heats referred to the pound basis become as follows:

$$c_p = \frac{6.93}{29} + \frac{1.2\,(10)^{-7}T^2}{29} = 0.239 + 4.138\,(10)^{-9}T^2,$$

and

$$c_v = \frac{4.945}{29} + \frac{1.2\,(10)^{-7}T^2}{29} = 0.1705 + 4.138\,(10)^{-9}T^2.$$

Hence, for $T = 492$, $c_p = 0.240$, $c_v = 0.1715$, and $\gamma = 1.400$, which verify the results of the preceding computation.

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Supplementary References. For references supplementing those given in this and other chapters, see The Engineering Index, published annually by the Am. Soc. of Mech. Engrs., for classified lists of articles that have appeared each year in periodicals, transactions, bulletins, reports, etc.; and for monthly lists refer to The Engineering Index Section of Mechanical Engineering.

CHAPTER VIII

THERMODYNAMIC PROCESSES OF GASES

76. Introductory.—In this chapter will be shown how the principles already established can be applied to gases which are undergoing the processes most commonly used in heat-power engineering. The quantities most generally needed are the amount of heat transferred to or from a gas and the work done during the processes under consideration, but it is also frequently necessary to determine the changes of pressure, volume, temperature, internal energy, heat content and entropy that occur. In the study of each case both the *pressure-volume* and the *temperature-entropy* diagrams will be employed, since, by their aid the subject can be very much clarified. Furthermore, until these two diagrams are well understood, it is difficult to obtain a proper knowledge of the cycles, which will be discussed later.

77. The Constant Volume Process.—(a) If any working substance has its volume held constant while heat is added to, or abstracted from it, the resultant process is a constant volume one. It will be assumed that, as far as the working substance is concerned, the process is a reversible one, and that the whole mass of gas is, therefore, passing through a series of equilibrium states that may be represented graphically, if desired, on any diagram such as the PV or $T\phi$.

(b) For a gas that is undergoing a constant volume process, let the initial and final states be represented by a and b respectively, then, from the principles already developed, and using the notation given in Sect. **18**, the following general relations may be written for w lb.:

From Charles' law,

$$\frac{P_b}{P_a} = \frac{T_b}{T_a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

From Eq. (8), since V is constant, the work done by the gas during the process is

$$Wk_{ab} = \int_{V_a}^{V_b} PdV = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

From Eq. (14), the heat transferred to the working substance is

$$Q_{ab} = w\int_{T_a}^{T_b} c_v dT. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

From the simple energy equation, Eq. (9a), the gain in internal energy is

$$\Delta I_{ab} = Q_{ab} - Wk_{ab} = Q_{ab} = w\int_{T_a}^{T_b} c_v dT. \quad . \quad . \quad . \quad (65)$$

From its definition, the entropy increase is

$$\Delta\Phi_{ab} = \int_{T_a}^{T_b} \frac{dQ}{T} = w\int_{T_a}^{T_b} \frac{c_v dT}{T}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

(**c**) If the gas is one that has c_v *constant* between the two temperatures T_a and T_b, then

$$\Delta I_{ab} = Q_{ab} = wc_v(T_b - T_a), \quad . \quad . \quad . \quad . \quad . \quad (67)$$

and

$$\Delta\Phi_{ab} = wc_v \log_e \frac{T_b}{T_a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

By substituting $\bar{c}_v$ for c_v, Eq. (67) can be used when the specific heat is variable, provided its mean value for the temperature range is known. A similar substitution in Eq. (68) involves only a slight error.

Note that if T_b is less than T_a, the gain of entropy becomes negative, which is consistent, as *the entropy of a substance is always decreased if heat is abstracted from it during any reversible process, and vice versa.*

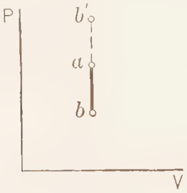

Fig. 34.

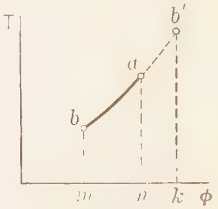

Fig. 35.

The Constant Volume Process.

(**d**) The pressure-volume and temperature-entropy diagrams for the process ab are given in Figs. 34 and 35, respectively. The former merely shows how the pressure varies while the volume remains constant. In the latter, $\Delta\phi_{ab}$ is represented by the distance mn. Also Q_{ab} is shown by the area $abmn$, since $dQ = Td\phi$ or $Q_{ab} = \int_a^b Td\phi$, the integral being the area under ab.

If the final temperature had been greater than the initial one, the corresponding state would be shown by some point such as b', with $\Delta\phi_{ab}' =$ distance nk and $Q_{ab}' =$ area $ab'kn$.

(e)[1] If $c_v = M + BT + CT^2$, ΔI_{ab} and Q_{ab} are obtained from Eq. (53*d*); and from Eq. (66),

$$\Delta\Phi_{ab} = w\int_{T_a}^{T_b}(M + BT + CT^2)\frac{dT}{T} = w\left[M \log_e T + BT + \frac{CT^2}{2}\right]_{T_a}^{T_b}, \quad (66a)$$

in which M, B, and C have the values given in Table III.

(f) For an example involving *variable specific heats,* suppose that 3 pounds of oxygen are heated at constant volume from $t_1 = 100°$ to $t_2 = 2000°$ F., and with

$$c_v = 0.154 + 3.75(10)^{-9}T^2,$$

as given in Table III.

Let it be desired to find $\Delta\phi_{1,\,2}$ and $\Delta I_{1,\,2}$, and to show the area representing $Q_{1,2}$ on the $T\Phi$-diagram. Then, from Eq. (66), or from Eq. (66*a*),

$$\Delta\Phi_{1,\,2} = \int_{T_1}^{T_2} wc_v\frac{dT}{T} = \text{the distance } om\text{, to scale, in Fig. 36.} \quad (a)$$

$$= \int_{560}^{2460} 3\left[0.154 + 3.75\,(10)^{-9}T^2\right]\frac{dT}{T}. \quad (b)$$

$$= \int_{560}^{2460} 3\left[\frac{0.154}{T} + 3.75\,(10)^{-9}T\right]dT. \quad (c)$$

$$= 3\left[0.154 \log_e T + 3.75\,(10)^{-9}\frac{T^2}{2}\right]_{560}^{2460} \quad (d)$$

$$= 3\left[0.154 \log_e \frac{2460}{560} + 1.875(0.00605 - 0.000314)\right] = 0.7164 \quad (e)$$

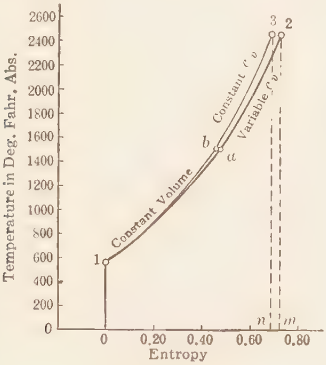

FIG. 36.

For any point a on the curve *1 2* in Fig. 36,

$$\Delta\Phi_{1,\,a} = 3\left[0.154 \log \frac{T_a}{560} + 1.875\,(10)^{-9}(T_a{}^2 - 560^2)\right]. \quad (f)$$

---

[1] In briefer courses the remainder of this section may be omitted.

Eq. (*f*) is obviously the equation for the curve. Substituting in it different temperatures T_a, the corresponding values of $\Delta\phi_{1,\,a}$ can be computed. Then, with the data thus obtained, and using the same scales that were adopted for the temperatures and for *om*, points can be plotted to determine the curve *1 2*. Thus, taking T_a as 1500, $\Delta\phi_{1,\,a}$ is found to be 0.4663, the corresponding point being *a* in the figure.

For the gain in internal energy Eq. (53*d*) may be used, or

$$\Delta I_{1,\,2} = \int_{T_1}^{T_2} wc_v dT = \int_{560}^{2460} 3[0.154 + 3.75\,(10)^{-9}T^2]dT \quad . \quad . \quad . \quad . \quad . \quad (g)$$

$$= 3\left[0.154T + 3.75\,(10)^{-9}\frac{T^3}{3}\right]_{560}^{2460} = 932.8 \text{ B.t.u.} \quad . \quad . \quad . \quad . \quad . \quad (h)$$

Since, for this process, $\Delta I_{1,\,2} = Q_{1,\,2}$ and as $Q_{1,\,2} = \int_1^2 Td\phi$, the area *1 2 m o* in Fig. 36 represents to scale both $\Delta I_{1,\,2}$ and $Q_{1,\,2}$.

(g) Now suppose that c_v is considered to remain *constant*, at the initial value it had before, while the gas is heated to a final state 3, such that $t_3 = 2000$, as in the previous case, and let it be desired to find $\Delta\phi_{1,\,3}$, $\Delta I_{1,\,3}$ and $Q_{1,\,3}$: For $t_1 = 100$,

$$c_v = 0.154 + 3.75\,(10)^{-9}(560)^2 = 0.1552. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A)$$

Then,

$$\Delta\Phi_{1,\,3} = wc_v \log_e \frac{T_3}{T_1} = 3 \times 0.1552 \log_e \frac{2460}{560}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (B)$$

$$= 0.690 = \text{distance } o\ n\text{, to scale, in Fig. 36;}$$

and

$$Q_{1,\,3} = \Delta I_{1,\,3} = wc_v(t_3 - t_1) = 3 \times 0.1552\,(2000 - 100) \quad . \quad . \quad . \quad . \quad (C)$$

$$= 885 \text{ B.t.u.} = \text{area } 1\ 3\ n\ o\text{, to scale, in Fig. 36.}$$

To determine any intermediate point *b*, substitution can be made in the equation of the curve. This equation is

$$\Delta\Phi_{1,\,b} = 3 \times 0.1552 \log_e (T_b/560). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (D)$$

Thus, for $T = 1500$, the same temperature that was used for point *a*, $\Delta\phi_{1,\,b}$ is found to be 0.459, and *b* in Fig. 36 is the corresponding point.

(h) From the above illustration it appears that if c_v is assumed constant, when heating this particular gas at constant volume from a temperature of 100 to a final temperature of only 2000° F., the calculated amount of heat required will be 5.1 per cent too small, and, similarly, the change in entropy obtained is 3.7 per cent less than it should be. If the final temperature had been 3000°, the errors would have been 9.4 per cent and 6.4 per cent respectively.

78. The Constant Pressure Process.—(a) Whenever a working substance is allowed to expand, or is compressed, under such conditions that its pressure does not change, it is said to be undergoing a *constant* *pressure*, *isobaric*, or *isopiestic* process. Clearly, on the PV-diagram, such a process must be a horizontal line, such as ab in Fig. 37.

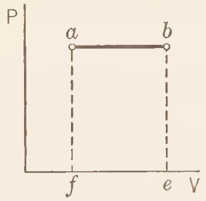

FIG. 37.—Constant Pressure Process.

(b) Consider w pounds of the gas heated at constant pressure, from T_a to T_b. Then:

From Charles' law,

$$\frac{V_b}{V_a} = \frac{T_b}{T_a}. \qquad (69)$$

From Eq. (8), since P is constant,

$$Wk_{ab} = \int_{V_a}^{V_b} PdV = P(V_b - V_a), \qquad (70)$$

or, since $PV = wRT$,

$$Wk_{ab} = wR(T_b - T_a) \qquad (71)$$

$$= \text{area } abef, \text{ to scale, in Fig. 37,}$$

this work being in foot-pounds, since P is in pounds per square foot, and V is in cubic feet.

From the definition of c_p,

$$Q_{ab} = w \int_{T_a}^{T_b} c_p dT. \qquad (72)$$

From Eq. (53b),

$$\Delta I_{ab} = w \int_{T_a}^{T_b} c_v dT. \qquad (73)$$

From Eq. (25),

$$\Delta \Phi_{ab} = \int_{T_a}^{T_b} \frac{dQ}{T} = w \int_{T_a}^{T_b} \frac{c_p dT}{T}. \qquad (74)$$

(c) *If c_p and c_v are constant,* the last three equations then become,

$$Q_{ab} = wc_p(T_b - T_a), \qquad (75)$$

and

$$\Delta I_{ab} = wc_v(T_b - T_a), \qquad (76)$$

and

$$\Delta \Phi_{ab} = wc_p \log_e \frac{T_b}{T_a}. \qquad (77)$$

(d) From the comparison of Eqs. (68) and (77) it is apparent that the change in entropy of a given weight of gas, when heated between the same temperature limits, will always be greater for the constant pressure process than for the constant volume one, because c_p is always larger than c_v. Thus nm is greater than nk in Fig. 38, in which ab represents the constant pressure addition of heat, and ab' is the corresponding constant volume process, T_b' being chosen equal to T_b. In this figure, Q_{ab} is shown by the area $abmn$, and Q_{ab}' by the smaller area $ab'kn$. The circumscribed symbols indicate that the corresponding characteristics are constant ones.

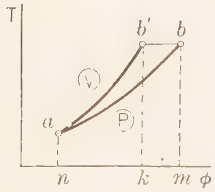

FIG. 38.

(e) If c_v and c_p vary as in Eqs. (51) and (52), $\Delta I_{1,2}$ can be obtained from Eq. (53*d*); and from Eqs. (72) and (74),

$$Q_{1,2} = w\left[M'T + \frac{B}{2}T^2 + \frac{C}{3}T^3\right]_{T_1}^{T_2}, \quad \dots \dots \quad (72a)$$

and

$$\Delta\Phi_{1,2} = w\left[M'\log_e T + BT + \frac{C}{2}T^2\right]_{T_1}^{T_2}, \quad \dots \dots \quad (74a)$$

Values of M', B, and C are given in Table III.

(f)[2] As in the previous example, consider the 3 lb. of oxygen heated from a temperature of 100° to a final temperature of 2000° F., but now let the *pressure*, instead of the volume, be held constant. Although the gas is assumed to be in the same initial state as before, the final state will be different; and, since it will be desirable to represent both processes on the same PV- and $T\phi$-diagrams, let the constant pressure process be designated as $1\,4$. For this gas it is known, from Table III, that $c_p = 0.216 + 3.75\,(10)^{-9}T^2$.

Then, with *variable* specific heat, from Eqs. (72*a*) and (74*a*),

$$Q_{1,4} = 3\left[0.216T + 3.75\,(10)^{-9}\frac{T^3}{3}\right]_{560}^{2460} = 1286.8 \text{ B.t.u.}, \quad \dots \quad (a)$$

and

$$\Delta\Phi_{1,4} = 3\left[0.216\log_e T + 1.875\,(10)^{-9}T^2\right]_{560}^{2460} = 0.9924. \quad \dots \quad (b)$$

(g) As a check, $Q_{1,4}$ may also be computed by the simple energy equation,

$$Q_{1,4} = \Delta I_{1,4} + Wk_{1,4}, \text{ in which } \Delta I_{1,4} = \int_{T_1}^{T_4} wc_v dT.$$

But, since

$$T_2 = T_4, \quad \Delta I_{1,4} = \Delta I_{1,2} = 932.8 \text{ B.t.u.}, \quad \dots \dots \quad (c)$$

---

[2] In short courses the remainder of this section may be omitted.

as before; and obtaining the value of R for this gas from Table II,

$$Wk_{1,4} = WR(t_4 - t_1) = 3 \times 48.4(2000 - 100)$$

$$= 275{,}800 \text{ ft-lb., or } 354 \text{ B.t.u.} \qquad (d)$$

Hence,

$$Q_{1,4} = 932.8 + 354. \qquad (e)$$

$$= 1286.8 \text{ B.t.u. thus checking Eq. } (a).$$

(**h**) Now, while the gas is heated at constant pressure from 1 to 5, with $T_5 = T_4$, let it be assumed that the *specific heat is constant* at the initial value it had in (e) of this section. Then, for $T_1 = 560$,

$$c_p = 0.216 + 3.75\,(10)^{-9}T^2 = 0.2172. \qquad (A)$$

Hence

$$Q_{1,5} = wc_p(t_5 - t_1). \qquad (B)$$

$$= 3 \times 0.2172\ (2000 - 100) = 1238 \text{ B.t.u.}$$

and

$$\Delta\Phi_{1,5} = wc_p \log_e\left(\frac{T_5}{T_1}\right). \qquad (C)$$

$$= 3 \times 0.2172 \times 1.48 = 0.9650.$$

Thus, if c_p is assumed constant, the calculated values of the heat added, and the change in entropy, are respectively 3.8 per cent and 2.76 per cent too small, for the conditions given in this case.

(i) For any point c on the curve *1 4*,

$$\Delta\Phi_{1,c} = 3\left[0.216 \log_e T + 1.875\,(10)^{-9}T^2\right]_{T_1}^{T_c}. \qquad (D)$$

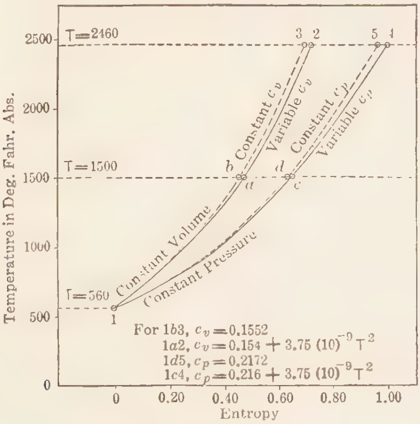

FIG. 39.

Hence, taking the intermediate temperature $T_c = 1500$, it is found that

$$\Delta\Phi_{1,c} = 0.6487.$$

The corresponding point is shown in Fig. 39.

Similarly, for the point d on the curve *1 5*,

$$\Delta\Phi_{1,\,d} = wc_p \log_e \frac{T_d}{T_1}; \quad \dots\dots\dots \quad (E)$$

and, for $T_d = 1500$, $\Delta\Phi_{1,\,d} = 0.641.$

(j) Figure 39 shows on the $T\phi$-diagram the results of the illustrative computations for the constant volume and constant pressure processes, with specific heats both variable and constant. The areas under the respective curves show quantitatively the relative values of the amount of heat added during the corresponding processes.

79. Isothermal Processes.—**(a)** Isothermal processes are those during which the temperature of the working substance remains constant. Such a process is shown in Figs. 40 and 41.

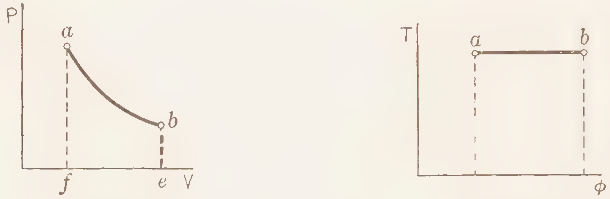

Fig. 40. Fig. 41.

Isothermal Process.

(b) It has been stated in Chapter VII that Boyle's law holds very closely for most of the "permanent" gases that are not cooled to very low temperatures. Hence, for such gases the isothermal process may be expressed by the equation, PV = *a constant*, which, on the PV-diagram, is represented by a rectangular hyperbola such as *ab* of Fig. 40.

Since

$$PV = P_aV_a = P_bV_b = \text{a const.},$$

then

$$\frac{P_a}{P_b} = \frac{V_b}{V_a} = r. \quad \dots\dots\dots \quad (78)$$

from Eq. (8),

$$Wk_{ab} = \int_{V_a}^{V_b} PdV = \int_{V_a}^{V_b} \frac{PV}{V} dV.$$

Or. since $PV = P_aV_a = const.$, and assuming P_a and V_a known,

$$Wk_{ab} = P_aV_a \int_{V_a}^{V_b} \frac{dV}{V} = P_aV_a \log_e \frac{V_b}{V_a}. \quad \dots \quad (79)$$

$$= P_aV_a \log_e \frac{P_a}{P_b} = P_aV_a \log_e r, \text{ (ft-lb.)}. \quad \dots \quad (80)$$

$$= wRT \log_e \frac{P_a}{P_b}, \text{ (ft-lb.).} \quad \ldots \ldots \quad (81)$$

$$= wRT \log_e \frac{V_b}{V_a} = wRT \log_e r, \text{ (ft-lb.).} \quad \ldots \quad (82)$$

$$= \text{area } abef, \text{ to scale, in Fig. 40.}$$

From Eq. (53*b*), since the temperature is constant,

$$\Delta I_{ab} = w \int_{T_a}^{T_b} c_v dT = 0. \quad \ldots \ldots \quad (83)$$

From the simple energy equation, Eq. (9*a*), and from Eq. (82),

$$Q_{ab} = \Delta I_{ab} + Wk_{ab} = 0 + \frac{wRT \log_e}{778}\left(\frac{V_b}{V_a}\right), \text{ B.t.u.}$$

$$= wART \log_e r, \text{ B.t.u.} \quad \ldots \ldots \quad (84)$$

(c) The last equation shows that during the isothermal expansion of any working substance whose internal energy depends upon its temperature, the heat supplied to the fluid must be exactly balanced by the work done by it during such an expansion. Or, if the gas is compressed isothermally, then heat must be abstracted from it at exactly the same rate as work is done upon it during such a compression.

(d) Note that Eq. (84) will always properly take care of the algebraic sign if a represents the initial state and b the final one for the particular "isotherm" under consideration. In this connection it should be remembered that $\log_e \frac{V_b}{V_a} = -\log_e \frac{V_a}{V_b}$.

(e) From Eq. (25), and knowing that T is constant, it follows that

$$\Delta\Phi_{ab} = \frac{1}{T}\int_a^b dQ = \frac{Q_{ab}}{T} = \frac{wART \log_e (V_b/V_a)}{T},$$

$$= wAR \log_e r. \quad \ldots \ldots \quad (85)$$

$$= \text{the distance } ab, \text{ to scale, in Fig. 41.}$$

(f)[3] For the entropy calculations of the previous problem, Sect. **78** (f), it is easy to obtain an additional check,—a procedure which is always desirable in any kind of engineering work. Since entropy depends only on the state of the substance, and not on paths, the gain in entropy between any two states may be computed by performing the integration $\int \frac{dQ}{T}$, along different paths. Thus, in this case, referring back to Fig. 39 it is obvious that $\Delta\phi_{1,4} = \Delta\phi_{1,2} + \Delta\phi_{2,4}$. But from Sect. **77** (f) $\Delta\Phi_{1,2} = 0.7164$; and from Eq. (85), for an isothermal,

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[3] This paragraph may be omitted in brief courses.

$$\Delta\Phi_{2,4} = wAR \log_e\left(\frac{V_4}{V_2}\right).$$

Then, since $V_2 = V_1$ and $P_1 = P_4$, it follows that $V_4/V_2 = T_4/T_1$. Hence,

$$\Delta\Phi_{2,4} = wAR \log_e \frac{T_4}{T_1} = \frac{3 \times 48.4 \times \log_e (2460/560)}{778} = 0.2760.$$

Therefore, $\Delta\Phi_{1,4} = 0.7164 + 0.2760 = 0.9924$, as before.

80. Polytropic Processes.[1] (**a**) Polytropic processes for any substance are defined as those which follow the equation

$$PV^n = \text{a constant}, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (86)$$

in which n is a constant. This is a very important equation to study, as the actual curves obtained from air compressors and internal combustion engines are of this type.

(**b**) The particular value that is given to the exponent n determines the nature of the expansion. Thus, if $n = 1$, the process becomes an isothermal one; if $n = 0$, then $PV^0 =$ a constant, or a constant pressure process results; if $n = \infty$, then $P^{\frac{1}{\infty}} V = a \cdot const.$, or the volume remains constant.

(**c**) In addition to Eq. (86), the gas is assumed to follow the characteristic equation of gases, viz., $PV = wRT$. Then, for any polytropic process, involving w pounds of gas, expanding or being compressed, from an initial state *1* to a final state *2*, it follows from Eq. (86) that $P_1V_1^n = P_2V_2^n$.

Hence

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n, \quad . \; . \; . \; . \; . \; . \; . \; . \quad (87)$$

or

$$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (88)$$

---

[1] For methods of solving exponential equations see Sect. **89**.

But, since $P_1 = wRT_1/V_1$ and $P_2 = wRT_2/V_2$, it follows that

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}\frac{V_2}{V_1},$$

or that

$$\left(\frac{V_2}{V_1}\right)^n = \frac{T_1}{T_2}\frac{V_2}{V_1}.$$

Therefore

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{V_2}{V_1}\right)^{n-1}, \qquad (89)$$

or

$$\frac{V_2}{V_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}. \qquad (90)$$

Then, combining Eqs. (88) and (90), there results

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{n-1}{n}}, \qquad (91)$$

or

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^{\frac{n}{n-1}}. \qquad (92)$$

(d) The *work done* by *any* kind of working substance expanding from *1* to *2* according to the law $PV^n = K$, must be

$$Wk_{1,2} = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{KdV}{V^n}. \qquad (a)$$

$$= K\int_{V_1}^{V_2} V^{-n}dV = K\left[\frac{V^{1-n}}{1-n}\right]_{V_1}^{V_2}. \qquad (b)$$

$$= K\left[\frac{V_2^{1-n} - V_1^{1-n}}{1-n}\right]. \qquad (c)$$

$$= \frac{P_2V_2^nV_2^{1-n} - P_1V_1^nV_1^{1-n}}{1-n}. \qquad (d)$$

$$= \frac{P_2V_2 - P_1V_1}{1-n} = \frac{P_1V_1 - P_2V_2}{n-1}, \text{ (ft-lb.)}. \qquad (93)$$

(e) Now, if the working substance is w lb. of a *gas that obeys the law* $PV = wRT$, then

$$Wk_{1,2} = \frac{wR}{n-1}(T_1 - T_2), \text{ (ft-lb.)}. \qquad (94)$$

(f) Eqs. (93) and (94) give the work done *by* the gas during the polytropic *expansion*. If the gas is compressed, the work will be negative, and the above equations will yield a negative result.

If a value of n is assigned, Eqs. (86), (87) and (88) can be used for drawing the PV-curve representing the process *1 2;* then, the area under this curve will represent this work.

(g) It is now desired to find the amount of heat that *must* be added to the gas in order that it may be made to follow the law $P_1V_1^n = P_2V_2^n$.

From Eqs. (9*a*), (53*b*) and (94) it follows that, *for gases in general,*

$$Q_{1,2} = \int_{T_1}^{T_2} wc_v dT + \frac{wAR(T_1 - T_2)}{(n - 1)}, \text{ (in B.t.u.)}. \quad . \quad . \quad (95)$$

If it can be assumed that c_v *is constant,* or if its average value is known between T_1 and T_2, this equation becomes

$$Q_{1,2} = wc_v(T_2 - T_1) + \frac{wAR(T_1 - T_2)}{n - 1}, \text{ B.t.u.}$$

But, from Eq. (49), $AR = (\gamma - 1)c_v$; hence,

$$Q_{1,2} = wc_v(T_2 - T_1) + \left(\frac{\gamma - 1}{n - 1}\right)wc_v(T_1 - T_2)$$

$$= w\left(\frac{n - \gamma}{n - 1}\right)c_v(T_2 - T_1), \text{ B.t.u.} \quad . \quad . \quad . \quad . \quad . \quad (96)$$

or

$$= w\left(\frac{\gamma - n}{n - 1}\right)c_v(T_1 - T_2), \text{ B.t.u.} \quad . \quad . \quad . \quad . \quad . \quad (97)$$

(h) *The gain in entropy* of the gas, between the states *1* and *2*, may be found from Eqs. (25) and (95).

Thus, *for gases in general,*

$$\Delta\Phi_{1,2} = \int_{T_1}^{T_2} \frac{dQ}{T} = w\int_{T_1}^{T_2} \frac{c_v dT}{T} - \left(\frac{wAR}{n - 1}\right)\int_{T_1}^{T_2} \frac{dT}{T}$$

$$= w\int_{T_1}^{T_2} \frac{c_v dT}{T} - \frac{wAR}{n - 1} \log_e \frac{T_2}{T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (98)$$

If c_v *is constant,* for the temperature range involved, then from Eq. (96), $dQ = w\left(\frac{n - \gamma}{n - 1}\right)c_v dT$. Hence, for this case,

$$\Delta\Phi_{1,2} = \int_{T_1}^{T_2} \frac{dQ}{T} = \int_{T_1}^{T_2} w\left(\frac{n - \gamma}{n - 1}\right)\frac{c_v dT}{T}$$

$$= w\left(\frac{n - \gamma}{n - 1}\right)c_v \log_e \frac{T_2}{T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (99)$$

(i)[5] *If the specific heat of the gas varies* according to Eq. (51), in which

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[5] This paragraph may be omitted in short courses.

$c_v = M + BT + CT_2$, then Eq. (95), for the *heat added* during a process, becomes,

$$Q_{1,\,2} = w\int_{T_1}^{T_2}(M + BT + CT^2)dT + \frac{wAR(T_1 - T_2)}{n - 1},$$

$$= w\left[M(T_2 - T_1) + \frac{B}{2}(T_2{}^2 - T_1{}^2) + \frac{C}{3}(T_2{}^3 - T_1{}^3)\right]$$

$$+ \frac{wAR(T_1 - T_2)}{n - 1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

Similarly, Eq. (98), for the *gain in entropy* during the process, becomes

$$\Delta\Phi_{1,\,2} = w\int_{T_1}^{T_2}\left(\frac{M + BT + CT^2}{T}\right)dT - \frac{wAR}{n - 1}\log_e\frac{T_2}{T_1}$$

$$= w\left[M\log_e\frac{T_2}{T_1} + B(T_2 - T_1) + \frac{C}{2}(T_2{}^2 - T_1{}^2)\right]$$

$$- \frac{wAR\log_e(T_2/T_1)}{n - 1}. \quad (101)$$

81. Adiabatic Processes in General.—(a) The word *adiabatic* means *no transfer of heat,* hence any process in which there is no heat transferred to or from the working substance is very properly called an *adiabatic process,* or more briefly, an *adiabatic.* Many authors, especially in England, prefer to limit the application of the term to those cases in which the processes are *reversible* ones,—i.e., to those without fluid friction or turbulence,—which also conform to the basic condition already mentioned. Many others, including the authors of this text, believe that the most satisfactory definition of the term is as follows: *An adiabatic is any process in which no heat is transferred to or from the working substance from any outside source.* From this definition it follows that there are both *reversible* and *irreversible* adiabatics.

(b) A *reversible adiabatic* is a constant entropy process, or, more briefly, an "*isentropic.*"[6] This follows from the definition of entropy combined with the definition of adiabatic. From the former, Sect. **58**, $\Delta\phi_{ab} = \int_a^b dQ/T$, with dQ taken along any *reversible* path connecting a and b. If it is now stipulated that the process ab is to be a reversible *adiabatic*, it follows that $dQ = 0$ and hence that this integral becomes zero; thus the entropy does not change during the process.

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[6] There might be imagined an irreversible process in which the turbulence is just sufficient to keep the entropy constant although there is a small transfer of heat from the fluid. Such a process is extremely improbable, and in this text the term *isentropic* will have the usual meaning, i.e. a *reversible adiabatic.*

As will be seen later, of all the processes, the reversible adiabatics are the ones during which the greatest amount of internal energy is made available for doing useful external work. Note that the term "*isentropic*" is a *general* one for a *reversible adiabatic,* and, therefore, may be applied equally well with liquids, vapors and gases. Its chief advantage is that there can be no confusion as to its meaning, which may not always be the case when the term adiabatic is encountered.

(c) During an *irreversible adiabatic,* as its name implies, there is no transfer of heat from an *outside* source to the working substance, or vice versa, but the process is one which proceeds with turbulence, or fluid friction, in consequence of which there is *heat developed within the working substance from the mechanical action of the fluid itself,* and hence the process is irreversible. From the law of the conservation of energy, it must follow that with such an irreversible adiabatic the fluid must contain within it a greater amount of energy at the end of the process than it would have had with a reversible adiabatic for the same pressure range, and consequently less energy must have been made available for doing external work. As a result of the increased internal energy of the fluid, its *specific volume,* and *entropy are greater* than they would have been if the process had been isentropic between the same pressure limits. Also the terminal temperature for the irreversible adiabatic will be higher than it would be for the corresponding reversible one, provided the substance is a gas.[7] Since both the internal energy and the specific volume are larger, it must follow, from the definition, that the *heat content is also greater* at the end of an irreversible adiabatic than for the corresponding isentropic.

82. Isentropic Processes of Ideal Gases.—(a) Consider w pounds of an ideal gas expanding, or being compressed, isentropically from an initial state *1* to a final state *2*. It is desired to find the relation existing between P, V and T and also the amount of work done during such an expansion or compression. Assuming that an isentropic process of an ideal gas must be one form of the polytropic, whose equation is PV^n = *a constant,* it only remains to find the value of n that will cause $Q_{1,2}$ to be zero. Since neither w, γ, c_v nor $\Delta T_{1,2}$ can be zero, from Eq. (97) it follows that n must equal γ in order to have this equation give $Q_{1,2} = 0$. Hence, γ must be the value of the exponent for the isentropic process. Using this exponent, the relation between the pressure and the volume for a *reversible adiabatic* expansion, or compression, *of an ideal gas* is given by the equation,

$$P_1V_1^\gamma = P_2V_2^\gamma, \quad \text{or} \quad PV^\gamma = a\ constant. \quad . \quad . \quad . \quad (102)$$

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[7] For vapors this statement requires certain modifications as will be seen later.

From this

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^\gamma, \quad \ldots \ldots \quad (103)$$

and

$$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}. \quad \ldots \ldots \quad (104)$$

Since the gas must also conform to the characteristic equation, $PV = wRT$, it follows that

$$\frac{T_1}{T_2} = \frac{P_1V_1}{P_2V_2}.$$

Substituting from Eq. (103) gives

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}; \quad \ldots \ldots \quad (105)$$

from which

$$\frac{V_2}{V_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}. \quad \ldots \ldots \quad (106)$$

Combining Eqs. (104) and (106) gives

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}, \quad \ldots \ldots \quad (107)$$

or

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}. \quad \ldots \ldots \quad (108)$$

It will be observed that Eqs. (102) to (108) are the same as Eqs. (86) to (92), except for the substitution of γ for n.

(b) A second method of deriving Eq. (102) is as follows: The simple energy equation, Eq. (9*a*), may be rewritten

$$dQ = dI + APdV. \quad \ldots \ldots \quad (a)$$

But $dI = wc_vdT$; $AP = w(\gamma - 1)c_vT/V$, since $P = wRT/V$ and $R = (\gamma - 1)c_v/A$; and, for an adiabatic, $dQ = 0$. Hence Eq. (*a*) becomes $0 = wc_vdT + w(\gamma - 1)c_vT\,dV/V$, or

$$-\frac{c_vdT}{T} = (\gamma - 1)c_v\frac{dV}{V}. \quad \ldots \ldots \quad (b)$$

Then

$$-\int_{T_1}^{T_2}\frac{dT}{T} = \int_{V_1}^{V_2}(\gamma - 1)\frac{dV}{V}, \quad \ldots \ldots \quad (c)$$

$$-\log_e\frac{T_2}{T_1} = (\gamma - 1)\log_e\frac{V_2}{V_1}, \quad \ldots \ldots \quad (d)$$

and

$$\log_e \frac{T_1}{T_2} = (\gamma - 1) \log_e \frac{V_2}{V_1}. \quad . \quad . \quad . \quad . \quad . \quad (e)$$

Thus, as in Eq. (105),

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (f)$$

Substituting this in $T_1/T_2 = (P_1V_1)/(P_2V_2)$ and reducing gives

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}, \text{ or } PV^{\gamma} = \text{constant, as in Eq. (102)}. \quad . \quad . \quad . \quad (g)$$

(c) *The work done by any working substance during an isentropic expansion is equal to the loss of internal energy of this substance.* This statement must be true for the following reasons: The simple energy equation, $dQ = dI + APdV$, holds for any working substance, and, since $dQ = 0$ for any isentropic, hence

$$Wk_{1,2} = \int_{V_1}^{V_2} APdV = -\int_{I_1}^{I_2} dI = I_1 - I_2. \quad . \quad . \quad (109)$$

Note that the result will be in the same units as those in which I_1 and I_2 are expressed, and that if the process is a *compression*, I_2 will be greater than I_1, thus giving a negative value of the *work done by the fluid.* Particular attention is called to the fact that Eq. (109) holds for *any vapor* as well as for *any gas.* However, the method of obtaining I_1 and I_2 for vapors will not be the same as for gases, as will be seen later.

(d) For an ideal gas expanding isentropically, the work done becomes, from Eqs. (109) and (43),

$$Wk_{1,2} = wc_v(T_1 - T_2), \text{ in B.t.u.} \quad . \quad . \quad . \quad . \quad . \quad (110)$$

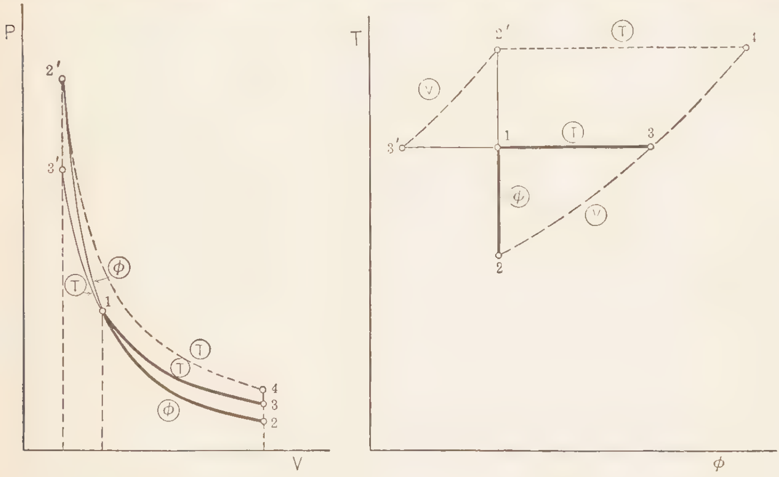
This same equation might have been obtained from Eq. (91) by substituting γ for n and reducing the result to B.t.u. For the case of gases with variable specific heats see Sect. **83**.

(e) The PV- and $T\Phi$-diagrams representing the *isentropic* and *isothermal expansions* of an ideal gas are shown in Fig. 42. The curve *1 2* represents the isentropic and *1 3* is the isothermal, drawn for the same ratio of expansion, or in other words, so that $V_2/V_1 = V_3/V_1$. The curves *1 2'* and *1 3'* represent the corresponding *compression* processes starting at the same initial state *1*, and with $V_2' = V_3'$. Note that the temperature falls during the constant entropy expansion and rises during the compression. Also observe that, on the $T\phi$-diagram, the constant volume lines *2 3* and *2'3'* are not parallel curves, because the temperatures involved are different. If the constant volume curve *2 3* should be extended until a state *4* is reached such that $T_4 = T_2'$, then the constant volume curve *3 4* would be parallel with the curve *3'2'*. The isothermal *2'4* is also shown on both diagrams.

83.[8] Isentropic Processes of Gases with Variable Specific Heats.—

(a) The previous section shows that, for ideal gases, the reversible adiabatic involves only rather simple exponential equations. These equations may also be used for real gases if the specific heats are nearly constant over the range of temperature encountered, such as occurs with air in the usual compressed air problem, or if the mean values of the specific heats can be determined for the temperature range involved.

(b) With internal combustion motors, and in many other cases, the temperatures are high and the fluid is a complex gas mixture. Both of these factors cause the specific heats to be very much larger at the high temperatures than at the relatively low ones. To make accurate calculations of the isentropic process for such cases involves somewhat complex relations between P, V and T, since the value of the exponent γ is not constant unless c_p and c_v do not change. However, although c_p and c_v are variable, their difference, AR, is constant, as was shown in Sect. 72; hence the equation $PV = wRT$ is still true. Also the equations $dQ = dI + APdV$, and $dQ = 0$ for adiabatic, and $dI = wc_vdT$ hold. Let c_v be represented by the general type of equation, $c_v = M + BT + CT^2$; then, for the isentropic, since $P = wRT/V$,



(a) (b)

FIG. 42.

$$-wc_vdT = APdV = \frac{wARTdV}{V},$$

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[8] In brief courses Sects. **83** to **88**, inclusive, may be omitted, with the possible exception of Sects. **84, 85** and **87**.

or
$$\frac{c_v dT}{T} = -\frac{ARdV}{V}.$$

Consequently, with c_v as given,

$$(M + BT + CT^2)\frac{dT}{T} = -AR\frac{dV}{V}.$$

Now if *1* represents the initial state of the gas, then for any secondary state during an isentropic expansion, or compression, it follows, by integration, that

$$M \log_e\frac{T}{T_1} + B(T - T_1) + \frac{C}{2}(T^2 - T_1^2) = -AR\log_e\frac{V}{V_1}. \quad (111)^*$$

Dividing each term of this equation by M, and by 2.3026 to reduce to the common base of logarithms, there results

$$\log\frac{T}{T_1} + \frac{B(T - T_1)}{2.3026M} + \frac{C(T^2 - T_1^2)}{4.6052M} = \frac{-AR}{M}\log\frac{V}{V_1}. \quad (112)^*$$

These equations between **temperature** and **volume**, being transcendental, cannot be solved directly if one of the temperatures is unknown, as is usually the case. Assuming T_1 to be given, one convenient method of solution is to choose different values of T and thus determine the corresponding ratios V/V_1. Then draw a curve between temperature and volume, or the volume ratio. This curve may then be read to find the final temperature corresponding to the final volume. This may be called the "Curve Plotting Method."

An alternative method is as follows: With T_1 known, and terminal ratio V_2/V_1 given, one can take a trial temperature T_2, substitute it in the equation and determine whether the resulting value of V_2/V_1 equals the given one. If there is a discrepancy, another temperature can be tried, and this procedure can be continued until as close an agreement as desired is obtained. This is the "Cut and Try Method," or "Trial and Error Method."

(c) To find the relation between **temperature** and **pressure**, it is now only necessary to substitute for V_1/V, in Eq. (111), its equivalent $\frac{T_1P}{TP_1}$, and there results another transcendental equation,

$$M\log_e\frac{T}{T_1} + B(T - T_1) + \frac{C}{2}(T^2 - T_1^2) = AR\log_e\frac{T_1}{T} + AR\log_e\frac{P}{P_1},$$

or

$$(M + AR)\log_e\frac{T}{T_1} + B(T - T_1) + \frac{C}{2}(T^2 - T_1^2) = AR\log_e\frac{P}{P_1}. \quad (113)$$

* With CO_2 in the gas the values of M, B and C change at $T = 2900$.

Reducing Eq. (113) to the form involving only the common base of logarithms, and dividing through by $(M + AR)$, there results

$$\log \frac{T}{T_1} + \frac{B(T - T_1)}{2.3026(M + AR)} + \frac{C(T^2 - T_1^2)}{4.6052(M + AR)} = \frac{AR}{M + AR} \log \frac{P}{P_1}, \quad (114)^*$$

This equation can be solved in the same manner as outlined in the previous page. It may be convenient to observe that the constant M' that occurs in Eq. (52) may be substituted for $M + AR$.

It is worth noting that, if B and C are each equal to zero, then $M = c_v =$ a constant; hence $\frac{AR}{M}$ becomes equal to $\gamma - 1$, and $\frac{AR}{M + AR}$ equals $\frac{\gamma - 1}{\gamma}$. Consequently, for $c_v = a$ *const.*, Eq. (112) reduces to $\frac{T}{T_1} = \left(\frac{V_1}{V}\right)^{\gamma - 1}$, and Eq. (114) becomes $\frac{T}{T_1} = \left(\frac{P}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$, which check the ideal gas equations, Eqs. (105) and (107).

(d) Since both of the specific heats are functions of temperature, the isentropic process for gases with variable specific heats cannot be expressed in terms of pressure and volume alone. However, by using the basic relation, $PV = wRT$, combined with the results of Eqs. (112) or (114), plotted in the form of a curve, the *pressure and volume relationship* may be found as needed for any particular case.

(e) The *work done* during an isentropic expansion, by a gas with variable specific heat, may be found from the facts that $Wk_{1,2} = I_1 - I_2$ as in Eq. (109), and that $I_2 - I_1 = w\int_{T_1}^{T_2} c_v dT$, in which, $c_v = M + BT + CT^2$. Thus considering *1* to be the initial state and *2* the final one,

$$Wk_{1,2} = -\int_{I_1}^{I_2} dI = -\int_{T_1}^{T_2} wc_v dT = -w\int_{T_1}^{T_2} (M + BT + CT^2)dT$$

$$= -w\left[M(T_2 - T_1) + \frac{B}{2}(T_2^2 - T_1^2) + \frac{C}{3}(T_2^3 - T_1^3)\right]. \quad (115)^*$$

Or, if the average value of c_v, or of the variable exponent, γ, can be determined, then the work done by the gas may also be obtained by using either of the following equations, corresponding to Eqs. (93) and (94).

$$Wk_{1,2} = \int_{V_1}^{V_2} PdV = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{wR(T_1 - T_2)}{\gamma - 1}, \text{ ft-lb.} \quad (116a)$$

$$= w\,\bar{c}_v\,(T_1 - T_2), \text{ B.t.u.} \quad (116b)$$

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* See footnote on preceding page.

(f) As an illustration of the use of preceding equations, consider their application to the products of combustion in an internal combustion motor in which there is an absolute temperature of 4500° F. at the beginning of expansion. Suppose it is known that $R = 54.0$ for the mixture of gases in the cylinder, and that $c_v = M + BT + CT^2$, with $M = 0.2021$, $B = 5.64\ (10)^{-7}$, and $C = 5.065\ (10)^{-9}$. Let it be desired to find the work done per pound of gas during an isentropic expansion in which the ratio of expansion, $\left(\frac{V_2}{V_1}\right)$, is 5, and also to determine the temperature curve throughout the expansion.

The temperatures, T, corresponding to different volume ratios (V/V_1), must be obtained first, and for that purpose it is convenient to use Eq. (112) rewritten as follows:

$$\log \frac{T}{T_1} + K_1(T - T_1) + K_2(T^2 - T_1^2) = K_3 \log \frac{V}{V_1}, \quad . \quad . \quad . \quad (117)\text{f}$$

in which $K_1 = \dfrac{B}{2.3026M}$, $K_2 = \dfrac{C}{4.6052M}$, and $K_3 = \dfrac{AR}{M}$.

Then, for this particular case,

$$K_1 = \frac{5.64\ (10)^{-7}}{(2.3026 \times 0.2021)} = 1.211\ (10)^{-6},$$

$$K_2 = \frac{5.065(10)^{-9}}{(4.6052 \times 0.2021)} = 5.45\ (10)^{-9},$$

$$K_3 = \frac{54}{(778 \times 0.2021)} = 0.3435.$$

Hence the set of values as given in Table IV are obtained.

From these results, the curve shown in Fig. 43 is readily plotted between the coordinates of absolute temperature and the ratio of expansion, and from this curve the temperature is found to be 2992° F., abs. for a ratio of expansion of 5.

(g) Had it been desired to find only this one temperature, the entire curve would not necessarily have been drawn, or the extra calculations made. Instead, the average value of γ might have been estimated for the given ratio of expansion, and the corresponding value of T'_2 guessed at or calculated by the relation

$$T'_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma - 1}.$$

Then, by taking two temperatures, one above and the other below this value of T'_2, and substituting in Eq. (117), the values of the corresponding ratios of volumes can be determined. A straight line joining these two points will cross the ordinate for volume ratio equal to 5 at a point which will give a close approximation to the true value of T_2.

(h) In order to assist in making estimates of γ for such cases, and also to show how it varies during the expansion period, its average value was calculated and is

TABLE IV

CALCULATIONS FOR THE TV-RELATION DURING ISENTROPIC EXPANSION OF A GAS WITH VARIABLE SPECIFIC HEATS

Initial Temperature = 4500° F. Abs.

(1) T_1	(2) T	(3) $\frac{T_1}{T}$	(4) $\log \frac{T}{T_1}$	(5) $T-T_1$	(6) $K_1(T-T_1)$	(7) $T^2-T_1^2$	(8) $K_2(T^2-T_1^2)$	(9) $\frac{(4)+(6)+(8)}{-K_3} = \log \frac{V}{V_1}$	(10) $\frac{V}{V_1}$	(11) Average Value of γ Such that $\frac{T_1}{T} = \left(\frac{V}{V_1}\right)^{\gamma-1}$
4500	4200	1.07	−0.030	−300	$-3.633\ (10)^{-4}$	$-2.61\ (10)^6$	−0.01423	0.130	1.35	1.225
	3900	1.153	−0.062	−600	$-7.266\ (10)^{-4}$	$-5.04\ (10)^6$	−0.02745	0.2625	1.83	1.236
	3600	1.25	−0.097	−900	$-1.0899\ (10)^{-3}$	$-7.29\ (10)^6$	−0.0397	0.401	2.52	1.241
	3300	1.363	−0.1345	−1200	$-1.4532\ (10)^{-3}$	$-9.36\ (10)^6$	−0.05105	0.545	3.51	1.246
	3000	1.50	−0.176	−1500	$-1.8165\ (10)^{-3}$	$-1.125\ (10)^7$	−0.0613	0.696	4.96	1.253

given in column 11 of the Table, and the corresponding curve is shown in Fig. 43. These average values of γ are seen to increase rapidly at the high temperatures, but for expansion ratios near 5 its value is usually not far from 1.25 for the products of combustion in high speed internal combustion motors.

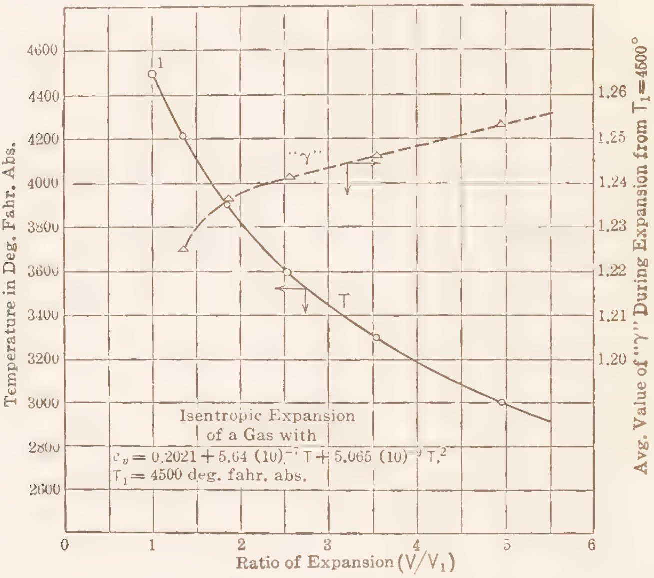

FIG. 43.—Variation of T and γ with (V/V_1).

(i) Knowing that $T_1 = 4500$ and $T_2 = 2992$, it only remains to compute the work done per pound of working substance during this expansion. From Eq. (115)

$$Wk_{1,2} = -w\left[M(T_2 - T_1) + \frac{B}{2}(T_2{}^2 - T_1{}^2) + \frac{C}{3}(T_2{}^3 - T_1{}^3)\right], \text{ in B.t.u.}$$

$$= -1\left\{0.2021(2992 - 4500) + \frac{5.64(10)^{-7}[(2992)^2 - (4500)^2]}{2}\right.$$

$$\left. + \frac{5.065}{3}[(2992)^3 - (4500)^3]\right\} = 415.4 \text{ B.t.u.}$$

As a check on this value, Eq. (116a) may next be used, with the average value of $\gamma = 1.253$, for a ratio of expansion of 5; thus

$$Wk_{1,2} = \frac{wR(T_1 - T_2)}{\gamma - 1} = \frac{1 \times 54(4500 - 2992)}{0.253} = 322{,}000 \text{ ft-lb.},$$

which is equal to 415 B.t.u., which checks the result obtained by the other method.

(j) Next, suppose it is desired to plot the *pressure-volume curve* representing the sentropic expansion of the preceding problem. Instead of assigning any numerical value for the initial pressure and volume, it will serve a more useful purpose to determine the curve in terms of p_1 and the expansion ratio. For any point on this curve, for which the expansion ratio is known, the pressure may be determined from

$$p = p_1\left(\frac{T}{T_1}\right)\left(\frac{V_1}{V}\right).$$

Thus, using the simultaneous values of V_1/V and T/T_1 from Table IV, the value of p in terms of p_1 can be found for this equation for these expansion ratios. The results are plotted in Fig. 44.

It should be carefully noted that this curve is drawn for a definite initial temperature, 4500°, and cannot therefore be used to determine *accurately* the pressure volume relationship for this gas expanding isentropically from any other initial temperature However, the results will be found useful in many cases, and will give very close approximations to the true values with this gas even though the initial temperatures are appreciably removed from 4500° F., abs.

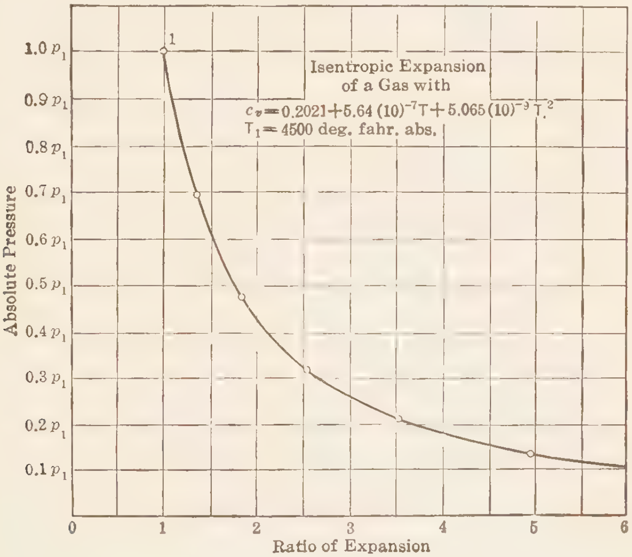

FIG. 44.

84. Irreversible Adiabatics.—(a) Whenever a working substance expands with turbulence and fluid friction it undergoes an irreversible process, as already shown in Sect. **50.** Hence, if such a process takes place in a closed vessel that is thermally insulated from other bodies, it becomes an irreversible adiabatic. Turbulence is always produced by permitting *uncontrolled or "free" expansion* of any fluid. This term means allowing the fluid to pass from any pressure to a lower one without attempting to utilize its available energy by means of a piston or nozzle. One illustration was discussed in Sect. **50**(**d**).

(**b**) Suppose an *ideal gas* is allowed to expand adiabatically from some known initial state to a lower pressure, and it is desired to find the difference in the final states of this gas if this expansion is perfectly controlled in one case and free in the other. Thus, two adiabatic expansions are involved, one reversible and the other irreversible to the highest degree possible. Let the initial state be represented by a, then P_a, V_a, and T_a are all known. Let the final states be represented by b for the reversible process and by c for the irreversible one.

For the *isentropic* process, from Sect. **82**, it follows that

$$V_b = V_a\left(\frac{p_a}{p_b}\right)^{\frac{1}{\gamma}}, \quad T_b = T_a\left(\frac{p_b}{p_a}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{and} \quad \phi_b = \phi_a.$$

Hence, the curve ab may be readily drawn on the PV- and $T\phi$-diagrams. It is shown in Fig. 45 for a ratio of pressures such that $p_a = 10p_b$.

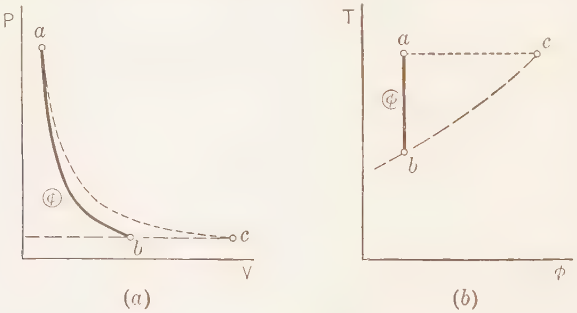

Fig. 45.—Isentropic and Free Expansions.

For the *free expansion*, it is known that the final pressure $p_c = p_b$, and from Joule's experiment, Sect. **69**, it follows that the temperature after the turbulence has subsided, at the end of a free adiabatic expansion of an ideal gas, is the same as initially, or in other words $T_c = T_a$. Hence the state c may be located, as shown in both diagrams, by the intersection of a constant pressure line through b with an isothermal through a. These lines bc and ac, shown dotted, are simply construction lines and do not represent the processes actually occurring, such as does the solid line ab for the reversible process. For the free expansion, the actual process cannot be shown on either diagram. However, the final state c, after equilibrium has been restored, can be determined, since, in general, for an ideal gas $\frac{P_aV_a}{T_a} = \frac{P_cV_c}{T_c}$; and since, for this case, $T_c = T_a$, hence $V_c = V_a(p_a/p_c)$.

(c) **The change in entropy during the irreversible adiabatic** *ac* may be determined by evaluating the integral $\int_a^c \frac{dQ}{T}$ along *any reversible* path connecting the points *a* and *c*. The gas in state *c* is in exactly the same condition which it would have been, had it expanded along a reversible isothermal from *a*, or had it been heated at constant pressure from *b*, although the heat required, the external work done, and the apparatus involved, are very different. Hence, from Eq. (85),

$$\Delta\Phi_{ac} = wAR \log_e \left(\frac{V_c}{V_a}\right) = wAR \log_e \left(\frac{p_a}{p_c}\right). \quad . \quad . \quad . \quad (118)$$

Then again, this state *c* might have been reached by expanding along the reversible paths *ab* and *bc*.

Then, from Eq. (77),

$$\Delta\Phi_{ac} = \Delta\Phi_{ab} + \Delta\Phi_{bc} = 0 + wc_p \log_e \frac{T_c}{T_b}.$$

But, as $T_c = T_a$ and $T_b = T_a\left(\frac{p_b}{p_a}\right)^{\frac{\gamma-1}{\gamma}}$, this equation becomes

$$\Delta\Phi_{ac} = wc_p \log_e \left(\frac{p_a}{p_c}\right)^{\frac{\gamma-1}{\gamma}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (119)$$

and, since $\left(\frac{\gamma-1}{\gamma}\right)c_p = AR$, it reduces to $\Delta\Phi_{ac} = wAR \log_e \frac{p_a}{p_c}$, as before. Any number of reversible paths might be chosen such that by following them the gas would be brought to the state *c* from *a*, and the value of $\int_a^c \frac{dQ}{T}$ will be found to give the same result, as that shown by Eq. (118) or (119).

(d) It is important to note that the free adiabatic expansion of an ideal gas gives the same final state as a reversible isothermal having the same initial state and the same final pressure; but this free expansion does no useful work and has a large increase in entropy, although no heat was added from an outside source.

Between these two extreme limits of the adiabatic expansion of a gas, the free and the isentropic, there are many actual expansions in engineering processes which approximate adiabatic conditions. Sometimes the turbulence is very slight, and consequently the entropy increases only a small amount, so the final state would be near *b* on the constant pressure curve *bc* of Fig. 45. On the other hand the degree of turbulence may be considerable, yet appreciably less than the maximum possible, hence the final state would be closer to *c* on the line *bc*. From

Fig. 45 observe how the final temperature and volume both become larger as the entropy increases for a given final pressure, such as *bc*.

85. Throttling or Wire Drawing in General.—(a) Whenever a fluid passes through a restricted opening, so that there is an appreciable reduction in its pressure, and much turbulence is produced, the process is commonly called "*throttling*" or "*wire drawing*." The usual conditions are such that the flow is steady, and for such cases the general energy equation, Eq. (21), and notation given in Sect. **44** apply. Furthermore, since the throttling usually takes place through a valve or an orifice in a thin plate, the amount of heat that may be transferred to or from the fluid is very small, and the process is, therefore, nearly, if not actually, adiabatic. The two sections, *1* and *2*, are so close together usually that the altitude z_1 may be considered equal to z_2. Since there is no possibility of delivering mechanical energy by any throttling device, $E_m = 0$; and, assuming the process to be adiabatic, then $E_h = 0$ and $R_r = 0$. Hence, for the adiabatic steady flow throttling process, Eq. (21) becomes

$$0 = H_2 - H_1 + \frac{w(v_2^2 - v_1^2)}{50{,}000},$$

or

$$H_1 - H_2 = \frac{w(v_2^2 - v_1^2)}{50{,}000}. \quad . \quad . \quad . \quad . \quad . \quad (120)$$

(b) Now, if the velocities on each side of the restriction are equal, or $v_1 = v_2$, then

$$H_1 = H_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (121)$$

This last equation is a general one for *any* kind of a fluid,—liquid, vapor or gas,—provided that the conditions of steady flow are maintained and also that the throttling is adiabatic with $v_1 = v_2$ and $z_1 = z_2$. Instead of giving all of these conditions, it is quite common, when dealing with gases and vapors, merely to say that a *throttling* process is one in which the *heat content* is constant; but it should be noted that the velocities may *not* always be equal, and that the radiation loss may need to be included, in which case the steady flow throttling process is not exactly one of constant heat content.

(c) Consider the case of an *ideal gas* being throttled under the conditions specified for Eq. (121). Then, $H_1 - H_2 = 0$. But, $H_1 = I_1 + AP_1V_1$ and $H_2 = I_2 + AP_2V_2$, and for an ideal gas, $I_1 - I_2 = wc_v(T_1 - T_2)$; hence

$$H_1 - H_2 = wc_v(T_1 - T_2) + A(P_1V_1 - P_2V_2) = 0,$$

$$= wc_v(T_1 - T_2) + wAR(T_1 - T_2) = 0.$$

Obviously, the only condition that will satisfy this equation, since w, c_v, A and R cannot be zero, is that $T_1 = T_2$.

In other words, the temperature of an ideal gas must be constant if its heat content remains constant. Hence, by measuring the decrease in temperature of any real gas for a given drop in pressure due to throttling adiabatically, that gas may be investigated, under various pressures and temperatures, to find its departure from this ideal gas law.

86. The Joule-Thomson Effect.—(a) In 1852, Joule and Lord Kelvin (Sir William Thomson) substituted a porous plug for the throttling valve, and, by insulating this plug carefully, they obtained a very close approximation to true adiabatic conditions. They tested a large number of gases by forcing them through this plug and measuring very accurately the resulting changes in temperature and pressure. They found that Joule's law for ideal gases, viz. that the change in internal energy of a given quantity of heat-insulated gas depends only on the change in temperature, and is independent of its pressure or volume,—does not hold for actual gases since a change of temperature does take place. The decrease in temperature per unit fall in pressure, with the heat content remaining constant, or the value of $\left(\frac{dT}{dp}\right)_H = \mu$, is a measure of the departure from the ideal, and is called the Joule-Thomson Coefficient or "cooling effect." With perfect gases it is zero. With real gases and vapors this coefficient serves not only as the indication of the variation from the ideal but also as a measure of the cooling that may be obtained during adiabatic throttling, which is important in connection with refrigeration and with the liquefaction of gases. Further, this coefficient is very valuable in determining the properties of gases and especially those of vapors, such as steam and ammonia.

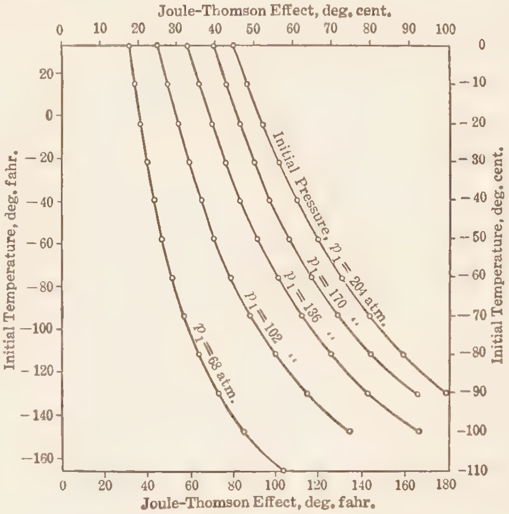

FIG. 46.—Joule-Thomson Effect with Air Expanding to Atmospheric Pressure.

(b) This classical experiment has been repeated by many other investigators on various gases and vapors. In general, the higher the temperature and the lower the pressure the less is the Joule-Thomson coefficient, or cooling effect, -or in other words, the smaller the value of μ the more nearly perfect is the behavior of the gas. As an illustration, the experimental results of Bradley and Hale,[9] obtained from air at various initial temperatures and pressures, are shown in Fig. 46.

87. Velocity through Nozzles.—(a) Consider that a fluid flows, by virtue of a drop in pressure, through a properly shaped nozzle, and does so without turbulence or friction and without any transfer of heat to or from the fluid. Such a nozzle would be called an ideal one, because in the actual case these losses cannot be reduced entirely to zero, even though they are often very small. Then, for the steady flow of a fluid through an ideal nozzle, with $z_1 = z_2$, Eq. (21) becomes

$$\frac{w(v_2^2 - v_1^2)}{2g \times 778} = H_1 - H_2 = w(h_1 - h_2),$$

or

$$v_2 = \sqrt{50{,}000\,(h_1 - h_2) + v_1^2}. \quad . \quad . \quad . \quad (122)$$

When v_1 is extremely small compared with v_2, as frequently occurs, it may be neglected, in which case,

$$v_2 = 223.7\sqrt{h_1 - h_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (123)$$

(b) For the nozzle without friction, turbulence or heat absorption, the expansion must be isentropic, hence, for the particular fluid involved, the value of $(h_1 - h_2)$ is calculated for isentropic expansion from a known initial state to some given secondary pressure, volume, or temperature.

(c) For **ideal gases** it was shown in Sect. **74** that the difference in heat content per pound of gas is $h_1 - h_2 = c_p(T_1 - T_2)$; hence, for such cases, with v_1 negligible,

$$v_2 = 223.7\sqrt{c_p(T_1 - T_2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (124)$$

But, for isentropic expansion, $T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$. Hence

$$v_2 = 223.7\sqrt{c_pT_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}. \quad . \quad . \quad . \quad . \quad (125a)$$

Thus, since $T_1 = P_1\overline{V}_1/R$ and $c_p/R = \gamma/[(\gamma - 1)778]$,

$$v_2 = \sqrt{2g\frac{\gamma}{\gamma - 1}P_1\overline{V}_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}. \quad . \quad . \quad (125b)$$

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[9] Physical Review, Vol. 29, 1909.

(d) For gases with variable specific heats the value of $h_1 - h_2 = (H_1 - H_2)/w$ may be obtained from Eq. (56) after finding T_2.

(e) **The probable velocities from actual nozzles** may be found by applying a proper " nozzle coefficient " to the velocity obtained by the foregoing equations. These coefficients depend on many factors, especially when vapors are involved, and will be discussed later in dealing with the flow of steam and in the further consideration of nozzles.* For air the coefficients are very high,—often approaching 99 per cent. If the value of the heat content, h_3, of the fluid leaving the actual nozzle can be obtained in any way, then from Eq. (21), with $z_1 = z_2$, $E_m = 0$, $E_h = 0$, and $R_c = 0$, it is apparent that

$$v_3 = \sqrt{50{,}000\ (h_1 - h_3) + v^2{}_1} = \text{coeff.}\ \sqrt{50{,}000\ (h_1 - h_2) + v_1{}^2}. \quad (126)$$

88. A General Method of Determining the Change of Entropy of Gases.—(a) Consider the initial state of a gas to be fixed by its pressure volume and temperature, say by p_1, V_1 and T_1. Then knowing the kind of a gas, its constant R may be determined, and consequently the weight, w, can be found. Let this amount of gas undergo any number of processes, some reversible and some irreversible, until a new state, 2, is reached. After equilibrium is established the values of p_2, and T_2 may be determined experimentally. If it is desired to compute the gain in entropy, $\Delta\phi_{1,2}$, it may be done as follows:

In general, $$dQ = dI + APdV = wc_vdT + wART\frac{dV}{V}.$$

Hence, $$\Delta\Phi_{1,2} = \int_1^2 \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{wc_vdT}{T} + \int_{V_1}^{V_2} wAR\frac{dV}{V}$$

$$= w\int_{T_1}^{T_2} \frac{c_vdT}{T} + wAR \log_e \frac{V_2}{V_1}. \quad . \quad . \quad . \quad . \quad (127)$$

which can be evaluated when c_v is known.

(b) *If c_v varies* according to the equation $c_v = M + BT + CT^2$, then Eq. (127) becomes

$$\Delta\Phi_{1,2} = w\Big[M \log_e \frac{T_2}{T_1} + B(T_2 - T_1) + \frac{C}{2}(T_2{}^2 - T_1{}^2) + AR \log_e \frac{V_2}{V_1}\Big]. \quad . \quad (128)$$

Or, if the *mean* value of the specific heat over the temperature range involved is known, Eq. (129), in the next paragraph, may be used for a very close approximation.

(c) For *ideal gases*, $c_v = a\ const.$, $\frac{P_1V_1}{P_2V_2} = \frac{T_1}{T_2}$, and $AR = (\gamma - 1)c_v$.

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* See Part II.

Then, $$\Delta\phi_{1,2} = wc_v \log_e \frac{T_2}{T_1} + w(\gamma - 1)c_v \log_e \frac{V_2}{V_1}. \quad . \quad . \quad (129a)$$

$$= wc_v \log_e \left(\frac{T_2}{T_1}\right)\left(\frac{V_2}{V_1}\right)^{\gamma-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (129b)$$

$$= wc_v \log_e \left(\frac{p_2}{p_1}\right)\left(\frac{V_2}{V_1}\right)^{\gamma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (129c)$$

$$= wc_p \log_e \left(\frac{T_2}{T_1}\right)\left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (129d)$$

Hence it follows that, for any gas in which c_v is constant, any one of these four forms of equations permits the ready calculation of the gain in entropy between the two states of the gas, provided the ratios, p_2/p_1, T_2/T_1 and V_2/V_1 are known. Many other forms of these equations may be developed, but those given will usually be found the most convenient ones to apply.

89. Methods of Solving Exponential Equations.—(a) Throughout this chapter many equations involving terms with an exponent are used. They also occur very frequently in many other kinds of engineering work, and may be solved by different methods, each one involving logarithms, either directly or indirectly. No one method can be said to be superior to all others, as the choice of the one to be adopted depends largely upon the purpose of the calculations.

(b) For the usual numerical results that are involved in thermodynamic processes, the calculations can generally be made most rapidly, and to a satisfactory degree of accuracy, by means of a 10-in. log-log slide rule, which permits the *direct reading* of fractional powers and roots of all numbers that are likely to be encountered. For example:—Let it be desired to find the value of r^n. By logarithms, $\log r^n = n \log r$; and $\log(\log r^n) = \log(\log r) + \log n$. Then using the slide rule, r is set on the log log scale, and by means of the slide the log of n is automatically added to it, thus obtaining directly the reading of the value of r^n by means of the log log scale.

Slide rules are also very convenient for determining the numerical values of common and hyperbolic logarithms of numbers, especially small ones, such as the ratios involved in the thermodynamic processes.

(c) If extreme accuracy is essential, or if the transcendental equations are involved, the use of a suitable table of logarithms becomes desirable.

(d) If a large number of values is to be found from an equation, it will usually be desirable to plot the corresponding curve, and this may be done to such a scale that results may be read to the required degree of accuracy. This curve may be constructed on the usual type of cross-section paper; but, if the equation is of the general form $PV^n = a$ *const.*, the logarithmic paper is most convenient since the resulting curve is then a straight line. The method of using logarithmic cross-section paper will now be considered.

(e) The equation $PV^n = K =$ *const.*, if solved by logarithms, takes the form $\log P = -n \log V + \log K$. Then letting $y = \log P$, $x = \log V$, and $k = \log K$, the last equation may be rewritten $y = -nx + k$. This is the equation of a straight line with negative slope n and y-intercept k. It is shown in Fig. 47 by KS, drawn to the ordinary *uniform scales* O_X and O_Y. The abscissa of any point A on KS, measured on the scale O_X, gives the logarithm of the value of V represented; its ordinate on scale O_Y is the logarithm of the value of P.

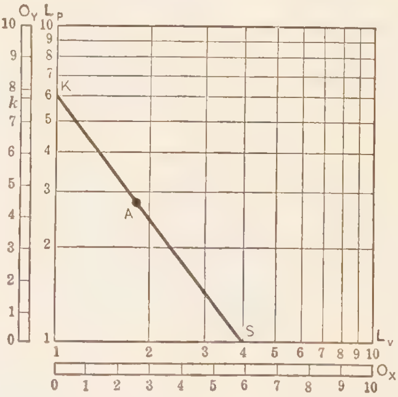


FIG. 47.—Logarithmic Chart.

If now new scales L_V and L_P are constructed in such a manner that the lengths 1–2 and 1–3, etc., represent the logarithms of 2, 3, etc.,—as is done on the scale of the slide rule,—then these **logarithmic scales** may be used for reading *directly* the numerical values of P and V corresponding to points on KS. To the scale L_P, the y-intercept of this line is the constant K, and to the *uniform* scale the slope is n. Since the values of P and V may be read directly on the logarithmic scales, the uniform scales are not usually given on charts of this character.

Any straight line can be located on the logarithmic chart if the PV values for two points are known; or if one point and the slope n are given; but in the latter case it must be remembered that the slope is laid off using the uniform scale. For one of the points it is sometimes convenient to use the y-intercept, K.

After the line has been drawn, the simultaneous values of P and V may be read, and these may then be used in drawing the PV curve on ordinary cross-section paper.

The chart in Fig. 47 is arranged for numbers between 1 and 10, but it may be used for numbers between .1 and 1, between 10 and 100, 100 and 1000, and so on, by merely changing the scales to suit.

When a wider range of numbers is under consideration (as from .1 to 100) a "checkerboard" composed of several similar logarithmic charts may be used. Thus in Fig. 48, let each of the squares contain such a chart and let the one surrounded by heavy lines correspond to Fig. 47, with KS reproduced. In the lower tier of charts the ordinates are for numbers from .1 to 1, in the middle tier they are from 1 to 10, and in the upper, from 10 to 100. The abscissas for the vertical columns progress, from left to right, by multiples of ten also.

The coördinates of a point anywhere on the checkerboard can be read directly on the proper scales. For example, point B has coordinates $P = 25$ and $V = .35$; for point C the value of P is 0.14 and V is 13.

As all the squares are cross-ruled the same, and differ only as to scales (and that by multiples of 10), it is evident that if $S'T'$ is drawn in the central chart in a position similar to that of ST in the square below, it can be used in place of the latter line provided points on it are read to a scale $\frac{1}{10}$ that used for KS. Similarly, $T''U'$, $K'Q'$, and $Q''M'$ may be drawn to correspond to TU, KQ, and QM respectively, and may be used instead of them with proper change in scale. Thus a single logarithmic chart may be used in place of the checkerboard. Obviously, when the curve crosses a horizontal boundary line the scale of ordinates changes; and when it crosses a vertical boundary the scale of abscissas changes.

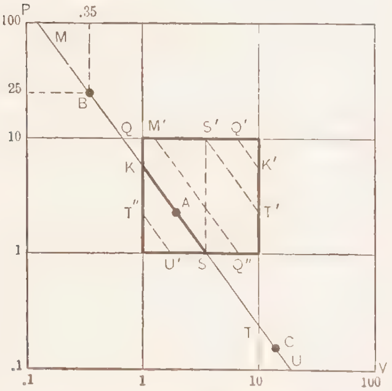


FIG. 48.—Checkerboard of Logarithmic Charts.

If $n = 1$, as in the case of the equilateral hyperbola, the slope of the line is $-45°$. If the exponent is greater than 1, the slope is steeper, and *vice versa*.

(f) An interesting application of the use of logarithmic cross-section paper in engineering work is found in *Clayton's Analysis*[10] of indicator diagrams of steam engines and internal combustion motors. When drawn on such paper, the actual expansion and compression curves are found to be substantially straight lines, because they represent processes that follow closely the law $PV^n = \text{const.}$ The actual values of the exponent n can be more accurately determined in this way than by any other means.

[10] "A New Analysis of the Cylinder Performance of Reciprocating Engines," by J. Paul Clayton, Bul. 58, Univ. of Ill. Eng'g Experiment Station.

When carefully plotted these logarithmic diagrams may also be used for

(1) Approximating the clearance volume;
(2) Detecting the leakage, if appreciable;
(3) Determining the piston positions for the various events;
(4) Approximating the steam consumption.

Illustrations of such diagrams, and Clayton's Analysis, will be given in the discussion of the Steam Engine.

(g) On the *PV-diagram*, the curve corresponding to $PV = const.$ is an equilateral hyperbola, and hence can be easily constructed. For drawing the curve through point A two methods will be given. **First Method**:—In Fig. 49, draw through the point A horizontal and vertical lines pp_1 and v_1v_1'; next, from the origin O draw any number of rays (such as Od) to intersect these lines (as at a and b); then horizontal and vertical lines drawn through these points of intersection will meet at points (such as B) on the desired curve. For expansion from point A, the rays fall below A; for compression, they fall above. **Second Method**:—Through A, Fig. 50, draw any number of lines, as bb_1, cc_1, etc.; make $b_1B = Ab$, $c_1C = Ac$, and so on; then the points A, B, C, etc., will be on the desired curve.

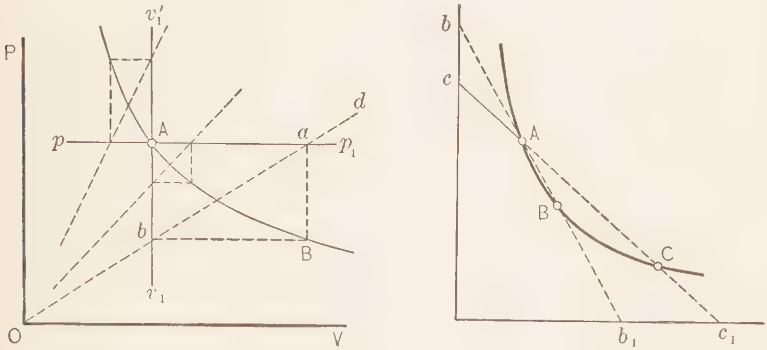


Fig. 49. Fig. 50.

Figs. 49 and 50.—Construction of Curve PV = Constant, on PV-Chart.

90. Reference Diagrams and Table.—(a) Isobaric, isometric, isothermal, isentropic and polytropic processes are met with so frequently in engineering calculations, that it is important to have a clear picture of their relative shapes on the PV- and $T\phi$-diagrams, whether expansions or compressions are involved. These processes are represented graphically in Fig. 51(a) and (b), on page 122, with the initial state of

TABLE V

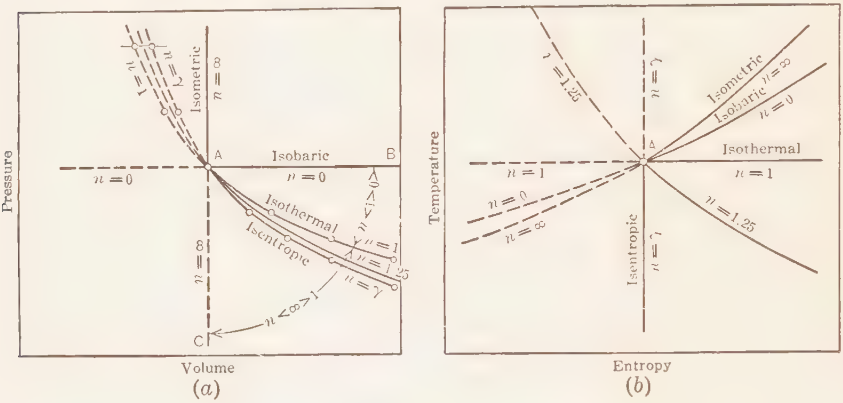
Convenient Formulas for Gases with Constant Specific Heats

Process	PV-Relation	Final Pressure	Final Volume	Final Abs. Temp.	Gain in Entropy	Heat Added to Gas, in B.t.u.	Work Done by Gas, in Ft-lb.
		P_2	V_2	T_2	$\Delta\Phi_{1,2}$	$Q_{1,2}$	$Wk_{1,2}$
Isobaric or Constant Pressure	$P_1V_1{}^{\circ}=P_2V_2{}^{\circ}$	P_1	$V_1\frac{T_2}{T_1}$	$T_1\frac{V_2}{V_1}$	$wc_p \log_e \frac{T_2}{T_1}$	$wc_p(T_2-T_1)$	$P(V_2-V_1)$
	$P=\text{Const.}$	$\frac{wRT_2}{V_2}$	$\frac{wRT_2}{P_2}$	$\frac{P_2V_2}{wR}$	$wc_p \log_e \frac{V_2}{V_1}$	$w\left(\frac{\gamma}{\gamma-1}\right)AR(T_2-T_1)$	$wR(T_2-T_1)$
Isometric or Constant Volume	$P_1V^{\infty}=P_2V_2{}^{\infty}$	$P_1\frac{T_2}{T_1}$	V_1	$T_1\frac{P_2}{P_1}$	$wc_v \log_e \frac{T_2}{T_1}$	$wc_v(T_2-T_1)$	0
	$V=\text{Const.}$	$\frac{wRT_2}{V_2}$	$\frac{wRT_2}{P_2}$	$\frac{P_2V_2}{wR}$	$wc_v \log_e \frac{P_2}{P_1}$	$w\left(\frac{1}{\gamma-1}\right)AR(T_2-T_1)$	
Isothermal or Constant Temperature	$P_1V_1=P_2V_2$	$P_1\frac{V_1}{V_2}$	$V_1\frac{P_1}{P_2}$	T_1	$wAR \log_e \frac{V_2}{V_1}$	$T(\Phi_2-\Phi_1)$	$P_1V_1 \log_e \frac{V_2}{V_1}$
					$wAR \log_e \frac{P_1}{P_2}$	$w(\gamma-1)c_vT \log_e \frac{P_1}{P_2}$	$wRT \log_e \frac{P_1}{P_2}$
	$\frac{P_1}{P_2}=\frac{V_2}{V_1}$	$\frac{wRT_2}{V_2}$	$\frac{wRT_2}{P_2}$	$\frac{P_2V_2}{wR}$	$w(\gamma-1)c_v \log_e \frac{P_1}{P_2}$	$wART \log_e \frac{V_2}{V_1}$	$w(K_p-K_v)T \log_e \frac{V_2}{V_1}$

Isentropic or Constant Entropy or Reversible Adiabatic	$P_1V_1^\gamma = P_2V_2^\gamma$ $\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}$	$P_1\left(\frac{V_1}{V_2}\right)^\gamma$ $\frac{wRT_2}{V_2}$	$V_1\left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}$ $\frac{wRT_2}{P_2}$	$T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$ $T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$ $\frac{P_2V_2}{wR}$	0	0	$\frac{P_1V_1-P_2V_2}{\gamma-1}$ $\frac{wR(T_1-T_2)}{\gamma-1}$ $wK_v(T_1-T_2)$
Polytropic	$P_1V_1^n = P_2V_2^n$ $\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}}$	$P_1\left(\frac{V_1}{V_2}\right)^n$ $\frac{wRT_2}{V_2}$	$V_1\left(\frac{P_1}{P_2}\right)^{\frac{1}{n}}$ $\frac{wRT_2}{P_2}$	$T_1\left(\frac{V_1}{V_2}\right)^{n-1}$ $T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$ $\frac{P_2V_2}{wR}$	$wc_v \log_e \frac{T_2}{T_1} + w(\gamma-1)c_v \log_e \frac{V_2}{V_1}$ $w\left(\frac{n-\gamma}{n-1}\right) c_v \log_e \frac{T_2}{T_1}$ $wc_v \log_e \frac{T_2}{T_1}\left(\frac{V_2}{V_1}\right)^{\gamma-1}$ $wc_v \log_e \frac{P_2}{P_1}\left(\frac{V_2}{V_1}\right)^{\gamma}$ $wc_p \log_e \frac{T_2}{T_1}\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$	$w\left(\frac{n-\gamma}{n-1}\right) c_v(T_2-T_1)$ $w\left(\frac{\gamma-n}{n-1}\right) c_v(T_1-T_2)$	$\frac{P_1V_1-P_2V_2}{n-1}$ $\frac{wR(T_1-T_2)}{n-1}$ $w(\gamma-1)K_v\left(\frac{T_1-T_2}{n-1}\right)$

a gas for which $\gamma = 1.4$ shown in each case by the letter A. The processes having an increase in volume or in entropy are represented by solid lines and the opposite cases are shown by broken ones.

(b) **A comparison of the expansion curves on the PV-diagram** (Fig. 51(*a*)) will show that *as the graph of the different expansions considered swings down through the quadrant BAC the exponent increases in size. The facts that any equation with $n < 1 > 0$ gives a graph less steep than the isothermal and that any equation with $n > 1 < \infty$ gives one steeper than the isothermal* are very useful. The idea of *steepness* in this statement is important, as, in general, one curve cannot be said to lie above or below another. For example, if the curves in the figure are continued as compressions, from the point A, their relations as to vertical position are reversed.



(*a*) (*b*)

FIG. 51.—Showing Effect of Value of n in Equation $PV^n = \text{Const.}$

(c) Any polytropic that has a value of n between 1 and γ must lie between the adiabatic and the isothermal on any graphical representation. Such a process is characterized by a *drop in temperature* during expansion although *heat is being added* to the gas, as will always be shown by its increase in entropy. Similarly, the corresponding compression is characterized by a *rise in temperature* although *heat is being abstracted* from the gas, as will always be shown by its decrease in entropy.

(d) In Table V are arranged in convenient form for reference the formulas that are very frequently needed when dealing with ideal gases, or with real gases for which the average values of the specific heats are known for the temperature ranges involved.

CHAPTER IX

THE INDICATOR DIAGRAM

91. The Indicator Diagram.—In connection with any type of reciprocating apparatus, such as engines or pumps, an "indicator" may be used to determine graphically the pressure-volume changes that occur within the cylinder. From the pressure-volume history of the variable amount of working substance within the cylinder, the work actually transferred between the piston and this working substance can be computed for any period desired.

92. The Indicator.—(a) The indicator is the instrument used to obtain the "*indicator diagram.*" There are in use to-day many different types, one of which is shown in Fig. 52. A card is mounted on the outside of a drum which is caused to oscillate in unison with the motion of the piston of the reciprocating apparatus. A pencil, which may be pressed against this card, is actuated by a small, spring-balanced piston subjected to the same pressure that exists in the main cylinder. Thus the card movement is proportional to the volume displaced by the piston of the apparatus being studied, while the pencil movement is proportional to the pressure which actuates the piston. The pencil movement is at right angles to the card movement and hence a pressure-volume diagram with rectangular coordinates is obtained. The heavy lines of Fig. 53 represent an indicator diagram characteristic of one common type of reciprocating apparatus.

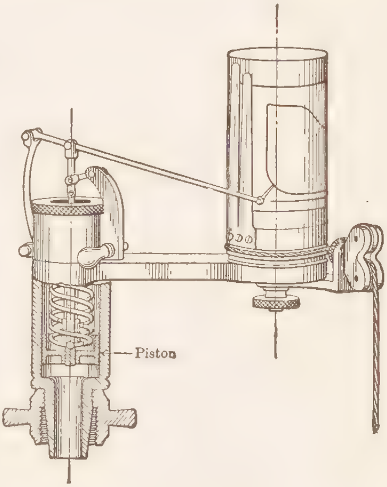


FIG. 52.—Indicator.

(b) If the card drum oscillates under the pencil while the indicator piston is subjected only to atmospheric pressure, a horizontal line, called the *atmospheric line,* will be drawn. Every indicator diagram should include this reference line in order to be complete.

93. Types of Pressure-Volume Diagrams.—There is a distinct difference between the pressure-volume diagram already studied in connection with thermodynamic processes and the indicator diagram. The former is constructed to represent changes of state of a *constant* weight of the working substance, whereas an indicator diagram represents the changes of state of the amount of working substance within the cylinder that *varies* while the diagram is being drawn. Therefore, in analyzing indicator diagrams, equations developed for a constant weight of substance should be applied only to those portions of the diagram that do not involve a variable amount.

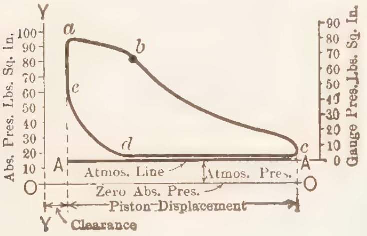


FIG. 53.

94. Determining Absolute Values from the Diagram.—(a) Before absolute pressures and volumes can be scaled from the diagram, the zero line of absolute pressure and the zero volume line must be located. The pressure represented by one linear unit of ordinate on the card, or the *pressure scale*, depends on the indicator spring and the size of indicator piston used. This scale being known, the zero absolute pressure line can then be located the proper distance below the atmospheric line to correspond to the barometric pressure that existed at the time the diagram was taken. The length of the diagram is proportional to the "*piston displacement*," which is defined as the volume displaced by the piston in one stroke, and which can be determined when the diameter of the cylinder and the length of the stroke of the piston are known. Having the piston displacement and the length of the diagram, the volume represented by each unit of abscissa of the diagram, or the *volume scale*, can be found. If the engine is double acting, the diagrams for both working ends may be drawn on the same card. Knowing the clearance at each end of the cylinder, as a per cent of the piston displacement, the zero volume line for each end of the card is located by adding to the length of the card the proper percentage for the clearance at that end.

(b) The zero absolute pressure and zero volume lines are shown in Fig. 53 by the axes *O–O* and *Y–Y*. With these axes located, the pressure and volume may be scaled for any point on the diagram.

95. Calculations from an Indicator Diagram.—(a) The **mean effective pressure** of an indicator diagram is a quantity widely used in designs or analyses relating to reciprocating apparatus. This quantity, which will be designated by p_m, is defined as the constant pressure which, acting on the piston for one stroke, would accomplish the same amount of work that is done in the actual number of strokes required for tracing the complete indicator diagram. Thus, this pressure is represented by the altitude of a rectangular diagram having the same area and length as the actual indicator diagram. The mean effective pressure can be found by dividing the area of the actual diagram by its length, and introducing the pressure scale to convert the resultant distance into the desired pressure. All engines require at least two strokes to complete an indicator diagram, and many internal combustion engines require four strokes.

(b) The **indicated work per cycle** is represented by the enclosed area of the diagram, and may be either the net amount of work delivered by, or that absorbed by, the apparatus during one cycle of events within the cylinder, depending upon whether the machine is an engine or a pump. The work in foot-pounds represented by each cycle of events of a cylinder end is

$$p_m \times a \times L,$$

where

p_m = the mean effective pressure in pounds per square inch,
a = the area of the piston in square inches, and
L = the length of the stroke of the piston in *feet.*

(c) The **indicated horsepower** is usually desired from an indicator diagram rather than the indicated work. Since an indicator diagram is traced separately for each working end of the cylinder of the apparatus, it is usually desirable to speak of the "indicated horsepower *per cylinder end.*" If there are n cycles per minute per cylinder end, then

$$\text{i.hp. per cyl. end} = \frac{p_m L a n}{33{,}000}. \quad . \quad . \quad . \quad . \quad . \quad (130)$$

The indicated horsepower, as is the case with the indicated work, may be either the power delivered *by* or *to* the piston.

(d) Knowing the scale of the spring, the number of cycles per minute, and the area and stroke of the piston, the only quantity needed from the indicator diagram for determining the indicated horsepower is the mean ordinate, which can be found by dividing the area of the indicator diagram by its length, as already shown.

96. Methods of Determining the Area of an Indicator Diagram.—(a) The area of an indicator diagram can be determined (1) by placing transparent "cross-section paper" over the diagram and counting the squares surrounded; (2) by using some such form of mechanical integrating instrument as the "planimeter"; (3) by applying the "method of ordinates"; or (4) by using some integration rule such as the "Trapezoidal Rule"[1] and "Simpson's One-third Rule."[2]

(b) One form of **planimeter** is shown in Fig. 54. It consists of two arms jointed together, one terminating in a "fixed point" which is a stationary pivot, while the other carries a "tracing point." The third support for the instrument is a point on the rim of a graduated wheel or "record roller." If the record wheel is set at zero and the tracing point is moved clockwise around the outline of the diagram and is returned to its original position, the area of the figure is given by the reading of the record wheel. The theory and use of planimeters is treated in books on Experimental Engineering.[3]

(c) In the **method of ordinates,** the length of the diagram is divided into a number of equal parts, with interval Δx, as in Fig. 55, and ordinates

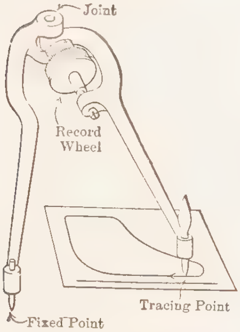


FIG. 54.—Planimeter.

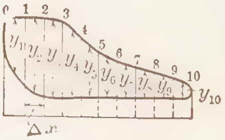


FIG. 55.

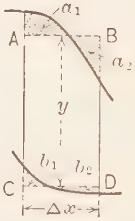


FIG. 56.

are drawn, as 1, 2, 3, etc., in the figure. Central intermediate ordinates are then drawn and the intercepts y_1, y_2, y_3, etc., are scaled and used as the mean heights of the elementary areas between ordinates. The area of the diagram is approximately $A = \Sigma y \times \Delta x$, and the mean ordinate is $y_m = \frac{\Sigma y}{(no.\ of\ ordinates)}$.

This method is not strictly correct, for the middle intercepts are not necessarily the mean heights of the elementary areas. These mean heights can be found quite accurately by the method shown in Fig. 56. Here lines AB and CD (not necessarily horizontal) are so drawn that areas a_1 and a_2 are equal and that $b_1 = b_2$. Then the distance y between the centers of these lines is the true mean height. The equality between areas a_1 and a_2 and between b_1 and b_2 can be estimated very accurately by eye.

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[1] For this rule see Kent's "Pocket Book."

[2] See Church's "Notes on Mechanics" or Kent's "Pocket Book."

[3] See Carpenter and Diederich's "Experimental Engineering."

97. Other Uses of Indicator Diagrams.—(a) In addition to giving the value of the indicated horsepower of the apparatus to which the indicator is attached, the indicator diagram is often used for other purposes, such as the determination of the time of opening and closing of the valves, and the effect of different types of valves on the resistance to the flow of the fluid through them. Also the character of the curves for the various parts of the diagram are frequently studied in order to obtain information regarding the transfer of heat to and from the working substance during the corresponding periods.

All of the above information is of value in the design and operation of various kinds of reciprocating pumps and heat engines. Also for internal combustion engines, the indicator diagram often yields additional valuable information concerning the rate of combustion and the time of ignition.

(b) For engines that operate at very high speeds, such as those for airplanes and automobiles, the ordinary indicator is not suited as the inertia effect of its moving parts becomes so pronounced as to preclude its use under these conditions. In such cases *special high speed indicators* are generally employed. In this type there is usually a small mirror, of negligible weight, which is moved in such manner that it causes a ray of reflected light to trace the indicator diagram on a ground glass screen or on a photographic film.

98. Conventional Indicator Diagrams.—In addition to the very valuable indicator diagrams which are taken from actual apparatus, there are others that are termed " conventional " indicator diagrams which are very useful for the purposes of analysis and design. Such diagrams are laid out on a drawing board according to certain accepted conventions and permit estimating the probable performances of various pieces of apparatus. The particular values of the generally accepted conventions will be discussed in the chapters dealing with the type of machine then under consideration.

CHAPTER X

COMPRESSED AIR[1]

99. The Compressed Air Plant.—In many kinds of work, compressed air serves as the most useful medium for the application of energy. In such cases, energy from some convenient source is used to operate an *air compressor* from which the compressed air passes to a *receiver*, or storage tank, in which nearly constant pressure is maintained. From the receiver, this air passes through a *pipe line* to the point, usually remote from the compressor, where it causes *air engines*, or *tools*, to deliver a certain amount of useful work. The volume of the receiver should be large enough to care for the variable demands for air so that the compressor may be operated at nearly constant speed.

The compressing of the air increases its temperature, but usually the cooling which the air undergoes in the receiver and transmission line will cause its temperature to become again nearly atmospheric by the time the air tools or engines are reached. Since, for reasons given later, it is often desirable to have hot air for the use of the air machines, a *preheater*,[2] or apparatus for transferring heat to the compressed air, is sometimes installed in the circuit to heat the air as it is delivered to the tools.

The purpose of this brief description of a Compressed Air Plant is to name the various pieces of apparatus in such an installation and indicate their functions. These component parts will be considered individually in the remainder of the chapter.

100. The Air Compressor.—Air compressors are generally of the reciprocating, or centrifugal types, but occasionally the rotary and jet types are used. In the broadest sense, they include all devices used for raising the pressure of air; but technically the term is generally applied only to apparatus which raise the pressure to a comparatively high value, say greater than 25 lb. per square inch gage. In extreme cases the pressure is increased to several thousand pounds per square inch.

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[1] If desired, this chapter may be postponed until later; but it serves well to show some simple applications of the preceding work.

[2] The term "reheater" is sometimes used. Since the air has already become heated in the compressor, such a term is logical. However, since there is the possibility of reheating the air after partial expansion in the engine, the term preheater is perhaps, more logical as indicating the heating taking place before the air is delivered to the engine.

Other devices, such as fans and rotary blowers, are really compressors but are seldom spoken of as such, principally because the pressures attained are so small that the main function may be considered to be the propelling of air rather than its compression.

The term *Blowing Engines*, or Blowers, is used to designate certain apparatus used for compressing air to pressures to between 10 lb. and 30 lb. per square inch above atmospheric for use in "blowing" cupolas and blast furnaces. These are properly air compressors but because of the low pressures many of the difficulties attending compression to higher pressures are not met in their design.

101. Free Air.—Since the requirements of air tools and machines are usually expressed in terms of the "volume of free air" required per minute, the determination of the size of an air compressor is also usually based on the amount of free air it must handle. As will be seen later, a given volume of moist air weighs less than the same volume of dry air at the same pressure and temperature. The pressure of a mixture of a gas and vapor is equal to the sum of the two individual pressures. The volume of *free air* is measured at the pressure of *dry* air and at the actual temperature at the entrance of the compressor intake.

102. The Ideal Reciprocating Air Compressor Having No Clearance.—(a) A large proportion of the air compressors of all sizes in use to-day are of the reciprocating type. The essential elements of an air compressor of this kind are shown in Fig. 57, *A* being the spring-closed inlet valve, which opens inwardly, and *B* the spring-closed discharge valve, opening outwardly. In the case shown there is *no clearance*, the piston just touching the cylinder head at one end of its stroke.

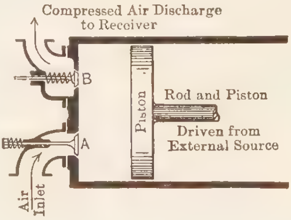
Compressed Air Discharge to Receiver
B
Piston
Rod and Piston
Driven from External Source
A
Air Inlet

Fig. 57.

(b) Initially, imagine the piston in contact with the cylinder head. By the application of an external force to the piston rod, the piston can be drawn to the right, and air will then enter the cylinder through valve *A* at suction pressure, according to the horizontal line *ab*, in Fig. 58, which represents the conventional indicator diagram for this compressor. This is the *suction period*.

(c) When the cylinder has been thus filled with air, at the pressure P_b, and the volume V_b, the piston may be driven back to the left. As soon as such motion starts, the valve *A* will be closed by the light spring shown, and the air entrapped in the cylinder will then be compressed according to some law, as shown by *bc*, the final volume being V_c. This is the *compression period.*

(**d**) When the piston has compressed the air to the pressure P_c the discharge valve will open (at *c*) and the continued motion of the piston will deliver the air at constant pressure, P_c, to the receiver, as shown by the line *cd*. Thus, the piston will end its stroke in contact with the cylinder head, having discharged at pressure P_c all the air received at pressure P_b. This is the *delivery period* and completes a cycle of the compressor.

103. Net Work Done on the Air in Compressor.—(a) In Fig. 58 the area *abgc* shows the work done *upon* the piston during the suction stroke by the entering air, *gbcde* represents that done *by* the piston *on* the air during the compression and delivery periods, and the net work delivered to the air is shown by the net area *abcd*.

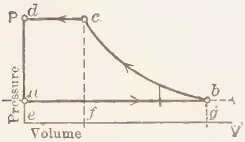

FIG. 58.

(**b**) A convenient expression for the *net work* done on the air during the *cycle* of an *air compressor* having *no clearance,* as shown by Fig. 58, can readily be developed. Thus, assuming polytropic compression, which is the most general case, the net work done on the air per cycle is

$$Net\ Wk_{cycle} = \text{Work}_{bc} + \text{Work}_{cd} - \text{Work}_{ab}$$

$$= \frac{P_cV_c - P_bV_b}{n-1} + P_cV_c - P_bV_b. \quad . \quad . \quad (131)$$

Reducing this expression to a common denominator, it becomes

$$Net\ Wk_{cycle} = \frac{n}{n-1}(P_cV_c - P_bV_b) = \frac{n}{n-1}(w_cRT_c - w_bRT_b). \quad (132)$$

With no leakage, $w_c = w_b$ = weight of air handled during one cycle, or unit of time, for which the diagram is drawn; therefore, the net work done on w_b lb. of air becomes

$$Net\ Wk_{w_b} = \frac{n}{n-1}w_bR(T_c - T_b).$$

Multiplying and dividing through by T_b, this becomes

$$Net\ Wk_{w_b} = \frac{n}{n-1}w_bRT_b\left(\frac{T_c}{T_b} - 1\right) = \frac{n}{n-1}w_bRT_b\left[\left(\frac{P_c}{P_b}\right)^{\frac{n-1}{n}} - 1\right] \quad (133)$$

$$= \frac{n}{n-1}P_bV_b\left[\left(\frac{p_c}{p_b}\right)^{\frac{n-1}{n}} - 1\right]. \quad (134)$$

(c) The advantage of reducing the equations to the forms given by (133) or (134) is due to the fact that these expressions involve only the terms that are likely to be most readily available. Thus, one would have given the suction pressure, p_b, and the volume V_b at the beginning of compression, or the weight w_b of air compressed per unit of time, and the temperature T_b. The delivery pressure p_c is known and the nature of the compression curve *bc* is determined when the value of the exponent n is chosen. In Eq. (134) the value of V_b is often taken as the volume of "free air" although the temperature and pressure at b may be slightly different from the corresponding values at the entrance to the compressor intake at *a*.

(d) There are two limiting conditions which may be imagined as existing during the compression period:

(1) No heat may be removed from the air during the process, in which case the compression will be *adiabatic* with a consequent rise in temperature. Unless otherwise stated, in this chapter the adiabatics will be considered reversible.

(2) The air may be cooled at such a rate that its temperature is kept constant, in which case the compression will be *isothermal*.

All actual cases generally fall between these limits, as will be seen later.

(e) If the compression is carried out *adiabatically* it means that the curve *bc* follows the law PV^{γ} = a constant, or that the exponent n has taken the particular value γ for the gas that is being compressed. Hence with adiabatic compression, the net work for w_b lb. is

$$Net\ Wk_{adiab.} = \frac{\gamma}{\gamma - 1} w_b R T_b \left[\left(\frac{P_c}{P_b} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$= \frac{\gamma}{\gamma - 1} P_b V_b \left[\left(\frac{P_c}{P_b} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]. \quad . \quad . \quad . \quad . \quad (135)$$

(f) If the compression process, *bc*, is *isothermal*, the general equations, Eqs. (133) and (134), cannot be used since they reduce to indeterminate forms. However, for such a case the net work can be readily determined. Thus, with isothermal compression,

$$Net\ Wk_{isoth.} = P_b V_b \log_e \frac{V_b}{V_c} + P_c V_c - P_b V_b.$$

But, from Boyle's Law, $P_c V_c = P_b V_b$; therefore,

$$Net\ Wk_{isoth.} = P_b V_b \log_e \frac{V_b}{V_c} = w_b R T_b \log_e \frac{V_b}{V_c}, \quad . \quad . \quad . \quad (136)$$

or

$$Net\ Wk_{isoth.} = P_b V_b \log_e \frac{p_c}{p_b} = w_b R T_b \log_e \frac{p_c}{p_b}. \quad . \quad . \quad . \qquad (137)$$

(g) In Fig. 59 are given two superposed diagrams, *abcd* and *abc′d*, both from the same compressor which is to compress to a delivery pressure P_d the amount of air which originally occupied the volume V_b when at atmospheric pressure. The compression line *bc* is an *isothermal* and the line *bc′* is an *adiabatic*.

It is evident from the figure that the diagram containing an adiabatic compression line encloses more area (*bc′da*) than that having isothermal compression (*bcda*), and, hence, that more energy from some outside source will be required per cycle. Obviously, the isothermal compression is the more desirable, the work saved over that with adiabatic compression being shown on the diagram by the area *cbc′*. The higher the compression pressure the greater is the ratio of the work done with adiabatic compression to that with isothermal.

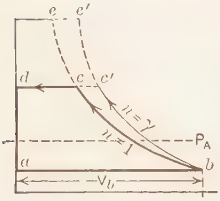

FIG. 59.

(h) With isothermal compression the temperature of the air at *c* and *b* is, of course, the same, but during adiabatic compression it rises according to Eqs. (105) and (107), the final value being,

$$T_c' = T_b\left(\frac{V_b}{V_c'}\right)^{\gamma-1} = T_b\left(\frac{p_c'}{p_b}\right)^{\frac{\gamma-1}{\gamma}}. \quad . \quad . \quad . \quad . \qquad (138)$$

If the air with pressure P_c' and volume V_c' is cooled at constant pressure it will attain a volume V_c when the initial temperature T_b is reached. This is approximately what happens in most real cases, for after the cooling has occurred the air is in the same condition as though it had been compressed isothermally. It is therefore advisable to strive for isothermal compression if this can be attained, or approached, without entailing greater outlay than the cost of the work area *cbc′*. The means whereby isothermal compression is approached in a real compressor will be considered later.

If air can be used immediately after leaving the compressor, that is, before it has any chance to cool, then adiabatic compression may be desired since a given weight of air is capable of doing more work the higher its temperature.

104. The Effect of Clearance.—(a) No real compressor can be operated with zero clearance, as there must always be a certain amount of mechanical clearance between the cylinder head and the piston to insure safe operation, and there are always valve passages of some sort between the valve faces and the inside of the cylinder, which increase the clearance volume.

(b) In Fig. 60 is given a conventional indicator diagram for an *air compressor with clearance.* The clearance volume is $V_{cl} = V_d$. At the end of the discharge period, the clearance contains V_{cl} cubic feet of air at the delivery pressure P_c. When the piston starts on the suction stroke the inlet valve will be held closed by this clearance air until the piston has moved far enough to allow this air to expand to suction pressure, according to some such process as *da*. This is known as the *reexpansion period.* When *a* is reached, the inlet valve will open and the remainder of the cycle will be similar to that already discussed for the compressor without clearance.

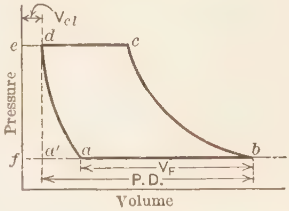

FIG. 60.

(c) The net work done on the air during this cycle is

$$Net\ Wk = \text{area } cefb - \text{area } adef,$$

which, from Eq. (134), becomes

$$Net\ Wk = \frac{n}{n-1}P_bV_b\left[\left(\frac{P_c}{P_b}\right)^{\frac{n-1}{n}} - 1\right] - \frac{n'}{n'-1}P_aV_a\left[\left(\frac{P_d}{P_a}\right)^{\frac{n'-1}{n'}} - 1\right]. \quad (139)$$

If it is assumed that the compression and reexpansion curves have the same exponent, i.e., $n = n'$, as is very nearly true in many cases, then the net work per cycle is

$$Net\ Wk = \frac{n}{n-1}P_b(V_b - V_a)\left[\left(\frac{P_c}{P_b}\right)^{\frac{n-1}{n}} - 1\right]. \quad . \quad . \quad . \quad (140)$$

$$= \frac{n}{n-1}w'RT_b\left[\left(\frac{P_a}{P_b}\right)^{\frac{n-1}{n}} - 1\right]. \quad . \quad . \quad . \quad . \quad . \quad (141)$$

In Eq. (140) the expression $(V_b - V_a)$ is the volume of free air passing through the compressor either during one cycle, or per unit of time, and in Eq. (141) w' is the corresponding weight.

(d) By comparing Eqs. (140) and (141) with Eqs. (133) and (134) it is immediately apparent that, on the basis of the conventional diagrams, *the clearance does not affect the net work delivered by a compressor* *piston while handling a given volume or weight of free air.* However, other things being equal, a larger clearance will be accompanied by increased loss due to mechanical friction, and more energy, therefore, will be required to drive the compressor, as will now be shown:

In Fig. 60, although the piston displacement, PD, is equal to $V_b - V_d$, the reexpansion of the clearance air limits the volume V_F of free air handled to $V_b - V_a$ and thus reduces very appreciably the capacity of a compressor of a given piston displacement. Hence, for a given capacity, a compressor with clearance must be *larger* than one without it; and, therefore, the *friction losses* and the *cost* of the machine will become greater as the clearance is increased.

(e) In order to compare the clearances of compressors of different sizes, *the ratio of the clearance volume to the piston displacement* is used. This is often expressed in per cent and may be made independent of the size of the compressor. For reasons already shown it is desirable to keep this clearance per centage as small as possible. It may vary from about $\frac{3}{4}$ per cent in large well-designed compressors to 5 per cent and over in small, or poorly-designed ones.

105. Real Single Stage Compressor Diagram.—(a) The *real compressor* differs from the ideal in many respects, chiefly because of clearance, because the cylinder and piston cannot be made of heat-resisting materials, because the valves cannot be made to operate perfectly, and because of the leakage, friction and inertia of the air being handled.

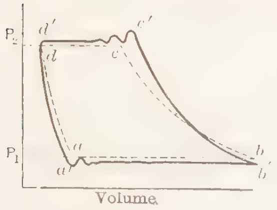

FIG. 61.

An indicator diagram obtained from a real compressor is shown in Fig. (61), superposed upon an ideal one for the same machine, the pressure of air being supposed to be raised from atmospheric pressure (P_1) to a receiver pressure equal to P_2.

(b) The ideal diagram is drawn for isothermal expansion of the clearance air and for isothermal compression of the entire amount of air in the cylinder. In the real case it is never possible to obtain isothermal compression with a reciprocating air compressor; instead, the compression line falls between the adiabatic and the isothermal and is expressible by the equation PV^n = constant, with values of n varying from about 1.2 in extremely favorable cases to about 1.35 under rather unfavorable conditions. To obtain such a curve it is necessary to cool the air during compression, by the methods which will be considered later. For present purposes it is merely necessary to note that the air, and therefore the cylinder walls, will become heated during compression.

(c) During expansion the clearance air will, in general, be in contact with walls which are at a higher temperature, hence it will receive heat during the process. Ordinarily the real expansion line for this air lies between an adiabatic and an isothermal, but in the average case it approaches more nearly to the latter. The real expansion line may then be assumed to have a shape and position similar to *da′* in the figure.

In the ideal compressor the admission valve would open as soon as the clearance pressure has decreased to atmospheric, but actually the pressure must drop somewhat lower to give an unbalanced pressure great enough to open the valve against its spring, its friction and inertia, and also to overcome the inertia of the air and the resistance to flow through the more or less restricted areas available.

(d) After the valve is open and the air is in motion there are generally several oscillations of the valve and the air column, as indicated by the wavy suction line, after which the pressure settles down to an average value sufficiently below the atmospheric pressure to cause the inflow. The oscillation of the valve is known as "fluttering."

From the fact that machines with the same piston displacements and the same clearance have been assumed in both cases it is evident that the actual *volume* of air in the cylinder at the end of the suction stroke must be the same in each case. In the real case, however, the air has a lower pressure than it has in the ideal, and, in general, its temperature will also have been raised slightly by the heated walls which are uncovered by the piston, thus the actual *weight* present will be less than the corresponding ideal.

(e) Starting at the point *b′*, instead of *b*, the air in the real case will be compressed according to some curve intermediate between the adiabatic and isothermal and the line will be therefore steeper than the ideal, as shown by *b′c′* in the figure.

(f) The discharge valve does not open until the pressure attained is slightly above that in the receiver and, after opening, it behaves much like the suction valve and for similar reasons. The discharge line actually obtained will then generally look something like *c′d′* instead of *cd*.

The discharge valve will obviously not close suddenly at the end of the stroke, consequently the corner at *d′* may be more or less rounded, the exact point at which expansion of the clearance air starts being rather difficult to determine.

(g) The net result of the operation of the real compressor has been to compress a smaller weight of air than that handled by the ideal machine and to require the expenditure of work in excess of the ideal, as shown by the greater area enclosed.

106. Volumetric Efficiency.—(a) One measure of the effect of clearance is the *volumetric efficiency* of the compressor. In all compressors, the volumetric efficiency is defined as the ratio of the volume of free air actually delivered per unit of time, to the piston displacement in the same unit of time. As the name implies, this measures the effectiveness of the use of the cylinder volume, and the term has no other direct significance. In the conventional indicator diagram shown by Fig. 60, the volumetric efficiency would be $\frac{V_F}{PD}$ or $\frac{V_b - V_a}{V_b - V_d}$.

(b) The volumetric efficiency of any actual compressor is not easy to obtain accurately, even though the machine is equipped for a careful test. Its determination requires the measurement of the air received, or discharged, in a given time, but usually the accurate measurement of the flow of large quantities of compressed air is somewhat difficult to accomplish.

(c) Various methods of *approximating* the volumetric efficiency from the indicator diagram are often used and, as long as the result is understood to be only an approximation, it is useful. In Fig. 62 is given in exaggerated form a real compressor diagram with the atmospheric line added. If there is no change of temperature of the working substance during suction and if the expansion of the clearance air and the compression of the mixture both follow the same law, the distance *ef* must measure the volume occupied by the clearance air, and *fg* is that occupied by the fresh charge when compression has progressed up to the point *g*. If it be further assumed that the drop in temperature from *f* to *a′* and the rise from *b′* to *g* are negligible, it may be said that the distance *fg* is a measure of the volume occupied by the charge when at atmospheric pressure and temperature. Then the *volumetric efficiency on an atmospheric line basis*, would be

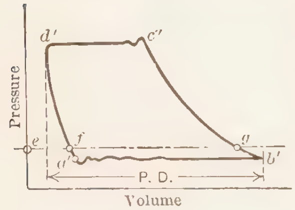

FIG. 62.

$$\frac{\text{distance } fg}{\text{distance representing the P.D.}} \quad . \quad . \quad . \quad . \quad (142)$$

107. Cooling.—(a) It has been shown in preceding sections that isothermal compression should, in general, prove more economical than adiabatic. It is practically impossible to attain isothermal compression in any real machine, but it can be more or less closely approximated and generally without involving excessive cost.

(b) If a compressor fitted with a metallic cylinder is operated very slowly, i.e., one or two cycles per minute, the heat generated by compression will be conducted away by the cylinder metal almost as fast as generated and in such case the compression could be made to approach an isothermal process as closely as desired, but it would involve the use of enormously large machines because of the slow operation. Some method must therefore be used which permits of operation at the highest feasible speeds.

(c) If any form of external cooling, such as radiation considered above, is to be used the proportions of the cylinder are important. That cylinder which exposes the greatest surface to the external cooling agent, per unit volume enclosed, will be the best so far as cooling is concerned. This would indicate the use of cylinders of small diameter and great length, i.e., "long stroke compressors," but such machines are always more expensive than short stroke mechanisms, consequently a commercial limit is set to cylinder proportions adopted.

(d) In practice a few types of compressors are cooled by radiation to the atmosphere, as, for instance, those used on locomotives for operating the air brakes. They are all comparatively small, are generally operated in strong currents of air and are at best rather inefficient. It is doubtful if the compression is appreciably better than adiabatic, the radiation serving simply to prevent overheating of the entire mechanism by storage of heat from cycle to cycle.

(e) Most commercial machines are water cooled. There are three distinct methods of applying cooling water, two or more of which may be, and generally are, used on the same machine. They are:—(1) *Injecting water* into the cylinders; (2) *water jacketing* the cylinders; and (3) compressing in stages and using water jacketed vessels, called "*intercoolers*." between cylinders.

(f) The **injection of water** into the compression cylinder has been rather extensively used in Europe but not in this country. If the water is introduced as a solid stream but little cooling is effected, the compression curve approximating the equation $PV^{1.35}$ to $PV^{1.37}$ = constant; but with a very fine spray the cooling effect is much greater, and values of the exponent n as low as 1.20 may be obtained.

(g) The introduction of water into the cylinder has the following disadvantages:—It generally increases the wear of cylinder and piston; the feeding devices are an almost constant source of trouble; and the air leaves the cylinder practically saturated with water, some of which precipitates when cooled in the receiver, but much remains in the air and later may cause trouble by freezing when the air is expanded in doing work.

(h) **Jacketing** the compressor cylinders with water does not introduce the difficulties considered above, but it is generally less efficient than water spraying unless it is very perfectly carried out. The cooled surface exposed to the compressed air is relatively small and heat is not conducted readily to these surfaces from air which is not close to them. Values of the exponent n equal to about 1.25 to 1.28 are generally attainable.

(i) **Intercooling** is the term applied to the method of abstracting heat from the compressed air as it passes from one cylinder to another of the multi-stage type of compressor. This will be discussed in the next section.

108. Multi-stage Compression and Intercooling.—(a) Fig. (63) (*a*) shows diagrammatically the arrangement of a two-stage ideal com-

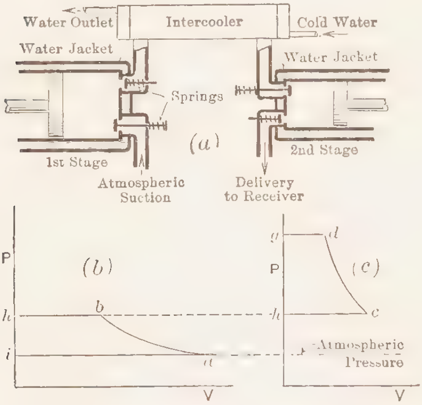

Fig. 63.—Diagrammatic Two-stage Compressor, and Conventional Diagrams.

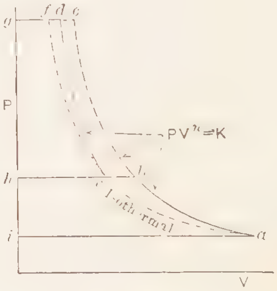

Fig. 64.

pressor of this type. Assuming no clearance, the conventional indicator diagram for the first stage is shown in Fig. 63 (*b*), and that for the second stage in Fig. 63 (*c*). In Fig. 64 are shown the two diagrams combined with the same pressure-volume coordinates,—a construction often used in studying the actual indicator diagrams of multi-stage compressors.

The air is compressed to an intermediate pressure in the first stage and then passes through the intercooler on its way to the second stage. The intercooler is an efficient heat interchanger in which cold water passes through a large number of tubes around which the air is flowing. The design of the intercooler is such that the pressure of the air drops only very slightly in passing through it. In the ideal case, the delivery pressure of the first stage is taken equal to the suction pressure of the second stage, which completes the compression to the desired receiver pressure.

(b) In Fig. 64 it is apparent that the intercooling is very effective, as the air is cooled from t_b to t_c, its original temperature. With warm circulating water, this would not be accomplished, but, on the other hand, with very cold water, it would be possible to undercool the air so that its temperature entering the second stage would be lower than atmospheric temperature.

Due to the water jackets on the cylinders, the compression line in each stage may be assumed to lie between the adiabatic and isothermal, hence the area *abcdgi* represents the net work done on the air. Without the intercooler in operation, the area of the combined cards would be *abegi*, and the area *bedc* thus represents the *very appreciable* **saving in work** *done by the compressor pistons due to intercooling.*

If the receiver pressure is high enough so that the saving will pay for the added complication, more than two stages may be employed.

(c) The disadvantage of multi-staging is the added equipment that is necessary. In addition to the intercooler and the high pressure cylinder of the two-stage compressor there is the low-pressure cylinder of nearly the same size as for the single-stage compressor of the same capacity. In general, if there were no clearance, leakage or throttling, so that the volumetric efficiency would be 100 per cent, the first stage of the multi-stage compressor could operate alone to deliver air at the receiver pressure and its capacity would be the same as the capacity of the multi-stage compressor running at the same speed. However, dividing the compression up into several stages and intercooling has a markedly beneficial effect upon the volumetric efficiency of a compressor for two reasons:—

First, since the temperature range in the low-pressure cylinder (1st stage) is reduced, the air is heated somewhat less during the suction stroke, hence a slightly greater actual weight will enter the cylinder than would be the case in a single cylinder operating between the extreme pressure limits. But, this effect is very small when compared with the other.

Second, less *weight* of air remains in the clearance of the low-pressure cylinder because the discharge pressure from that cylinder is lower than with a single-stage compressor, which would be of the same size and have the same clearance volume. This will be made clear by Fig. 65 in which the idealized diagrams of a three-stage compressor with clearance are *abcd*, *a′b′c′d′* and *a″b″c″d″*. It will be observed that the expansion of the clearance air in the low-pressure cylinder decreases the volume of the fresh charge by the small amount $V_a - V_d$; whereas if the compression had all been carried out in this one cylinder the clearance air at pressure p_e and volume V_e would have expanded to a volume V_f, at the suction pressure p_f, thus decreasing this charge by the very large amount $V_f - V_a$. This serious loss is one worth preventing, if commercially feasible. The improvement in the volumetric efficiency due to multi-staging is usually sufficient to offset the ill effects of added complication, for the actual multi-stage compressor will have a greater capacity than would the first stage operating alone.

It is obvious from the diagrams and preceding paragraphs that the larger the total pressure range, the greater in every way will be the advantages of multistage compression.

(**d**) For pressures less than 50 lb. per square inch, gage, reciprocating compressors are usually built single stage. Two stages are commonly used for pressures from 100 to 150 lb. per sq. in., and three-, four- and even six-stage machines are sometimes used for very high pressures.

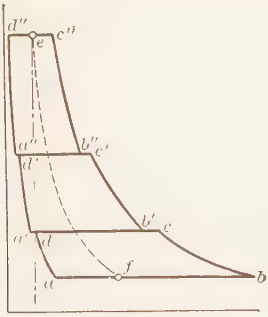

Fig. 65.

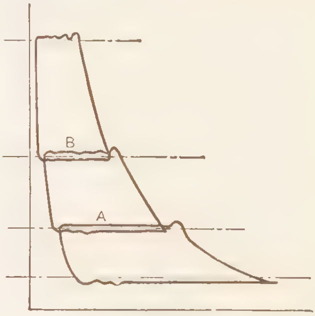

Fig. 66.

(**e**) The differences between the actual diagrams and the ideal ones of each cylinder of a multistage compressor are similar to those which have been discussed for the single-stage compressor. When superposed they look something like Fig. 66, in which the atmospheric, intercooler, and receiver pressure are indicated by horizontal dash lines. In each case the air is drawn into a cylinder at a pressure below that at which it exists outside of the cylinder and is discharged at a pressure higher than that maintained in the vessel receiving the air. This results in an overlapping of the areas in the center of the diagram, giving two loops, A and B, which very evidently represent work lost in forcing the air from one cylinder to the other.

(**f**) The better the action of the valves and the larger the passages through ports, intercooler, pipes and such, the smaller will the areas of these loops become, the upper and lower lines tending to become coin-

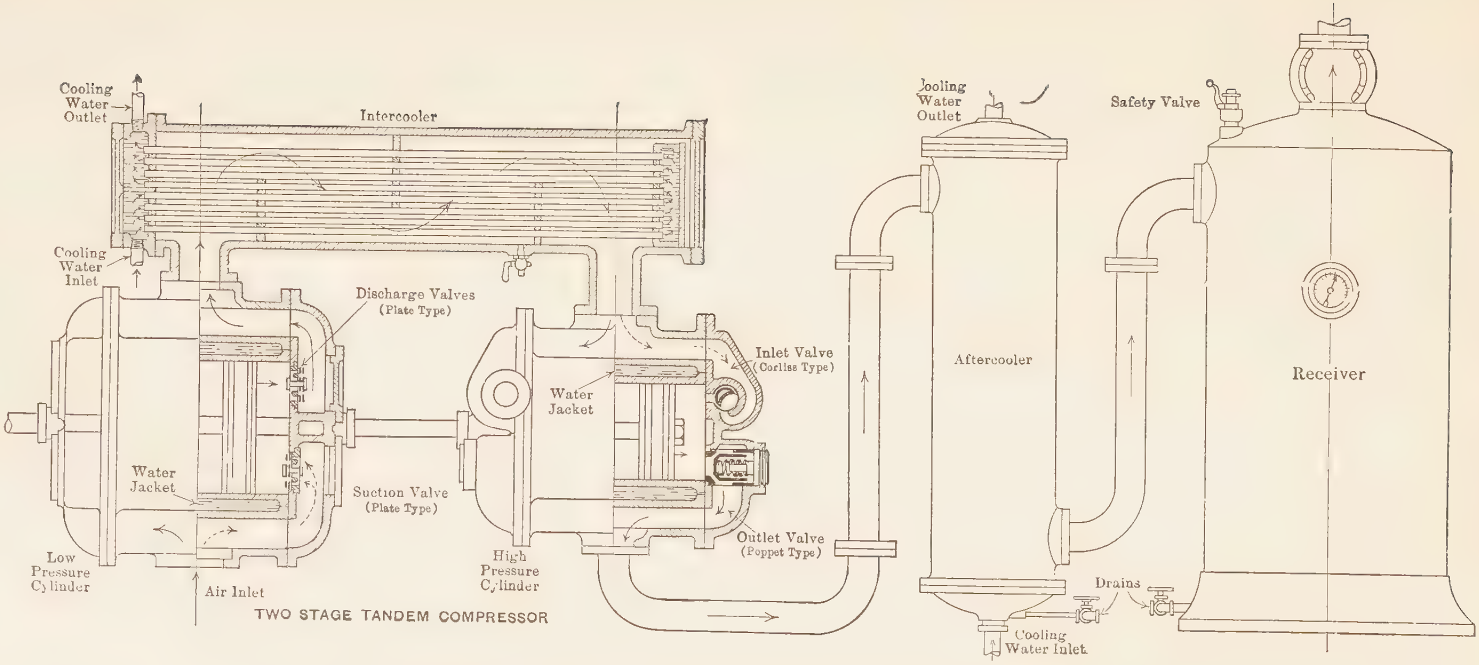

FIG. 67.—Principal Elements of a Two-stage Compressor Plant.

cident. In very well-designed compressors this lost work is so small as to be almost if not quite indeterminate. If large clearance is permitted, then liberal valve areas become possible so that the compressor may run at higher piston speeds without serious throttling losses. In this regard the larger clearance permits a smaller and cheaper machine, hence the designer has to use a clearance volume that is a compromise between the extremely large and small ones.

109. Typical Arrangements.—(a) Compressor plants and their equipment may be arranged in many different ways. Fig. 67 shows one typical layout. In this there is a two-stage compressor with cylinders in tandem and with one type of intercooler between. The method of driving the compressor is not indicated. Steam engines or electric motors are ordinarily used for this purpose.

(b) Between the compressor and the receiver is shown an *aftercooler*. This is arranged similarly to an intercooler, and is placed between the compressor and the transmission line. Its object is to cool the air sufficiently to precipitate most of the moisture in it, in order to avoid trouble from freezing in the pipe lines and tools. It is provided with a drain for removing the moisture that collects. The receiver is also provided with a drain, and has a safety valve and pressure gage.

(c) Reciprocating compressors are generally designed to be *double acting*, unless excessively high pressures (say over 500 lb. per square inch) are involved. Ordinarily there are no unusual difficulties to be overcome in accomplishing this, and the consequent reduction in size, weight and cost make the double acting type the preferable one. The *valves* of a compressor should have liberal passage areas and should open and close rapidly so as to offer little resistance to the flow of the air. They should be so located that the passages between them and the cylinder will be as small as possible in order to minimize the clearance space. Some valves are mechanically actuated, like the Corliss valve shown in Fig. 67; others are opened automatically by the difference in the pressures on their opposite faces, and are closed in a similar manner, assisted by weak springs. Light-weight automatic valves of the plate and poppet types are also illustrated in the figure.

(d) There are many different arrangements of cylinders and pistons possible in multi-stage compressors, the two most usual ones being the *Tandem* and the *Duplex*. In the first, one cylinder is placed behind the other, the pistons being on the same rod, as in Fig. 67. In the second, the cylinders are side by side as in Fig. 68. This latter figure also illustrates the application of a motor drive.

110. Blowing Engines.—Blowing engines, or blowers, are similar to reciprocating air compressors in principle, but they are generally built to handle relatively very large quantities of air at comparatively low pressures, say 10 to 20 lb. per square inch above atmosphere. Comparatively little attention need be given to cooling under such conditions because the pressure is so low that very little work can be saved by such means. Moreover, the compressed air is generally heated before being used, so that any cooling during compression would call for an expenditure of heat to raise the temperature immediately afterward.

Because of the large volumes of air to be handled considerable difficulty is generally met in designing efficient valves, particularly if operated at high speeds. As a result there are many different types of both inlet and discharge valves in use, some operating automatically under the action of springs and air pressure, some mechanically controlled and some partly automatically and partly mechanically operated.

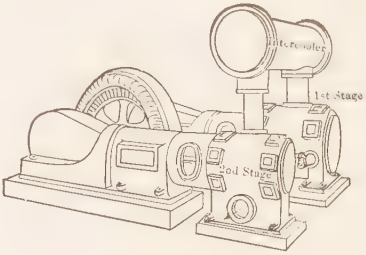

Fig. 68.—Duplex Compressor Driven by an Electric Motor.

Blowing engines are used to supply air for blast furnaces, and are therefore usually driven by gas engines, as the furnaces furnish more than enough gas for this purpose.

111. Lubrication and Explosions.—In lubricating reciprocating compressors it is desirable to use very sparingly a high-grade oil. If too much oil, or an unsuitable kind, is used, it increases the danger of having disastrous explosions in the receiver and pipe lines. It appears probable that the lubricating oil used is decomposed thus yielding highly explosive gases when mixed with the compressed air. The most effective remedy thus far proposed seems to be the use of an arrangement of receiver from which the gases and vapors are continuously swept, thus preventing the accumulation in pockets, or strata, of explosive mixtures of oil vapor and air.[3]

112. The Centrifugal Compressor.—(a) In principle the centrifugal compressor, or turbo compressor, is similar to the centrifugal pump. In each case the machine consists of a number of impellers rotating at high speed inside a suitable casing, as in Fig. 69. The fluid enters the impeller with a definite pressure, temperature and velocity. As a result of the rapid rotation the fluid leaves the impeller with an increased pressure, temperature and velocity. It is desirable to convert as large a part of this velocity head into pressure head as possible, and stationary vanes, called *diffusers*, are often used for this purpose. They are so made that the fluid will be received from the impellers and pass through them with a minimum amount of *turbulence.* From the diffuser the gas is led by an S-shaped passageway to the entrance of the next impeller.

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[3] See "The Prevention of Explosions in Air Receivers," by William F. Parish and William B. Smith Haley in "Power," June 30, 1925, p. 1948.

(**b**) In the case of the centrifugal pump the fluid is nearly incompressible and hence there is very little change in its temperature or specific volume. However, with centrifugal compressors the fluid is compressed to a small fraction of its original volume and consequently there is a large increase in its temperature, unless it is cooled very effectively by some special means.

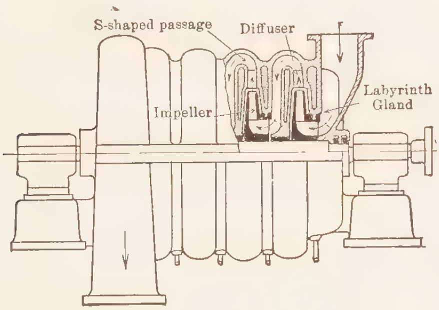

Fig. 69.—Multistage Centrifugal Compressor.

(**c**) For the same reasons as previously given, it is desirable to cool the air during compression, and this should be accomplished by the simplest means possible. It is usually done by *water jackets* surrounding the diffusers; and in the case of the multi-stage type, *intercoolers* are placed between certain groups of stages.

(**d**) The number and size of impellers used depend upon the amount of air to be handled, the pressure desired and the speed at which the compressor is to operate. Often centrifugal compressors are run at very high rotative speeds by means of steam turbines, or by electric motors with speed increasing gears. This combination gives a unit of extremely small size compared with a reciprocating compressor of the same capacity, especially when handling large volumes of air at low pressures. This class of work is the one for which the turbo-compressor is best suited, but this type of compressor is sometimes used for pressures as high as 100 or even 150 lb. per square inch.

(**e**) Since there are no valves in the turbo compressor the fluid flows through it, when operated at constant speed, in accordance with the conditions given in Sect. **43** for steady flow. Hence,

$$\left.\begin{matrix}\text{Energy}\\ \text{to drive}\\ \text{compressor}\end{matrix}\right\} = \left\{\begin{matrix}\text{Energy}\\ \text{gained by}\\ \text{fluid}\end{matrix}\right\} + \left\{\begin{matrix}\text{Energy}\\ \text{given to}\\ \text{jacket water}\end{matrix}\right\} + \left\{\begin{matrix}\text{Energy}\\ \text{lost by}\\ \text{radiation and}\\ \text{conduction}\end{matrix}\right.$$

Then, if w = weight of fluid flowing through compressor, in lb. per sec.
w_j = weight of jacket water used, in lb. per sec.
t_e = temperature of jacket water entering,
t_0 = temperature of jacket water leaving,

and the other notation as given in Sect. **44**, it follows that, for *any fluid*, the horsepower to drive the compressor is

$$\text{hp.} = \frac{w}{0.707}\left[(h_2 - h_1) + \frac{v_2^2 - v_1^2}{2g \times 778} + \frac{z_2 - z_1}{778}\right] + [w_j(t_0 - t_e) + R_c]/0.707. \quad (143)$$

(f) If the fluid flowing through the compressor be any gas that obeys *closely* the laws of an *ideal gas*, such as *air at the usual compressor temperature*, then, from Eq. 55*b*, $h_2 - h_1 = c_p(t_2 - t_1)$. Hence, for such gases, it follows that horsepower to drive the compressor is

$$\text{hp.} = \frac{w}{0.707}\left[c_p(t_2 - t_1) + \frac{v_2^2 - v_1^2}{50{,}000} + \frac{z_2 - z_1}{778}\right] + [w_j(t_0 - t_e) + R_c]/0.707. \quad (144)$$

Usually z_1 and z_2 are nearly equal, and R_c is relatively very small, so they are often ignored. Frequently the velocity term in the above equation is also small and it is often neglected because the value of weight may not always be measured accurately enough to justify the inclusion of such an insignificant term.

113. The Air Engine.—(a) Compressed air is used commercially in many different ways but most widely in engines for the production of power, the air serving as the working substance. At first sight it seems an uneconomical method of producing power as the air compressor must be driven by an engine or motor of some sort which apparently might better be used directly to produce the power desired, rather than to suffer the additional losses incurred in a compressor and in utilization of the compressed air in a second engine.

(b) Such reasoning is generally sound for conditions where the desired power can be conveniently generated at the point of utilization by a prime mover. There are, however, many cases where this cannot be done. Where a number of small engines are to be operated at widely scattered points and where electrical motors are not suitable, compressed air engines find a field to which they are admirably suited. Compressed air can easily be transmitted over great distances without serious loss. Steam, on the other hand, cannot be efficiently transmitted very far because of the resulting condensation; and mere than this, steam engines of small size are very inefficient, and the high temperature at which they operate renders them unsuitable when handling is required. The working substance of internal combustion engines can be transmitted as easily, if not more easily, than compressed air, but the complicated valve and ignition mechanisms, the high temperature and the hot noxious exhaust gases make them less desirable than compressed air engines for a number of purposes. Compressed air is more suitable for many kinds of work with small motors than is electricity, because the air can be conveyed by pipe or hose over all sorts of construction work with a minimum amount of labor and expense and with no danger from fire.

(c) Thus, compressed air engines are widely used in mining and quarrying operations and for the driving of small portable tools in shops and such. In Paris there is installed a central compressor station which distributes compressed air, much as gas is distributed in this country, and air is used by the consumers for operating small plants just as electricity is used here.

(d) The indicator diagram of an ideal air engine, which is without clearance and is provided with sufficient piston displacement to allow the *complete expansion* of the air, is shown by Fig. 70. During the period *1–2*, high pressure air is admitted from the pipe line to the cylinder. During *2–3*, the weight of air so admitted is allowed to expand, according to the law $PV^n = a$ *constant*, to atmospheric pressure, p_3. Then the return stroke of the piston exhausts the air from the cylinder, at constant pressure, according to the process *3–4*.

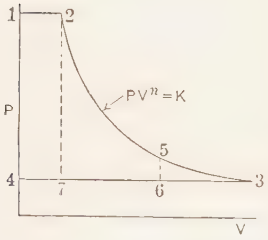

Fig. 70.

The enclosed area *1 2 3 4* measures the net work done upon the piston, during the engine cycle, by the weight of air involved. Summing up the work done by the air during the processes of the cycle, this net work amounts to

$$Wk_{1234} = P_2V_2 + \frac{P_2V_2 - P_3V_3}{n - 1} - P_3V_3$$

$$= \frac{n}{n - 1} w_2RT_2\left[1 - \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}}\right]. \quad . \quad . \quad . \quad (145)$$

(e) Although air turbines, or turbine tools, may permit the complete expansion of the air, reciprocating engines are designed for incomplete expansion, or for no expansion whatever. *Incomplete expansion* would be represented by the indicator diagram *12564* in Fig. 70. It is obvious that, for the same weight of air admitted, the net work done per cycle is less, by an amount represented by the area *5 3 6*, than if the air had expanded completely. However, the piston displacement is reduced from V_3 to V_6 and the resulting reduction in frictional losses in the actual engine will counter-balance to some extent the net work that is lost due to the incomplete expansion. Another factor in favor of the incomplete expansion is the reduced cost of the machine due to the reduced piston displacement. Again referring to Fig. 70, it is evident that the net work *done by the air* per cycle amounts to

$$Wk_{12564} = P_2V_2 + \frac{P_2V_2 - P_5V_5}{n-1} - P_6V_6. \quad . \quad . \quad (146)$$

(**f**) An engine whose indicator diagram is represented by the area *1 2 7 4* in Fig. 70 obviously makes no use whatever of the expansion of the air within the cylinder and operates on the *non-expansive* cycle. Such an engine, or tool, is characterized by very poor economy in the use of the air supplied, but has the advantage of developing the maximum power per unit of piston displacement and of weight. The net work of this cycle is, as seen from Fig. 70.

$$Wk_{1274} = (P_2 - P_7)V_2. \quad . \quad . \quad . \quad . \quad . \quad (147)$$

(**g**) One of the most important applications of compressed air is found in air tools, such as chipping hammers, riveters, drills, etc. They usually operate on the non-expansive cycle because they must be of relatively light weight and portable, cheap in first cost, and simple in construction. Their small size and intermittent operation also reduces the necessity for high economy. The other cycles of operation are important, however, because economy is an important item in some services in which the air consumption is steady and of considerable magnitude.

(**h**) In practice, the air generally enters an engine at about atmospheric temperature, and during the actual expansion it becomes cooled, in some cases to such an extent that the moisture in it freezes and leads to difficulties. The temperature decrease in any real engine will not be equal to the adiabatic, because heat will be supplied to the engine cylinder from the surrounding atmosphere and will tend to make the expansion somewhat more nearly isothermal. If the engine could be operated with extreme slowness the expansion would very closely approach a true isothermal.

(i) This approach toward isothermal expansion is advantageous, when using air at about atmospheric temperature, for the following reasons:

(1) It tends to prevent the deposition, in the form of ice, of the moisture accompanying the air, thus tending to prevent the resultant troubles with lubrication and stoppage of valves and passages.

(2) It increases the work made available, as the area under an isothermal expansion is greater than that under the steeper adiabatic between the same two pressures.

(3) It increases the mechanical efficiency of the engine by keeping the lubricating oil at a higher temperature.

(4) It reduces by a very small amount the weight of air required per cycle as there is less tendency to cool the entering air.

(5) It permits the use of a slightly smaller machine for the same output.

(j) It is interesting to note that, from the viewpoint of the conservation of energy, isothermal operation is not as advantageous as adiabatic. The object of using the air expansively is to utilize some of the internal energy of the working substance which enters the engine. If the expansion is isothermal no work can be done at the expense of such energy; on the contrary, heat equivalent in quantity to the work done during the expansion period must be supplied from an external source. With an adiabatic expansion, however, all of the work done during such an expansion will be at the expense of the *internal* energy of the gas.

The apparent discrepancy between these two cases is due to the fact that during the isothermal expansion it is assumed that the required amount of heat is supplied from the atmosphere, and that it costs nothing, and may, therefore, be freely used without decreasing the commercial efficiency of the process.

(k) Air engines are found to operate more quietly, and therefore more satisfactorily, when the exhaust valve closes before the end of the stroke, thus trapping some air which is then compressed into the clearance. Such operation causes a loss of diagram area and therefore a loss of work from a given size of cylinder running at a given speed. It may, however, result in a saving in two ways in the amount of air used per horsepower hour:

(1) When compression is not used, the air admitted must first be mixed with that in the clearance until the full admission pressure is attained; after that, the entering air becomes available for driving the piston.

(2) Compression tends to raise the temperature of the walls, cyl-

inder head and piston and thus to decrease the cooling effect upon the incoming air.

114. Preheating. (a) In most cases, compressed air engines and the compressors supplying the working fluid are generally widely separated. It has already been shown that so far as the *compressor* is concerned the cooler the air the better. A cool supply means a larger capacity for a given machine and efficient cooling during operation means a smaller amount of work required. The same thing is true for the " receiver," or storage tank, and for the pipe line carrying the air to the engine, for the cooler the air the smaller can these parts be for a given quantity of air.

(b) Conditions are, however, quite different so far as the *engine* is concerned. The warmer the air, within reason, the better.

If the compressed air could be heated at constant pressure before entering the engine, it would expand according to Charles' law. A given volume of heated air admitted to the engine would represent a smaller actual weight, but would be able to deliver the same amount of work as a larger weight of colder air, and there would be the added advantage that there would be less danger of the moisture freezing at the end of expansion; also the friction losses in the reciprocating air engine will be considerably reduced because of the higher temperature of the cylinder oil.

(c) Such heating of the air is known as " preheating " and the devices in which it is effected are called " preheaters." It is actually used in places where the transmission piping is of great length and also where the engine units are few and of large size. It is found in practice that the running expense for the fuel supplied for preheating is less than the extra charges against the larger compressor and pipe line which would otherwise be used.

115. Air Turbine Tools.—Recently there has been developed a line of compressed air tools operated by air turbines, instead of by reciprocating engines. They are particularly well suited for applications requiring high rotative speed, such as drills, or grinders. They possess the advantage of light weight, simplicity and no vibration. Furthermore, their air consumption is relatively low, due to the fact that the air is used expansively in the nozzle, even though the turbine may be a very small one, such as would be used for small tools. After long use these turbines may have the same air consumption as originally, since the wear does not affect the parts between which leakage may occur.

116. Performance of Air Compressors.—(a) For reciprocating compressors the indicator diagram taken from any actual machine enables the indicated horsepower to be obtained when the compressor is handling a known amount of air, usually measured in terms of the volume of free air per minute. The horsepower calculated for this same amount of air, with isothermal compression, and for constant suction and delivery pressures, is called the *isothermal horsepower.* The ratio of the isothermal horsepower to the actual indicated horsepower is commonly designated as the *efficiency of compression,* or *compression efficiency.* Clearly the compression efficiency will be decreased by leakage and inefficient cooling.

(b) The *mechanical efficiency* of the compressor is the ratio of the i.h.p. of the air cylinders to the horsepower required to drive it. The air i.h.p. includes the power used in overcoming the *fluid friction* due to the resistance of valves and passages. The power needed to drive the compressor exceeds the air i.h.p. by the amount absorbed in *mechanical friction.*

(c) The product of the mechanical efficiency and the compression efficiency gives the true measure of the performance of the compressor and may therefore very properly be called the *compressor efficiency.*

(d) The *overall efficiency,* or the *overall isothermal efficiency,* is the term that most owners of compressors are usually interested in, because it represents the ratio of the isothermal horsepower of the compressor to the horsepower supplied by the engine, turbine or motor used to drive the machine. This term includes all mechanical losses in the mechanism used for transmitting this power to the compressor. In the case of the steam-driven reciprocating compressor it has been common practice to take the i.hp. of the steam cylinders as the input to the compressor, whereas, strictly speaking, the mechanical losses of the steam end should be deducted in order to obtain the true input to the compressor.

(e) The *overall adiabatic efficiency* is sometimes used as a standard when comparing the performances of compressors, especially centrifugal ones. The term has the same meaning as the one defined in the previous paragraph except that the air horsepower is calculated for *adiabatic* compression instead of isothermal.

(f) An indication of the magnitude of the losses occurring in a modern centrifugal air compressor is shown in Plate V of the Appendix. The American Society of Mechanical Engineers' Test Code for Displacement Compressors and Blowers, and the Code for Centrifugal and Turbo Compressors, are recommended for use when commercial tests are involved.

CHAPTER XI

GAS CYCLES

117. Definition of a Cycle.—(a) If a given quantity of a working substance be used in any kind of an engine so as to obtain *all the work possible* under given circumstances, the same amount of work cannot be again obtained under the *same circumstances* unless the substance is first returned to the same *initial condition.* Thus a pound of water falling a given distance will develop a certain amount of work, and that work will be the greatest obtainable under the circumstances when the water falls to the lowest possible point. To again develop the same amount of work with the same pound of water, it must first be raised to the height from which it originally fell. Or, similarly, if one pound of gas does work by expanding adiabatically from a temperature T_1 to a temperature T_2, which is the lowest possible under the conditions, it cannot again do the same amount of work in the same way until its temperature is again raised to the initial value T_1.

(b) In order to deliver work *continuously,* as is generally required, there are obviously only two possible methods of operation: either (1) *the working substance must be periodically returned to initial conditions,* or (2) *new quantities of working substance must be supplied* at regular intervals. The latter is the simpler and is often used, but it is not Nature's method. If man uses falling water to develop power and allows the water to run to waste at the lower level, Nature immediately begins to lift it by evaporation due to the sun, hence sooner or later it will be again available. If man burns carbon in air, getting hot CO_2 and N_2, and then, after obtaining work by lowering the temperature, discharges the gas as useless, Nature through the agency of plant growth decomposes the cold CO_2 into C and O_2, after which they can again be combined to evolve the same amount of heat energy as before.

Thus, without man's agency, all working substances periodically return to the same starting conditions, that is, pass through *cycles.*

A **cycle** *is any series of operations which periodically brings the working substance back to its initial condition.*

It is customary to speak of **Open** and **Closed Cycles**, but there are really no open cycles in engineering. If the engineer carries a working substance through any series of changes which does not return it to initial conditions, Nature kindly closes the cycle for him.

(c) One difficulty here confronts the beginner: Experience shows that it requires at least as much energy to pump water between levels as can be obtained from it in flowing down again; this being true, how is man to obtain available work from a substance if equal work has to be done to raise the material to starting conditions? There are two solutions which amount to the same thing in the end:

1. Allow Nature to do the pumping, or
2. Imitate Nature in finding some way of pumping that does not require the expenditure of the useful form of energy which has been obtained from the cycle.

When a heat engine is used, *heat energy is available* but *mechanical energy is sought.* Most of the methods in use for returning the working substance to initial conditions in such cases depend upon the use of a small amount of the generated mechanical energy and a large amount of the available heat energy, obtained by burning some kind of fuel for this purpose; or, they employ some group of processes which are the substantial equivalent of this.

118. Method of Analysis of Any Cycle.—There are many varieties of *heat engine cycles*, but regardless of the working substance, or the particular kinds of processes making up the cycle, there are always the same fundamentals involved in the analysis of any ideal cycle.

(a) *Heat is supplied,* from some outside source, to the working substance during one or more processes. This heat, which it is convenient to designate by Q_1, is the only supply of energy received by the working substance from any source outside of the engine.

(b) During certain parts of the cycle it is necessary to *reject heat* from the working substance to some outside body that is at a temperature low enough to absorb it, just as water is rejected from a water wheel at the tail race, at low elevation, after it has given up some, *but not all,*[1] of its energy to the wheel in passing through it. Let the heat rejected per cycle be represented by Q_2.

(c) The cycle considered, being an ideal one, an *ideal heat engine* is involved. This means that there is no loss by leakage, none by any kind of friction of the fluid or mechanism, no radiation, conduction or convection losses of heat, and no loss by free expansion or turbulence. In other words, all compressions and expansions of the working substance except those in which Q_1, Q_2, or regenerators,[2] are involved must be reversible adiabatics. All processes are considered to take place in such a manner that the working substance is in equilibrium and the curves representing these processes may therefore be drawn on any diagram, such as the PV and $T\phi$.

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[1] In fact it is very common for this same rejected water to give up more energy in passing from the tail race to the sea than it gave up to the water wheel in passing through it; but such energy as may be obtained from the water after leaving the water wheel is not available for use in this wheel at the particular location under consideration.

[2] The explanation of regenerators will be given later, in Sect. **125**.

(d) From the law of the conservation of energy, combined with **(a)**, **(b)** and **(c)** above, it must follow that the difference between the heat supplied and the heat rejected during the ideal cycle represents the amount of *heat transformed into mechanical energy*, or *the net work of the ideal cycle*, which may also be spoken of as the "*available energy*" as it means the energy available for doing work in an ideal engine operating on the specified cycle. The symbol, Wk_i, will be used to represent the net work of the ideal cycle, hence

$$Wk_i = Q_1 - Q_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (148)$$

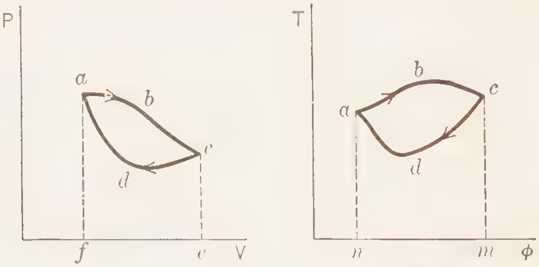
(e) The *efficiency of an ideal engine is always expressed by the ratio of the net work of the cycle to the heat supplied during the cycle.* This definition follows from the basic conceptions of efficiency, and of heat engines. Efficiency is a general term used by engineers to express the ratio of *useful return* to *expenditure.* When dealing with ideal engines both of these terms refer only to energy, because such engines can never be built and their cost cannot therefore be a factor, although in real engines to approach the ideal, one of the big factors involved is the cost of making, maintaining and operating such a machine. Since the heat engine is a machine to transform into useful mechanical energy as much as possible of the heat supplied for the purpose, it must follow that the *efficiency of the ideal engine*, or the *efficiency of the ideal cycle* is

$$e_i = \frac{Wk_i}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}. \quad . \quad . \quad . \quad . \quad (149)$$

Clearly this equation shows that for high cycle efficiency the ratio $\frac{Q_2}{Q_1}$ must be made as small as possible. The best way of doing this, for different types of engines operating under a variety of conditions, constitutes a very important problem that is best approached by the beginner by analyzing a number of cycles that are used as ideals for comparison when considering the results obtained from real engines.

(f) The *pressure-volume* and the *temperature-entropy* diagrams are always of great assistance in making the analysis of a cycle. As an illustration, consider any cycle, such as *abcd* in Fig. 71(a) and (b), which may be drawn as soon as the processes constituting the cycle are definitely specified. Applying the principle established in Sect. **40**, it follows that, during processes *ab* and *bc*, $\int_a^{c} {}_b\, Pdv =$ area *abcef*, which represents the total positive work done on the piston by the working substance during the entire cycle. Throughout the remainder of the cycle, processes *cd* and *da*, the piston is doing work upon the gas; hence $\int_{c}^{\substack{a\\d}} Pdv$ = area *cdafe*. Thus, the *net work* delivered to the piston during the entire cycle is the difference of these two areas, or *abcda*.

Similarly, applying to the $T\phi$-diagram of Fig. 71(*b*) the principles given in Sect. **58**, it follows that the heat addition must occur during the processes *ab* and *bc*, as it is only during these that the entropy of the working substance is increasing; and that heat abstraction takes place only during the processes *cd* and *da*. In other words, $Q_1 = \int_{a}^{\substack{c\\b}} Td\phi$ = area *abcmn*, and $Q_2 = \int_{c}^{\substack{a\\d}} Td\phi$ = area *cdanm*. Consequently the difference, $Q_1 - Q_2$, or the net area *abcd* on the $T\phi$-diagram, must be the amount of *heat that is transformed into work during the cycle.* Observe that, on the $T\phi$-diagram, the areas representing Q_1 and Q_2 must extend down to absolute zero of temperature.



(*a*) (*b*)

FIG. 71.

(g) It is important to note that, in general, the value of the heat supplied during an entire cycle is very different from the positive work done during the same cycle. In other words, the heat represented by the area *abcmn* on the $T\phi$-diagram is usually larger than the heat equivalent of the area *abcef* on the PV-diagram. Consequently, when expressing the efficiency of a cycle by means of a ratio of areas, these areas should be taken on the $T\phi$-diagram. Thus, for the cycle shown in Fig. 71(*b*), the efficiency = area *abcda* ÷ area *abcmna*,—which may be quite different from area *abcda* ÷ area *abcefa* in Fig. 71 (*a*).

119. The Carnot Cycle for Gases.—(a) This cycle, named from Sadi Carnot, the man who in 1824 first investigated it, represents the best that can possibly be done in the conversion of heat energy into

mechanical energy.[3] It cannot be carried out in any actual engine, but it is of great value *as a criterion of the maximum result approachable.*

(b) For performing such a cycle with *gas*, in addition to having the working substance, it is necessary to have certain apparatus, specified below. The working substance may be any gas far enough removed from its point of liquefaction to obey the laws already developed for gases.

The necessary apparatus is shown in Fig. 72 and may be described as follows:

U is a body at high temperature T_1 and so arranged that this temperature remains constant despite withdrawal or addition of heat energy. An ordinary furnace with a controllable fuel and air supply approximates these conditions. Similarly, a hot liquid enclosed in a large, tight vessel under high pressure will give off a considerable amount of heat with only a very small drop in temperature. The body U will hereafter be called the **Hot Body.**

X is a body at temperature T_2, lower than T_1, and this temperature T_2 remains constant despite addition or removal of heat energy. A vessel jacketed with flowing water at temperature T_2 would approximate these conditions. The body X will hereafter be called the **Cold Body.**

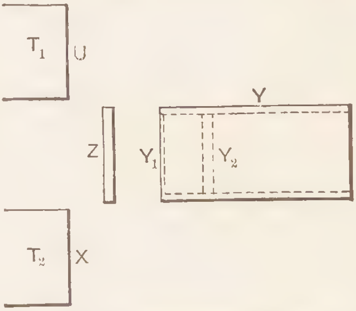


Fig. 72.—Machinery of Carnot Engine.

Y is a cylinder, Z is a removable plate which may be used to cover the end of the cylinder, and Y_2 is a frictionless piston. These parts are made of material that will neither absorb nor conduct heat. Y_1 is a cylinder head made of material that offers no resistance to the flow of heat.

Operation of Carnot Engine

(c) Imagine, first, that w pounds of gas are inclosed in the cylinder Y in condition a, as shown at a in Fig. 73(a) and (b), T_a being equal to T_1, the temperature of the hot body.

(1) Remove cover Z, apply the hot body to the conducting head Y_1, and allow the gas to *expand isothermally* to some lower pressure P_b. When this pressure has been reached the volume will have become V_b and the entropy has increased by the amount $\Delta\phi_{ab}$ as shown. The necessary heat supply, Q_1, must have come from the hot body and is

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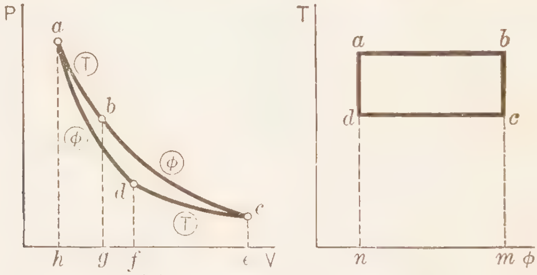
[3] This statement must not be interpreted to mean that no other cycle can do as well; it means only that no other cycle can do better. It will be shown later that there are a number of ideal cycles equally efficient as energy transformers.

represented by the area *abmn*, since *ab* is the only process during which heat is supplied.

(2) Next remove the hot body, apply the nonconducting cover Z, and allow the gas to *expand* by a reversible *adiabatic* until its temperature has dropped to T_c, equal to T_2, the temperature of the cold body. The gas will then be in the condition represented by the state c as shown by each diagram.

(3) Again remove the cover Z, then apply the cold body X, and drive the piston back, *compressing* the gas *isothermally* to some higher pressure P_d whose value will be considered in the next paragraph. Since the temperature of the gas is to remain constant at T_2 during the compression from c to d, heat must be absorbed by the cold body from the gas at the same rate that work is done upon the latter during this period. As this is the only period during which heat is abstracted from the working substance, Q_2 must be represented by the area *cdnm*.



(*a*) (*b*)

FIG. 73.—The Carnot Cycle.

(4) For the fourth and completing operation, remove the cold body, replace the nonconducting head Z, and drive the piston back, *compressing* the gas by a reversible *adiabatic* until its temperature has again risen to that of the hot body, which was the starting temperature of the cycle. To close the cycle, the pressure and volume must return to P_a and V_a when T_1 is reached. This can only be attained if the isothermal compression is stopped at such a point d that the subsequent isentropic compression will return the gas to the starting condition, a.

Analysis of the Carnot Cycle

(d) The area *abceha* represents work done *by* the gas, while the area *cdahec* represents work done *upon* the gas. The net work resulting from one cycle is shown by the inclosed area *abcd*. If the pressure is plotted in pounds per square foot and the volume in cubic feet, the net area *abcd* will be in foot-pounds.

Note that this cycle consists of two isothermals joined by two isentropics. The isothermal T_1 is an expansion with ratio $\frac{V_b}{V_a} = r$, and the isothermal T_2 is a compression with ratio $\frac{V_c}{V_d} = r'$. These two ratios must be equal because, by Eq. (105),

$$\frac{T_1}{T_2} = \frac{T_b}{T_c} = \left(\frac{V_c}{V_b}\right)^{\gamma-1}$$

and

$$\frac{T_1}{T_2} = \frac{T_a}{T_d} = \left(\frac{V_d}{V_a}\right)^{\gamma-1}$$

giving

$$\frac{V_c}{V_b} = \frac{V_d}{V_a}, \qquad \text{or} \qquad \frac{V_b}{V_a} = \frac{V_c}{V_d};$$

hence

$$r = r'.$$

The mathematical expression for net work done per cycle can be obtained by using the formulas already developed for reversible isothermal and adiabatic processes. The results are tabulated below.

Process	Curve	Heat Received by the Gas		Work Done by the Gas	
		In B.t.u.	Area	In Ft.-Lbs.	Area
Isothermal Expansion	*ab*	$wART_1 \log_e r$	*abmn*	$wRT_1 \log_e r$	*abgh*
Adiabatic Expansion	*bc*	0	0	$\frac{wR}{\gamma - 1}(T_1 - T_2)$	*bceg*
Isothermal Compression	*cd*	$-wART_2 \log_e r$	$-$*dcmn*	$-wRT_2 \log_e r$	$-$*dcef*
Adiabatic Compression	*da*	0	0	$-\frac{wR}{\gamma - 1}(T_1 - T_2)$	$-$*adfh*
		Heat Transformed		Net Work Done	
Carnot Cycle	*abcd*	$wAR(T_1 - T_2) \log_e r$ . . . Eq. (150*a*)	*abcd* on $T\phi$-diagram	$wR(T_1 - T_2) \log_e r$ . . . Eq. (150*b*)	*abcd* on *PV*-diagram

(e) The *efficiency* of the Carnot Cycle may be derived very readily in three ways as follows:

$$\text{Eff.} = \frac{Q_1 - Q_2}{Q_1} = \frac{wART_1 \log_e r - wART_2 \log_e r}{wART_1 \log_e r} = \frac{T_1 - T_2}{T_1}; \quad (151a)$$

or, expressing the heat supplied and the net work in foot-pounds,

$$\text{Eff.} = \frac{\text{Net Work}}{\text{Heat Supplied}} = \frac{wR(T_1 - T_2)\log_e r}{wRT_1 \log_e r} = \frac{T_1 - T_2}{T_1}; \quad . \quad (151b)$$

or, referring to the $T\phi$-diagram of the Carnot Cycle, which must always be a rectangle for *any working substance*, since it is always composed of two isothermals and two isentropics, it follows that

$$\text{Eff.} = \frac{\text{area } abcd}{\text{area } abmn} = \frac{(T_1 - T_2)(\phi_b - \phi_a)}{T_1(\phi_b - \phi_a)} = \frac{T_1 - T_2}{T_1}. \quad . \quad (151c)$$

Objection is often made to the expressions of efficiency just developed because it seems as though the engine ought to be credited with the heat given to the cold body. The fallacy of this appears when it is understood that the heat given to the cold body leaves the engine at a *low temperature*, T_2, whereas to operate the engine heat must be available at *a high temperature* T_1. The heat rejected to the cold body *could not*, therefore, *be directly used again* in the same engine, and hence mechanical engineers have agreed to consider the heat rejected to the cold body as wasted as far as the engine is concerned.

To make this heat available again for use in the same engine, it would have to be returned to the engine by way of the hot body at the temperature T_1, but *experience shows that heat will not flow of its own accord from any body to one at a higher temperature.* From the discussion which follows, it will be seen that *at least as much mechanical energy would be consumed in causing such a flow as could be obtained by using this heat in a heat engine.* It will be discovered that this is all in accord with the Second Law of Thermodynamics.

The case is analogous to that in which water leaving a water wheel is pumped again to the original height in the attempt to utilize the energy still possessed by the water when leaving the wheel. Obviously, in this case the energy leaving the wheel with the effluent water is of no further use to that particular wheel, and exactly the same thing is true of the heat energy leaving the Carnot engine.

(f) Fig. 74 is intended to show graphically the energy changes of the Carnot cycle. If vertical distances between heat reservoirs T_1 and T_2 in the figure represent temperature, and widths of streams represent quantities of energy, the sense of the foregoing discussion becomes graphically evident.

The dotted part of the figure shows how a part of the effluent energy might be used if another cold body with temperature T_3, lower than T_2, could be obtained.[4] The ultimate limit to this arrangement would be an engine having a cold body with temperature at absolute zero.

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[4] The possibility of the existence of cold body T_3 immediately suggests the use of only one engine operating between temperatures T_1 and T_3. There is no theoretical objection to this, but sometimes when analogous schemes are tried with real engines a number of practical considerations dictate the use of several engines in series, as above, rather than one engine working through the entire temperature drop. The reasons will be considered later.

It is of interest to note that, if this limiting case could be produced, *the Second Law of Thermodynamics would no longer be true* because the last engine of the series would reject no heat, having reduced the temperature of its working substance to absolute zero. All the heat entering the group of engines could then be completely and continuously converted into mechanical energy. *It is obviously an impossible proposition,* because of the absurd assumption that any body could be maintained indefinitely at absolute zero of temperature without the expenditure of work in a continuous process of refrigeration, even after granting the possibility of reaching absolute zero—something that has never yet been done.

120. Reversibility of the Carnot Engine, and Available Energy.—(a) In the preceding section it has been shown that no matter what the working substance may be the efficiency of this heat engine cycle is $\frac{(T_1 - T_2)}{T_1}$. It is now very important to remember that T_1 is not only the maximum absolute temperature attained by the working substance during the entire cycle, but it is also the absolute temperature of the hot body, or the source of heat. Similarly, T_2 is not only the minimum absolute temperature of the working substance, but it is also the absolute temperature of the cold body. With these facts in mind and the discussion of availability and reversibility with heat transfers, as given in Sect. **52**, it follows that *the transfer of heat to or from the working substance in the Carnot Cycle is in a reversible manner;* or, in other words, the two isothermal processes are reversible and may therefore proceed equally well in either direction. The two adiabatic processes are by hypothesis assumed to be reversible ones since there is to be no turbulence within the working substance at any time. **The entire cycle must therefore be reversible;** that is, it must be possible to operate the machine, starting at the point *a* in Fig. 73 and following the cycle backwards in the direction *adcb*.

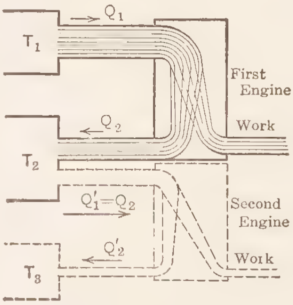

FIG. 74.—Energy Flow in Carnot Engine.

There is no reason why the gas cannot *expand adiabatically* from *a* to *d*, and then *isothermally,* at temperature T_2, from *d* to *c*. During the latter process it would *absorb heat* Q_2 from the cold body If the necessary mechanical energy is available the gas can be *compressed adiabatically* from *c* to *b*, and then *isothermally,* in contact with the hot body, to the starting point. During the isothermal compression the gas *must give to the hot body the amount of heat* Q_1 *exactly equal to that previously removed during the direct operation.*

(b) In the PV-diagram, Fig. 73(a), the work done by the gas during the two expansions must be represented by the area *adceh,* and that done on the gas during the two compressions must be shown by the area *ecbah.* The net result must then be the *absorption of external work equal to that given out* in the direct cycle and represented by the area *adcb.* Also, from Fig. 73(b), the area *dcmn* is now the heat absorbed from the cold body and the area *banm* is the heat delivered to the hot body.

Tabulation of the results of operation, first direct and then reversed, gives:

Direct Operation.		Reversed Operation.
Heat absorbed from hot body =	Q_1	= Heat delivered to hot body.
Heat discharged to cold body =	Q_2	= Heat absorbed from cold body.
Mechanical energy delivered =	$(Q_1 - Q_2)$	= Mechanical energy absorbed.

Thus the Carnot engine reversed can remove heat at low temperature from the cold body and, by receiving *a certain quantity of mechanical energy,* can deliver the sum of these two energies to the hot body as heat at high temperature. It is therefore a heat pump.

(c) From the above discussion it is evident that a Carnot engine is able to convert continuously into mechanical energy some of the heat supplied to it, and this can be accomplished only by permitting a drop in temperature of the working substance within the engine,—just as a water wheel can deliver mechanical energy only by permitting a drop in elevation of the water passing through the wheel. The net work of the cycle, or *the available energy,* is seen to be directly proportional to the difference in temperature between the hot and cold bodies,—just as the energy available from a waterfall is dependent upon the difference in elevation between the water entering the penstock and the tail race. It now remains to be seen whether any other heat engine cycle can be devised to yield a greater amount of net work, or, in other words, have a higher efficiency when operating between the same temperature limits.

121. Efficiencies of All Reversible Engines.—(a) There are many possible types of reversible and irreversible ideal engines. It will now be proved that, when working between the same temperature limits,—i.e., receiving heat from a hot body at the same temperature as that supplying the Carnot engine and rejecting heat to a cold body at the same temperature as that used with the Carnot engine,—(1) *no engine whatever can have higher efficiency than the Carnot engine; and* (2) *the efficiency of all reversible engines equals the efficiency of the Carnot engine.*

(b) To prove (1): Assume that any engine, *A*, is more efficient than the Carnot engine, *C*. Obviously *A* could deliver more mechanical energy than could *C*, although receiving the same amount of heat; and the heat rejected to the cold body would evidently be less by *A* than by *C*, by an amount equal to the difference between the quantities of mechanical energy delivered by *A* and *C*.[5]

Let *A*, operating as an engine, drive *C* reversed, that is, as a heat pump. This is shown diagrammatically in Fig. 75, in which the width of stream is supposed to be a measure of the energy flow. From the combination there would result an excess of mechanical energy, E_2, which could be used outside the system. This excess mechanical energy would be exactly equal to the *only heat supplied the system*, that is, to E_2' given by the cold body. Therefore the combination could *continuously* convert into mechanical energy *all* the heat supplied it; but this would be contrary to human experience as expressed in the Second Law of Thermodynamics. Since the assumption that *A* is more efficient than *C* results in this impossibility, it follows that that assumption must be incorrect, and that no heat engine, reversible or irreversible, can have an efficiency greater than that of the Carnot engine. Hence statement (1) is correct.

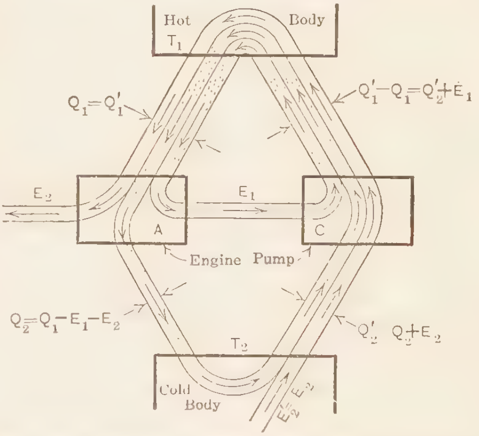

Fig. 75.—Diagram to Show that No Engine Can Have a Greater Efficiency than the Carnot.

(c) To prove (2), that if the engine *A* is reversible it must have the same efficiency as the reversible Carnot engine *C* working between the same temperature limits, imagine it to have an efficiency less than that of *C*.[6] Being reversible, it can be used as a heat pump driven by *C*. Then, if the pump can be less efficient than the engine, the same impossible conditions again appear, merely interchanged. Hence, neither engine can be more efficient than the other, so the efficiencies of all reversible engines working between the same temperature limits must be the same, which proves (2).

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[5] Because heat received = heat discharged + mechanical energy delivered. With the left member of the equation constant, neither term of the right member can vary except at the expense of the other.

[6] It has already been proved that its efficiency cannot be greater than that of *C*.

122. Comparison of Carnot Engine and Real Engine.—(a) The Carnot engine as described above is evidently only an ideal mechanism, for the material assumed for the parts does not exist and a perfect apparatus could not be constructed. Even if it could be constructed its cost would be prohibitive as the *rate* at which heat is transferred from one body to another depends upon the difference in temperature of these two bodies, which means that a reversible isothermal is the limit of slowness in this regard. The slower the rate of heat transfer the larger the size of the engine and consequently the greater the cost. It is possible, however, to approach such ideals and they may therefore be regarded as limiting cases for actual constructions. Comparing the real engines with the Carnot as a standard gives a measure of the perfection of attainment.

In any actual engine, the piston, which itself meets with frictional resistance, is connected to a friction-burdened mechanism. In the real engine, provision must also be made for storing part of the energy delivered during the expansion, to be returned for the compression of the working substance, and this storage and return always involves a slight waste. In the reciprocating engine, for instance, this energy-storing device is usually a fly-wheel, and a small portion of the energy stored is lost in friction and windage.

(b) Obviously there must be the following losses in any real engine:

1. There must be a very considerable difference in temperature maintained in bodies between which heat is to be transferred rapidly.
2. Some of the heat received from the hot body must be lost as heat through conduction and radiation by the material of the engine.
3. Some of the mechanical energy delivered to the piston must be lost by friction in the mechanism of the engine.
4. Some of the energy stored for compressing the working substance must be lost by friction during its storage and its return.

123. Criterion of Maximum Efficiency.—(a) A study of the classical cycle devised by Carnot one hundred years ago shows very clearly what must be the objective in order to obtain the maximum amount of available energy from any ideal heat engine working between any two given temperatures. It may be stated as follows: That an ideal[7] engine may have the maximum possible efficiency, $\frac{(T_1 - T_2)}{T_1}$, when receiving heat from a body at temperature T_1 and rejecting heat to a body at temperature T_2, it is necessary that—

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[7] The definition of an ideal engine as given in Sect. **118** (**c**) should be carefully reviewed at this time.

(1) *All heat received from the hot body be taken when the working substance has the same temperature as that body; and*

(2) *All the heat rejected to the cold body be given it when the working material has the same temperature as that body.*

(b) Even though the above criterion is extremely difficult to approach when building actual engines the result of Carnot's wonderful work is always the basic principle upon which the engineer tries to design various types of engines,—although he must violate these principles to a considerable degree in all cases, because of other important factors, two of which are size and cost of the real engine for a given power output. Just how the various types of engine cycles operate and how they compare with the Carnot and why they possess certain advantages and disadvantages will be considered in the following pages.

124. Refrigerating and Warming Machines.—(a) With the Carnot cycle carried out in the reversed order from that of the engine, there result two possible types of ideal machines, refrigerating and warming, that are of engineering importance. Since the cycle is reversed, mechanical energy is no longer delivered *by* the apparatus, but instead must be absorbed in order to keep the machines in operation. The **refrigerating machine** is operated for the *purpose of abstracting heat from a body that is to be maintained at a temperature lower than its surroundings.* The **warming machine**, first suggested by Lord Kelvin[8] in 1852, is one for *delivering heat to a body that is to be maintained at a temperature higher than its surroundings.* Both of these types of machines may be called **Heat Pumps.**

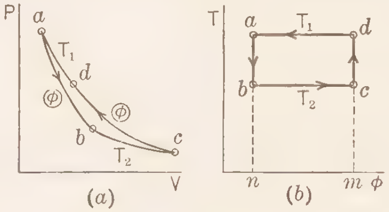

FIG. 76.

(b) [9] In the case of the reversed Carnot engine, see Fig. 76, the working substance absorbs heat from the cold body while both are at the same temperature, T_2, and, similarly, it delivers heat to the hot body while both are at the same temperature, T_1. This isothermal heat transfer represents the limit of perfection and, of course, it could never be realized in an actual machine,—but it serves as an ideal for comparison.

(c) Instead of speaking of the efficiency of the reversed Carnot engine, it has become common practice to use the term "**coefficient of performance,**" *c.o.p.* For the *refrigerating machine,* referring to Fig. 76, this measure of performance is

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[8] See Collected Papers of Lord Kelvin, Vol. 1, p. 515, or Proceedings of Phil. Soc. of Glasgow, Vol. III, p. 269.

[9] In brief courses it may be desirable to omit the remainder of this section.

$$\text{c.o.p.} = \frac{\text{Desired result}}{\text{Energy}\,^{10}\text{ that must be expended to attain this result}}$$

$$= \frac{\text{Heat absorbed from the cold body}}{\text{Mechanical energy required to operate machine}}$$

$$= \frac{\text{area } bcmn}{\text{area } abcd} = \frac{T_2(\phi_c - \phi_b)}{(T_1 - T_2)(\phi_c - \phi_b)} = \frac{T_2}{T_1 - T_2}. \quad . \quad (152)$$

For the *warming machine*

$$\text{c.o.p.} = \frac{\text{Heat delivered to the hot body}}{\text{Mechanical energy to operate machine}}$$

$$= \frac{\text{area } danm}{\text{area } abcd} = \frac{T_1(\phi_d - \phi_a)}{(T_1 - T_2)(\phi_d - \phi_a)} = \frac{T_1}{T_1 - T_2} \quad . \quad (153)$$

Since it is always desirable to use as small an amount of energy to drive the machines as possible, it follows that the larger the coefficient of performance of both these ideal heat pumps the better. The coefficient is often much greater than unity, and its value increases as T_1 and T_2 approach each other,—which is just the opposite from the conditions for highest efficiency in the case of the Carnot engine. Actual refrigerating machines operating on a cycle somewhat different from the Carnot are in common use to-day and will be discussed more fully in the chapter on Refrigeration.

(d) **Warming machines** are not common at present, but with further development of certain of the apparatus needed, and under very unusual conditions, they may become commercially feasible for producing small temperature increases. For instance, such heat pumps are sometimes used in connection with evaporating apparatus,[11] and, under very exceptional circumstances, they might be used for warming buildings in certain localities where the climate is mild. Their application for the latter purpose can be more easily understood after first considering a hydraulic analogy, such as will now be given.

(e) Assume, for illustration, that at a point A, which has an elevation z_1 of 4000 ft. above sea level, there is a continuous but small supply of water which has to be conserved as much as possible, and at point B,

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[10] This refers to energy that must be paid for, not to that which is obtained without cost.

[11] See The Heat Pump in Evaporating Plants, Brown-Boveri Review, Mch. 1922.

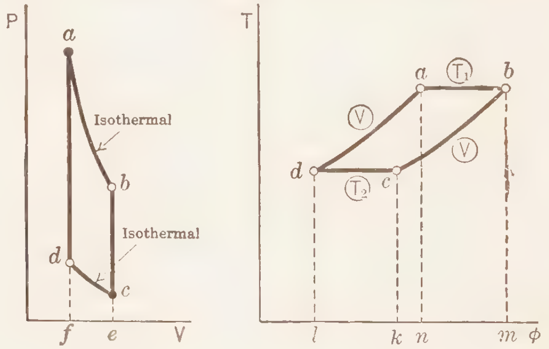
at an altitude z_2 of 510 ft., there is need for a large amount of water; and, further, suppose that at C, ten feet below B, or at the elevation z_3 of 500 ft., there is a very large river. But for the restriction imposed, B might, of course, receive directly from A all the water needed. However, under the conditions prevailing, the supply must be provided with as little water as possible taken from A. This can be accomplished by using water from A in a water wheel located at elevation z_2,—and discharging into B,—and utilizing the energy developed by the wheel to drive a pump at level z_3 to raise water from the liberal source C and deliver it at the desired height, the lift being only 10 ft. Since the head pumped against is relatively low, a very large delivery can be accomplished with the expenditure of only a small amount of the more valuable water from A; and such procedure would be desirable even though the efficiencies of the water wheel and pump were rather poor.

(f) Similarly, using absolute temperatures, suppose that, as the results of burning expensive fuel, heat is developed which is available from the products of combustion at a temperature T_1 of 4000 deg. fahr., and that it is desired to keep the air in a very large building B at a temperature T_2 of 510 deg. while that outside is at $T_3 = 500$ deg. Obviously, by burning a large amount of fuel directly in the building B the necessary heating can be accomplished, and, at first sight, this would appear to be the most efficient method. However, the air (at 500 deg.) outside the building contains an unlimited supply of heat, and a heat pump can be used for causing the desired 10 deg. increase in temperature. To drive this warming machine, mechanical energy may be obtained from an engine utilizing some of the heat obtained from burning fuel, and the heat in the exhaust from the engine can also assist in the warming process. As the ten-degree temperature increase is relatively small, a very large amount of energy for heating can be made available in this way from the great quantity of air passing through the machine, and this is accomplished with the expenditure of less fuel than is required in the direct method of heating, even though the performance of the actual warming machine is poor compared with the ideal reversed Carnot engine.

(g) Thus, from the standpoint of the utilization of heat-energy, the warming machine has attractive possibilities. However, from a consideration of the complication involved, the great cost of equipment, the very large size of apparatus, and the annoyance of operating a heat pump, there is much to be overcome before the warming machine can become a serious competitor of the common methods of heating buildings in most localities. As better designs of pumps and compressors are developed and as the cost of fuel goes up, the greater will be the possibility of such machines becoming commercially feasible. Other applications of heat pumps will be considered later.

125. The Constant-Volume Regenerative, or Stirling, Cycle.—(a) In this cycle, which was first used by Stirling in 1827 and which is shown in Figs. 77 (*a*) and (*b*), the gas receives heat from the hot body and rejects heat to the cold body along the isothermals *ab* and *cd* respectively, exactly as in the case of the Carnot engine. The two adiabatics of this latter case are, however, replaced by the two constant volume lines, *bc* and *da*, hence $\frac{V_c}{V_d} = \frac{V_b}{V_a} = r$.

The line *bc* is supposed to be obtained by allowing the working substance to transfer heat to a body so arranged that it stores that heat in such a manner that (1) each part of the body is always at the same temperature as the contiguous gas, (2) the temperature of each part remains constant, and (3) each increment of heat after storing is maintained at the temperature of reception. The line *da* is supposed to be obtained by the return of the stored heat to the gas by a reversal of this operation.



(*a*) (*b*)

Fig. 77.—Constant-Volume Regenerative Cycle (Stirling).

(b) Such a heat storing and restoring body is known as a **Regenerator** and in its perfect state is of course purely ideal. It may be approximated, however, by a long pipe of heat-insulating material filled with wire gauze or equivalent, and with temperature T_1 at one end and T_2 at the other. As hot gas flows through in the direction T_1T_2 it will impart heat to the walls and filling at a progressively decreasing temperature and give the process *bc*; while *da* may be obtained by causing gas to flow through the regenerator in the opposite direction.

(c) Since the regenerator acts only as a storage reservoir by means of which a definite quantity of heat is alternately received from and returned to the working substance, it must be carefully noted that the fresh supply of heat, Q_1, in each cycle is obtained only during the isothermal expansion *ab*, and similarly the rejection of heat occurs only during the isothermal compression, *cd*. Hence the *net work* of the Stirling Cycle may be obtained from Eq. (84) and the $T\phi$-diagram, Fig. 77(*b*), as follows:

$$\begin{aligned} Q_1 &= \text{area } abmn = wART_1 \log_e r, \qquad \text{B.t.u.,} \\ Q_2 &= \text{area } dckl = wART_2 \log_e r \\ \hline \text{Net work} = Q_1 - Q_2 &= \text{area } abcd = wAR(T_1 - T_2)\log_e r, \text{ B.t.u.} \qquad (154a) \end{aligned}$$

This net work may also be readily determined from Eq. (82) and the PV-diagram, Fig. 77(*a*), as follows:

$$\begin{aligned} Wk_{ab} &= \text{area } abef = wRT_1 \log_e r, \qquad \text{ft-lb.,} \\ Wk_{bc} &= 0 \\ Wk_{cd} &= \text{area } cdfe = -wRT_2 \log_e r \\ Wk_{da} &= 0 \\ \hline \text{Net work} = Wk &= \text{area } abcd = wR(T_1 - T_2)\log_e r, \text{ ft-lb.} \quad . \quad . \quad (154b) \end{aligned}$$

(d) The efficiency of the Stirling Cycle is

$$\frac{Q_1 - Q_2}{Q_1} = \frac{wAR(T_1 - T_2)\log_e r}{wART_1 \log_e r} = \frac{T_1 - T_2}{T_1}. \quad . \quad . \quad . \quad (155)$$

This is the same efficiency as the Carnot for the same temperature limits, a result that would be expected from the facts that *perfect regenerators* have been assumed and that the isothermals are both reversible since the working substance is at the same temperature as the body with which it is in thermal contact. With perfect regenerators there is no loss of available energy since the working substance is progressively at the same temperature as that part of the regenerator with which it is in contact, hence the two constant volume processes are also reversible for this case, and the whole cycle is therefore reversible. Consequently the cycle efficiency must be the same as the Carnot.

(e) The outline of the machinery necessary for carrying out such a cycle is shown in Fig. 78. The cylinders, Y and Y_1, and the hot and cold bodies, U and X, are similar to those used in the Carnot engine. The tube R is the ideal regenerator and its contained volume is assumed to be negligible compared with that of either cylinder.

Imagine the piston in Y_1 at the bottom of the cylinder and that in Y at the top, as the result of the expansion *ab*, Fig. 77. Y is thus filled with a gas in the condition shown at *b*. Now drive the right piston down, thus forcing the gas through the regenerator, and allow the left piston to rise at just the rate necessary to keep constant the *total volume of the gas*. During this process the regenerator will absorb heat and its temperature will grade from T_b at the right to T_c at the left. When the piston in Y has reached the bottom of its stroke all the gas will be in Y_1, the piston in the latter will be at the top of the stroke, and the constant-volume line *bc* will have been produced. Now hold the right piston stationary, bring the cold body up to cylinder Y_1 and force the piston into this cylinder, until the volume occupied by the gas is that shown at *d*, in Fig. 77. This will produce the isothermal compression. Free the piston in Y, continue the downward motion of that in Y_1 until it reaches the bottom of its cylinder, and simultaneously allow the right piston to rise as much as is necessary to keep the volume constant. This will give the line *da* of the diagram, and the gas in passing through the regenerator will rise in temperature from $T_d (= T_c)$ to $T_a (= T_b)$. Finally fix the left piston in its position, bring the hot body up to cylinder Y, and allow the gas to expand isothermally from *a* to *b*, completing the cycle.

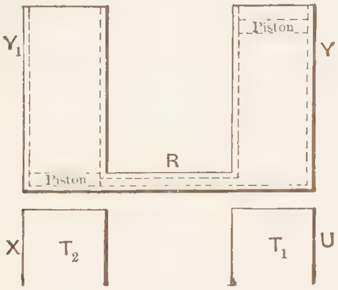


FIG. 78.—Machinery of Constant-Volume Regenerative Cycle.

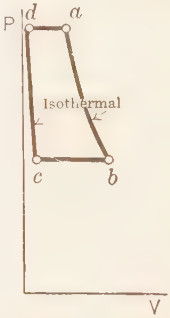


FIG. 79.—Constant-Pressure Regenerative Cycle (Ericsson).

126. The Constant-Pressure Regenerative, or Ericsson, Cycle.—This cycle, shown in Fig. 79, differs from the one last considered only in the fact that the regenerative processes are carried on at constant pressure instead of at constant volume. Substantially the same mechanism may be used as in the last case, and the ideal cycle, being reversible, must have the same efficiency.

127. The Otto Cycle, or *Beau de Rochas Cycle*.—(a) This cycle is one of the most important in use to-day as nearly all high speed internal combustion motors involve its use. Beau de Rochas described it in 1862, and fourteen years later Otto used it in an engine and demonstrated its great value. Although both men have a just claim to the cycle, for simplicity, it is commonly called the Otto Cycle.

(b) The cycle is shown in Fig. 80, and, as outlined by deRochas is carried out with *four strokes* of the piston, in the following manner:

(1) During the first or *suction stroke* a fresh charge of fuel and air, mixed in the proper proportion for burning, is sucked into the cylinder,—as shown by the line 1–2.

(2) The *compression stroke* then takes place, compressing the charge from the suction pressure at 2 to that at 3, according to the line 2–3. The mixture is then ignited and *combustion* occurs with rising pressure, the volume remaining practically constant, as shown by the line 3–4.

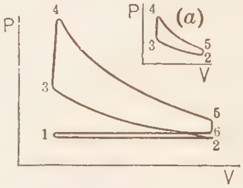


FIG. 80.

(3) The *expansion stroke* follows with the gases expanding according to line 4–5. The exhaust valve then opens and *release* occurs, the resulting drop in pressure, with volume practically constant, being shown by line 5–6.

(4) The *exhaust stroke*, line 6–1, then completes the cycle. During it the piston drives the greater part of the burnt gases out of the cylinder, thus scavenging it and leaving it ready to receive the fresh charge for the next cycle.

(c) In the *actual* Otto engine, as soon as the exhaust valve opens there is a very sudden drop of pressure within the cylinder as the " free expansion," 5–6, of the exhaust takes place. During this period the piston is near the end of the stroke, thus its movement is small and the indicator diagram would therefore show a sudden drop in pressure at nearly constant volume and consequently very little work would be done on the piston. The pressure within the cylinder is usually somewhat above atmospheric during the exhaust stroke as there is considerable " throttling " through the exhaust passages. Similarly, during the suction stroke the pressure is less than atmospheric due to the throttling through the inlet passages. It is part of the designer's problem to reduce these throttling losses as much as possible. At the limit, the lines 1–2 and 6–1 would coincide with each other and with the atmospheric line; and, under these circumstances, the suction and exhaust strokes would have no influence on the net work done, or on the efficiency, except for certain small losses of heat by radiation or conduction. The cycle 2–3–4–5–2 can be carried out with two strokes, as in Fig. 80(*a*), but in this case other means must be provided for injecting the charge and for scavenging the cylinder, and the efficiency is thereby reduced somewhat below that obtained with the four-stroke cycle. The arrangement and functioning of actual engines using the two- and four-stroke cycles will be considered in a later chapter.

(d) In the *ideal* Otto engine, the same working substance is considered as remaining always in the cylinder and as being carried through the entire cycle repeatedly, thus eliminating the exhaust and suction strokes. The compression and expansion strokes are assumed to be isentropic processes. Instead of having the constant volume combustion occur within the cylinder, an *equivalent amount of heat* is considered as *transmitted* to the confined gas while the piston is at the end of the compression stroke. Furthermore, instead of having free expansion of the gas just before the beginning of the exhaust stroke, heat is abstracted at constant volume from the charge until the pressure drops to atmospheric. Thus, with two strokes of the piston, this series of operations gives a pressure-volume diagram showing the *same amount of net work delivered* to the piston as occurs in the case of either the two- or four-stroke cycle having internal combustion, free expansion at the beginning of exhaust, adiabatic processes during the compression and expansion strokes, and no throttling. Hence, the heat supplied per cycle, the net work and the efficiency of the ideal cycle just outlined must be true standards of comparison for the actual Otto engine using either a two- or four-stroke cycle. Obviously, the net work of such an ideal cycle is the available energy from a given supply of heat used in an engine of this type.

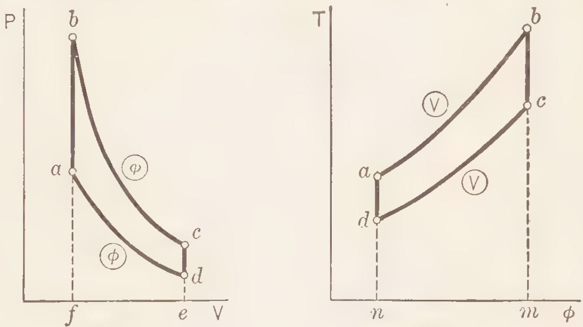
The gas in the cylinder of the ideal engine is considered to be in thermal equilibrium at all times so that its state may be completely specified at any instant. The diagrams of the ideal cycle must therefore consist of *two constant volume* lines and *two isentropics*, as shown by *abcd* in Fig. 81.

(e) Engineers are in agreement regarding the *processes* involved in the ideal Otto cycle, as just outlined, but they are not of the same opinion as to what should be considered as the *working substance* when making the calculations of the efficiency of the ideal cycle.

(f) For many years air, with $\gamma = 1.4$, has been taken as the working substance. Using any constant value of γ makes the solution of the ideal cycle very simple. However, the actual values of this quantity for air vary over a considerable range, as is shown in Fig. 33. In this text the ideal cycle using air will be designated as the "**Cold Air Standard**" if γ is taken as 1.4, and as the "**Hot Air Standard**" if γ is taken as constant, but less than 1.4, the value to be determined later in this chapter.

(g) A still more accurate criterion is the ideal cycle with a working substance the same as that of the actual mixture in the cylinder of the real engine. This may be referred to as the "**Real Mixture Standard.**"

Its disadvantage is the complication involved in determining the composition of the real mixture and in solving the necessary equations. The actual mixture will depend upon the ratio of air to fuel, the composition of the fuel, the amount of burnt gases in the cylinder when the fresh charge is admitted and on other factors. A method of analysis, with complete numerical results for one definite mixture, is given in Sect. **129**, but for short courses it may not be desirable to include all of it. In general, the "cold air standard" will give a cycle efficiency very much higher than that obtained by using the real mixture. On the other hand the "hot air standard" will be shown to be in very close agreement with the "real mixture standard."



(a) (b)

FIG. 81.—Ideal Otto Cycle.

128. The "Air Standard" Otto Cycle. — (a) Consider air having c_p, c_v and γ constant throughout the cycle, although their values depend upon the decision as to whether the "hot" or "cold" standard is used. Then, from the $T\phi$-diagram in Fig. 81*b* and from Eq. (67), it follows that, for w pounds of air,

$$Q_1 = \text{area } abmn = wc_v(T_b - T_a), \text{ B.t.u.}$$

$$Q_2 = \text{area } dcmn = wc_v(T_c - T_d).$$

or,

$$\text{Net work} = Q_1 - Q_2 = \text{area } abcd = wc_v(T_b - T_a - T_c + T_d), \text{ B.t.u.} \quad . \quad (156a)$$

Also, from Eq. (110) and the PV-diagram, the work done by the gas during each process is

$$Wk_{bc} = \text{area } bcef = wK_v(T_b - T_c), \text{ ft-lb.,}$$

$$Wk_{cd} = \qquad = 0,$$

$$Wk_{da} = \text{area } dafe = wK_v(T_d - T_a),$$

$$Wk_{ab} = \qquad = 0.$$

or,

$$\text{Net work} = Wk_{cycle} = \text{area } abcd = wK_v(T_b - T_a - T_c + T_d), \text{ ft-lb.,} \quad . \quad (156b)$$

which checks the value obtained in Eq. (156*a*).

(**b**) The efficiency of this cycle is

$$e_{t(air)} = \frac{Q_1 - Q_2}{Q_1} = \frac{wc_v(T_b - T_a - T_c + T_d)}{wc_v(T_b - T_a)} = 1 - \frac{T_c - T_d}{T_b - T_a}. \quad . \quad (157)$$

Or, from Eq. (105), this becomes

$$e_{t(air)} = 1 - \frac{T_b\left(\frac{V_a}{V_d}\right)^{\gamma-1} - T_a\left(\frac{V_a}{V_d}\right)^{\gamma-1}}{T_b - T_a} = 1 - \left(\frac{V_a}{V_d}\right)^{\gamma-1}. \quad . \quad . \quad . \quad (158)$$

(**c**) Now, since V_a and V_d are respectively the volumes at the end and beginning of compression, both may be expressed in terms of the piston displacement (*P.D.*). Thus, $V_a = c(P.D.)$ and $V_d = (1 + c)(P.D.)$, where c is the ratio of the clearance volume to the piston displacement. Substituting in Eq. (158), the efficiency becomes

$$e_{t(air)} = 1 - \left[\frac{c(P.D.)}{(1 + c)(P.D.)}\right]^{\gamma-1} = 1 - \left(\frac{c}{1 + c}\right)^{\gamma-1} \quad . \quad (159a)$$

$$= 1 - \frac{1}{r^{\gamma-1}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (159b)$$

where r is the compression ratio $\frac{V_d}{V_a}$, or $\frac{1 + c}{c}$, or the expansion ratio $\frac{V_c}{V_b}$.

Equations (159*a*) and (159*b*) show that *the greater the ratio r, or the smaller the clearance of the engine, the larger is the efficiency of this ideal Otto cycle.*

(**d**) For a definite value of γ it is very important to observe that this ideal efficiency is dependent *only* on the expansion or *compression ratio.* However, in the actual engine the material compressed is a combustible mixture, hence there is a *limit to the compression ratio* that can be used, since, if it is too large, the resulting temperature rise will cause spontaneous ignition before the point a is reached. The *throttling of the charge during suction,* which reduces the pressure at the beginning of compression below atmospheric without changing r, *does not reduce the ideal cycle efficiency,* although it will be shown later that it does decrease the mechanical efficiency, and it would certainly reduce the power output of the real engine, since a smaller weight of the charge w would be drawn in. The discussion and equations given in this section also apply for any gas having a constant value of γ.

129.[12] **The "Real Mixture Standard" Otto Cycle.**—(**a**) As in the previous case, this cycle consists of two constant volume processes and two constant entropy ones, as shown by *abcd* in Fig. 81. Now, however, the working substance is not an ideal gas and therefore c_v and γ are not constant, but depend upon the temperature.

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[12] This section may be omitted in brief courses.

(**b**) Before the PV- or $T\phi$-diagrams can be drawn to scale, and the net work or the efficiency be found, it is necessary to determine the values of the pressure, volume and temperature at the end of each process. Then the value of $\Delta\phi_{ab}$ and $\Delta\phi_{cd}$ may be computed, as in Sect. **77.** The point d, at the beginning of compression, is usually the starting point in the analysis. The pressure p_d of the mixture is commonly taken as atmospheric at the place in which the actual engine is located, and the temperature, t_d, may be assumed as 200° F., or $T_d = 660$, as this is a fair value for the actual temperature of the mixture in the cylinder at the beginning of compression in the real engine. The ratio of compression, or expansion, r, may always be found when the clearance and piston displacement of the actual engine are known.

(**c**) The composition of the mixture in the cylinder is assumed to change during the cycle and to approximate that in the actual engine. Thus, during compression, the mixture consists of the fuel, air, and the part of the burnt gases which remains from the preceding cycle; while during the remainder of the cycle it is made up of the products of combustion.

The higher heating value, $H.H.V.$, of fuel, in B.t.u. per lb., is experimentally obtained by allowing the combustion of a known weight of fuel to proceed completely and then finding the heat that must be removed to cool the resulting *products of combustion* back to the room temperature.[13] Thus, the heat supplied when burning w_f lb. of fuel per cycle is

$$Q_1 = w_f(H.H.V.) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (160)$$

Referring to Fig. 81, the *net work* per cycle is seen to be

$$\begin{aligned} Net\ Wk &= Wk_{da} + Wk_{ab} + Wk_{bc} + Wk_{cd} \\ &= (I_d - I_a) + 0 + (I_b - I_c) + 0 = I_d - I_a + I_b - I_c. \quad (161) \end{aligned}$$

From Eqs. (160) and (161) it follows that the efficiency of the cycle is

$$e_{t(mtx)} = \frac{net\ work}{Q_1} = \frac{I_d - I_a + I_b - I_c}{Q_1} \quad . \quad . \quad . \quad . \quad (162)$$

(**d**) Before numerical values of the internal energies I_a, I_b, I_c, and I_d can be obtained, a datum at which the internal energy is arbitrarily taken as zero must be chosen. The calculations are simplified by taking this datum at absolute zero, and assuming that the specific heat equation applies to a temperature as low as that—an assumption that will not affect the results since *differences* in internal energies are the only quantities to be obtained. Under these conditions, with $T_0 = 0$, the internal energy for any given state, such as 1, is

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[13] For a brief discussion of the use of higher and lower heating values, see Sect. **136.**

$$\text{If } T_1 < 2900°, \quad I_1 = w\int_{T_0}^{T_1} c_v dT = w\int_{T_0}^{T_1} (M + BT + CT^2)dT$$

$$= w\left[MT_1 + \frac{B}{2}T_1^2 + \frac{C}{3}T_1^3\right]. \quad . \quad . \quad . \quad . \quad . \quad (163a)$$

$$\text{If } T_1 > 2900°, \quad I_1 = I_{2900} + \left[MT + \frac{BT^2}{2} + \frac{CT^3}{3}\right]_{2900}^{T_1}. \quad . \quad (163b)$$

(e) A *numerical example* will serve to show the calculations necessary to determine the heat supplied, net work, and efficiency of this ideal Otto cycle. Suppose that gasoline, having higher and lower heating values of 20,000 and 18,675 B.t.u. per lb., respectively, is used in an engine having a compression ratio, r, of 5 to 1; that fifteen pounds of air are supplied per pound of gasoline, this "15 to 1 mixture" being about the proper proportions for perfect[14] combustion; and that the products of combustion consist of, H_2O = 8.4 per cent, CO_2 = 19.4 per cent, and N_2 = 72.2 per cent, by weight. For the water vapor it will be assumed that $c_v = 0.353 - 1.533(10)^{-5}\,T + 2.35(10)^{-8}\,T^2$ and $R = 85.8$; and for gasoline[15] $c_v = 0.3188 + 3.333(10)^{-4}\,T$ and $R = 13.53$. For convenience in making the calculations, assume that the cycle is carried out with one pound of working substance. With $r = 5$ it is a reasonable assumption that 10 per cent of the mixture present during compression are products of combustion left from the preceding cycle. During *compression*, therefore, the pound of mixture consists of 0.1 lb. of products of combustion, 0.8438 lb. of air and 0.0562 lb. of gasoline.

First, it is necessary to determine the values of M, B, and C in Eq. (163). For the *compression mixture* with $T < 2900$, it is found from Eqs. (59) and (61) that

$$R = 50.33, \quad M = 0.1799, \quad B = 2.032(10)^{-5}, \quad \text{and} \quad C = 3.7275(10)^{-9};$$

and for the *products of combustion*,

with $T < 2900°$, $M = 0.1799$, $B = 1.591(10)^{-5}$, and $C = 2.425(10)^{-9}$,

with $T > 2900°$, $M = 0.2021$, $B = 5.64(10)^{-7}$, and $C = 5.065(10)^{-9}$,

and, irrespective of temperature, $R = 54.01$.

Using these values in Eq. (163) the internal energies can be obtained. The resulting equation for the compression process with $T < 2900°$ is shown graphically in Fig. 82(a), and that for the expansion of the products of combustion with $T > 2900°$ is given in Fig. 82(b), for later use.

Now, assuming a suction temperature of $T_d = 660°$ F. abs., then, from Fig. 82(a),

$$I_d = 123.7 \text{ B.t.u. per lb.}$$

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[14] This term means that there is no excess of either fuel or oxygen.

[15] Derived from values given in Bul. 139, Univ. of Ill. Experiment Station.

Having T_d, and knowing the ratio of compression, V_d/V_a, and the constants, substitution may be made in Eq. (112) to obtain T_a at the end of the isentropic compression. However, this equation cannot be solved directly. It is represented in Fig. 83 by the curve labeled "compression," from which, for $r = 5$, it is found that $T_a = 1120°$ F. abs. This temperature used with Fig. 82(*a*) shows that at the end of compression,

$$I_a = 215.8 \text{ B.t.u. per lb.}$$

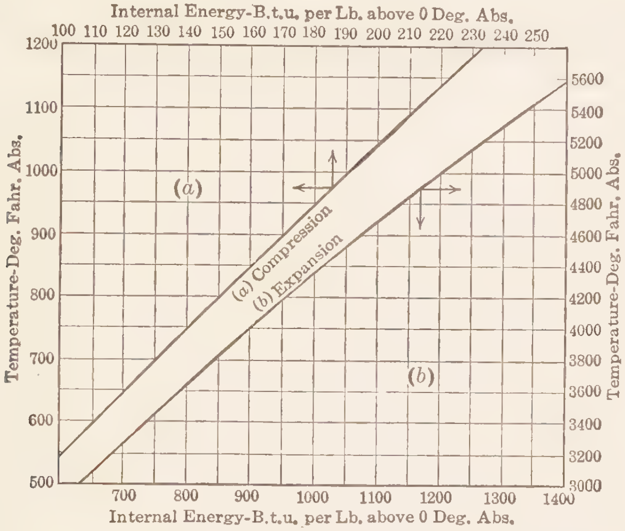

FIG. 82.—The Relation between Internal Energy and Absolute Temperature.

(*a*) For fresh mixture of 10% residue, 84.38% air, and 5.62% gasoline.
(*b*) For burnt products having 8.4% H_2O, 19.4% CO_2, and 72.2% N_2.

With combustion now occurring, the *amount of heat supplied per cycle*, on which the thermal efficiency is based, is, from Eq. (160),

$$Q_1 = w_f(H.H.V.) = 0.0562 \times 20{,}000 = 1124 \text{ B.t.u.}$$

However, the energy available for producing a rise in temperature of this mixture is equal to

$$Q_t = w_f(L.H.V.) = 0.0562 \times 18{,}675 = 1050 \text{ B.t.u.}$$

in which $L.H.V.$ is the lower heating value per pound of fuel.[16]

Note that Q_t is considered as heating the *products of combustion*—not the unburnt charge. If the mixture of burnt gases were cooled to the temperature $T_a = 1120°$, which existed at the end of compression, its internal energy would be, from Eq. (163),

$$I'_a = 1\left[0.1799 \times 1120 + \frac{1.591}{2}(10)^{-5} \times (1120)^2 + \frac{2.425}{3}(10)^{-9} \times (1120)^3\right]$$

$$= 212.6 \text{ B.t.u. per lb.}$$

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[16] For discussion of the use of this heating value, see Sect. **136**.

Then, adding Q_t to I'_a will give the internal energy at b, combustion having taken place during the process $a\ b$. Thus,

$$I_b = I'_a + Q_t = 212.6 + 1050 = 1262.6 \text{ B.t.u. per lb.}$$

It should be noted that had the value of $I_a = 215.8$ been used, instead of calculating $I'_a = 212.6$, the final result would have been changed but one-quarter of 1 per cent and considerable labor saved. However, the use of I'_a is more consistent with the conception of the heating value of a fuel.

From Fig. 82(b) the corresponding value of the temperature is found to be

$$T_b = 5165° \text{ F. abs.}$$

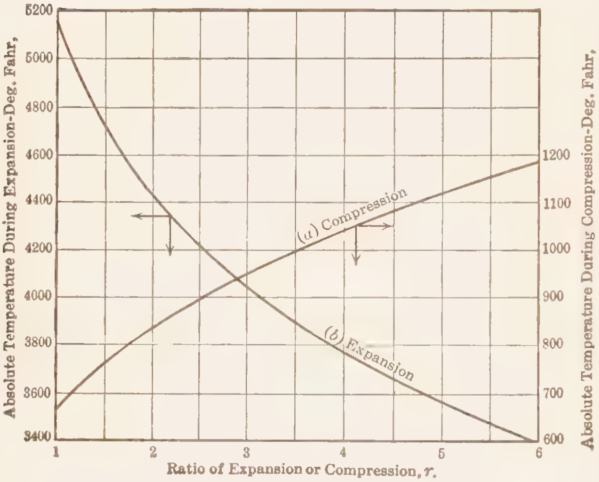

FIG. 83.

This temperature is of particular interest as it is a measure of the *maximum* value attained in the real engine using the assumed mixture.[17]

In order to get the temperature at c it is necessary to use Eq. (112), but this time for the products of combustion. The corresponding curve is shown in Fig. 83 and it is labeled "expansion." For $r = 5$, this curve shows that

$$T_c = 3558° \text{ F. abs.}$$

Using this temperature, from Fig 82(b)

$$I_c = 778 \text{ B.t.u. per lb.}$$

Then, from Eq. (161)

$$Net\ Wk. = I_d - I_a + I_b - I_c = 123.7 - 215.8 + 1262.6 - 778 = 392.5 \text{ B.t.u. per lb;}$$

and, from Eq. (162), the *efficiency* is

$$e_{i(mix.)} = Net\ Wk./Q_1 = 392.5/1124 = 0.349, \text{ or } 34.9 \text{ per cent.}$$

(f) It is interesting to compare this efficiency with that of the corresponding "Cold Air Standard Cycle." For the latter, from Eq. (158), using $\gamma = 1.4$,

$$e_{i(cold\ air)} = 1 - \left(\frac{V_a}{V_d}\right)^{\gamma-1} = 1 - \left(\frac{1}{5}\right)^{0.4} = 47.5 \text{ per cent,}$$

which is over one-third larger than $e_{i(mix.)}$. Thus, since very high temperatures are involved in the real Otto engine, using air with

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[17] The actual temperature attained in the real engine will be less than this for reasons that will be considered later.

$\gamma = 1.4$ in the analysis of the ideal cycle is obviously unjust to the real Otto engine, and hence the "Cold Air Cycle" is not a very satisfactory standard for comparison of the actual performances of such engines.

(g) On the other hand, by using the "Hot Air Standard" with $\gamma = 1.3$ quite satisfactory results are obtained for the ideal cycle, if the actual engine with which the comparison is to be made is using gasoline, oil, or coal gas. Thus, for this case

$$e_{i(hot\ air)} = 1 - \left(\frac{1}{r}\right)^{0.3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (164)$$

This equation yields an efficiency of 38.2 per cent for the case in which $r = 5$, as compared with 34.9 per cent, the value obtained from the real mixture calculations. Had γ been taken as 1.267 the efficiency of the "hot air standard" would have checked exactly the real mixture value for this case.

While it has been found that Eq. (164) gives fairly satisfactory results for the fuels and conditions mentioned, note, however, that the value of γ may be different for other fuels; also, that it changes as the mixture ratio departs from the so-called "theoretical mixture," which is the one that has just the right amount of air to supply the oxygen needed to burn the particular fuel used. As an indication of how both the ratio of air to fuel, and the compression ratio, affect the ideal efficiency, some of the results of Tizard and Pye[18] are shown in Fig. 84.

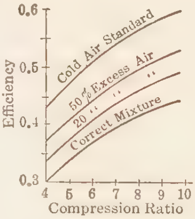

FIG. 84.

130. The Diesel Cycle.

130. The Diesel Cycle.—(a) This cycle is of great importance as Diesel engines are used quite extensively in stationary and marine practice. Though relatively costly, they develop very high actual economies and can use very low-grade fuel oils. A Diesel-engine power plant and its manner of functioning were discussed very briefly in Sect. **31**. This type of engine is comparatively recent, the first successful one having been constructed by Dr. Rudolph Diesel in 1897, after several years of experimentation.

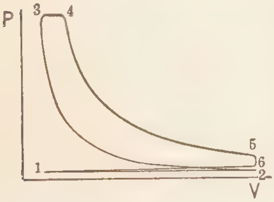

FIG. 85.

(b) The actual Diesel cycle may be carried out with either two or four strokes of the piston. For the latter case, the PV-diagram is shown in Fig. 85, in which the following sequence of events takes place:

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[18] The Character of Various Fuels for Internal Combustion Engines, in The Automobile Engineer, Feb., March, and April, 1921.

(1) During the first, or *suction stroke,* shown by the line *1–2,* air only is drawn into the cylinder.

(2) The *compression stroke* then takes place, compressing this air according to the process *2–3.* The ratio of compression, (V_2/V_3), and the resulting pressure at *3,* are much greater than is feasible in the Otto cycle. The temperature of the compressed air at *3* must be well above the temperature that will cause ignition of the fuel when it is introduced after the end of the compression stroke—no external igniting device being provided.

(3) From *3* to *4* in the *expansion stroke,* fuel is injected, usually by means of a " fuel pump " and a small proportion of high pressure " injection air," which also helps to atomize the fuel. In some of the later and modified designs of engines of this general type the fuel is injected without the use of the air, thus eliminating the auxiliary high-pressure air compressor that is otherwise needed. In either case it is intended that the fuel shall be injected at such a rate that the combustion that results when it unites with the hot air in the cylinder is maintained at substantially constant pressure. The exact time at which this fuel injection starts depends upon the kind of fuel and many other factors to be discussed later. From *4* to *5* the burnt gases expand during the remainder of the expansion stroke. The exhaust valve then opens and a free expansion occurs. The gases remaining in the cylinder have a drop in pressure at approximately constant volume, according to *5–6.*

(4) The *exhaust stroke,* line *6–1,* then completes the cycle of operation of the engine.

(**c**) The *ideal Diesel cycle* will now be considered. The heat supplied is to be added during a constant pressure process, thus causing a certain amount of expansion, its magnitude depending upon the quantity of heat so used. The remainder of the expansion is isentropic, as is all of the compression. The rejection of heat is considered to be at constant volume, for the reasons discussed in Sect. **127** (**d**). Such a series of processes represents the greatest amount of net work that can be delivered to the piston of a Diesel engine having *a given compression ratio and a known supply of heat.* This ideal cycle requires only two strokes, and it may be used as a standard of comparison for both the two- and four-stroke types of actual engines.

(**d**) In the real Diesel engine the amount of working substance in the main cylinder is not constant throughout the cycle, and this is because the fuel and injection air are not included in the suction and compression processes; but the compression of the injection air requires the expenditure of energy and therefore this air may be considered in the ideal case as being compressed in the main cylinder. The work of pumping the liquid fuel into the cylinder of the actual engine is such a small item that its equivalent need not be considered in the ideal cycle.

(e) As in the case of the Otto cycle, the analysis of the ideal Diesel cycle may be carried out on the basis of either of the two "*Air Standards*" or on the "*Real Mixture Standard.*" The latter results in a more complex analysis, but in a very much closer approach to actual conditions. With both of the Air Standards a constant weight of air may be considered throughout the cycle, but with the Real Mixture Standard the variable weights must be introduced. In each case, however, it is essential that the same amount of heat be supplied per cycle, a condition that enables certain important relations to be established between these standards, as will be shown later.

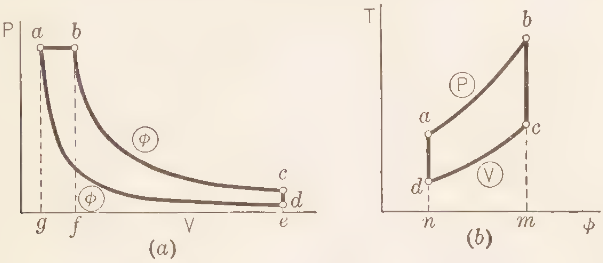

FIG. 86.—Ideal Diesel Cycle.

131. The Ideal "Air Standard" Diesel Cycle.—(a) Consider ideal air, having c_p and c_v constant, to be the working substance in the Diesel cycle. Since c_p and c_v are to be constant it follows that γ must be constant—but not necessarily equal to 1.4, its value for air at low temperature. Then, using Eqs. (75) and (67) and referring to Fig. 86, for w pounds of air,

$$Q_1 = \text{area } abmn = wc_p(T_b - T_a), \text{ B.t.u.}$$

$$Q_2 = \text{area } dcmn = wc_v(T_c - T_d)$$

$$\textit{Net Work} = Q_1 - Q_2 = \text{area } abcd = wc_p(T_b - T_a) - wc_v(T_c - T_d), \text{ B.t.u.} \quad . \quad (165)$$

This same result for the net work may be obtained also by making a summation of the work done during all of the processes involved.

(b) The efficiency of this cycle is

$$e_{i(air)} = \frac{Q_1 - Q_2}{Q_1} = \frac{wc_p(T_b - T_a) - wc_v(T_c - T_d)}{wc_p(T_b - T_a)}$$

$$= 1 - \frac{1}{\gamma}\left[\frac{T_c - T_d}{T_b - T_a}\right]. \quad . \quad . \quad . \quad . \quad . \quad (166)$$

This form of the equation is not suitable for general use, as the temperatures T_b and T_c depend upon the ratio of expansion involved during the constant pressure process *ab*. This ratio is often called the "*cut-off ratio*," as in the actual engine it represents the ratio of the volume at the instant the injection of fuel is stopped to the volume at the end of compression, or to the clearance volume. Then, if r_c be used as the symbol for the cut-off ratio, it follows that for the ideal cycle (see Fig. 86) $r_c = V_b/V_a$. Let r_a = "*adiabatic compression ratio*" $= V_d/V_a$. Then, with an ideal gas of constant weight considered as the working substance, it follows from Eqs. (69) and (105) that

$$T_b = r_c T_a,\ T_c = T_b\left(\frac{V_b}{V_c}\right)^{\gamma-1} = T_a\frac{r_c{}^\gamma}{r_a{}^{\gamma-1}},\ \text{and}\ T_d = \frac{T_a}{r_a{}^{\gamma-1}}.$$

With these values substituted in Eq. (166) there results

$$e_{i(air)} = 1 - \frac{1}{r_a{}^{\gamma-1}}\left[\frac{r_c{}^\gamma - 1}{\gamma(r_c - 1)}\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (167)$$

This equation shows that the efficiency of the Ideal Air Standard Diesel cycle depends upon the compression ratio r_a, the cut-off ratio r_c and γ. With a given cut-off ratio, the efficiency becomes larger with increased compression ratios, because the expansion after cut off is thereby made greater. For this same reason the ideal cycle efficiency is increased by decreasing the cut-off ratio. A smaller value of r_c corresponds to a reduced load on the actual engine, and, consequently, to a greater ratio of air to fuel in the engine cylinder. Furthermore, Eq. (167) also shows that the efficiency is increased by using a working substance with a high value of γ.

(c) It is now desirable to determine how the cut-off ratio r_c may be obtained in order to use Eq. (167) for measuring by this standard the performance of an actual engine, or for comparing the results of this equation with those obtained by using the Real Mixture Standard. Since Eq. (167) shows that the efficiency of the Ideal Air Standard cycle is independent of the weight of air considered, let the weight of air used in this cycle be taken the same as the total weight of the mixture at cut off in the actual case with which the comparison is to be made. Then the following equations may be written, since the ideal and actual cycles are to be compared on the basis of the same supply of heat that is available for producing a rise in the temperature of the working substance.

Let w_f = weight of fuel in pounds used per cycle in the actual case;
w_A = weight of air used per pound of fuel in the actual engine (This includes all injection air, if any);

w_B = weight of burnt gases remaining in the cylinder per pound of fuel in the actual case;

$L.H.V.$ = the lower heating value of the fuel, in B.t.u. per lb.;

$Q_t = w_f(L.H.V.)$ = the energy available per cycle for producing a rise in the temperature of the mixture in the actual engine;[19]

Q_1 = the heat supplied per cycle in corresponding ideal air case.

Then, per cycle, it follows that,

$(1 + w_A + w_B)w_f$ = weight of working substance,

consequently

$$\begin{aligned} Q_1 &= (1 + w_A + w_B)w_f c_p(T_b - T_a) \\ &= (1 + w_A + w_B)w_f c_p(r_c - 1)T_a. \quad . \quad . \quad (168) \end{aligned}$$

But since Q_1, is to be equal to Q_t, then

$$(1 + w_A + w_B)w_f c_p(r_c - 1)T_a = w_f(L.H.V.).$$

$$\therefore r_c = 1 + \frac{L.H.V.}{(1 + w_A + w_B)c_pT_a}. \quad . \quad . \quad . \quad . \quad (169a)$$

or

$$r_c = 1 + \frac{L.H.V.}{(1 + w_A + w_B)c_pT_d\, r_a{}^{\gamma-1}}. \quad . \quad . \quad (169b)$$

As illustrations of the effect of changing w_A and γ in Eqs. (167) and (169), consider the cases given in Table VI.[20]

TABLE VI

IDEAL DIESEL CYCLE EFFICIENCIES AS AFFECTED BY γ AND WEIGHT OF AIR PER POUND OF FUEL

$L.H.V.$	w_A	w_B	T_d	r_a	γ	c_p	r_c	$e_{i(air)}$
18650	13.5	0.45	660	15	1.40	0.240	3.66	0.533
18650	13.5	0.45	660	15	1.35	0.264	3.79	0.481
18650	13.5	0.45	660	15	1.30	0.297	3.84	0.427
18650	25	0.8	660	15	1.40	0.240	2.49	0.581
18650	25	0.8	660	15	1.36	0.259	2.54	0.539
18650	25	0.8	660	15	1.35	0.264	2.56	0.530
18650	100	3.0	660	15	1.40	0.240	1.38	0.638

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[19] The $L.H.V.$ is used for the reasons stated in Sect. **136**.

[20] Fixing γ also determines c_p for the air, since $c_p = \gamma R/(\gamma - 1)778$.

The combination of Eqs. 169(*b*) and 167 enable the ideal cycle efficiency of the Diesel to be calculated for comparison with actual test data. It is for this reason that these equations are so important. In order to choose a suitable value of γ for any given conditions, it is essential that a complete study of the "Real Mixture Standard" shall have been made. After completing the work of the next section it will be seen that *for full load conditions* the best value of γ is close to 1.35.

132.[21] **The "Real Mixture Standard" Diesel Cycle.**—(a) For this standard, as with the similar one for the Otto engine, the working substance is chosen to approximate the mixtures actually present before and after combustion; the variation in the specific heats of the mixtures is taken into account; and at the beginning of compression it may be assumed that the temperature is $t_d = 200°$ F., or $T_d = 660$, and that the pressure P_d is atmospheric.

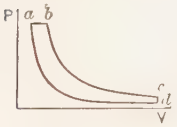

FIG. 87.

(b) If the higher heating value of the fuel used is *H.H.V.* B.t.u. per pound, then, if w_f pounds of fuel are burned per cycle, the *heat supplied* by it is

$$Q_1 = w_f(H.H.V.). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (170)$$

The *net work* of the cycle is the summation of the work done on the piston during the various processes; or, referring to Fig. 87,

$$\begin{aligned} Net\ Wk &= Wk_{da} + Wk_{ab} + Wk_{bc} + wk_{cd} \\ &= I_d - I_a + AP_a(V_b - V_a) + I_b - I_c + 0 \\ &= I_d - (I_a + AP_aV_a) + (I_b + AP_bV_b) - I_c \\ &= I_d - H_a + H_b - I_c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (171) \end{aligned}$$

It should be noted that each of these terms in Eq. (171) depends upon the weight of substance involved at its particular state.

From Eqs. (170) and (171) it follows that the *efficiency* of this cycle is

$$e_{i(mix)} = \frac{\text{net work}}{Q_1} = \frac{I_d - H_a + H_b - I_c}{w_f(H.H.V.)}. \quad . \quad . \quad (172)$$

(c) The internal energies needed are obtained from Eq. (163), and the heat contents H_a and H_b can be calculated from Eq. (56*b*), again using 0° F. abs. as the arbitrary datum. Thus, with $T_0 = 0$, the heat content for any state, 1, is

$$H_1 = w_1\left[M'T + \frac{BT^2}{2} + \frac{CT^3}{3}\right]_{T_0}^{2900} + w_1\left[M'T + \frac{BT^2}{2} + \frac{CT^3}{3}\right]_{2900}^{T_1}. \quad (173)$$

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[21] This section may be omitted in short courses.

(d) The rather complex computations required for the analysis of this cycle are best illustrated by a numerical problem. For example, suppose that it is required to find the ideal cycle efficiency of a Diesel engine which operates with a compression ratio, r_a, of 15 to 1; that the fuel oil used has higher and lower heating values of 19,150 and 18,650 B.t.u. per pound, respectively; that 25 pounds of air per pound of oil are supplied and that under such conditions the products of combustion are composed of 11.8 per cent CO_2, 4.0 per cent of H_2O, 73.9 per cent of N_2, and 10.3 per cent of O_2, by weight. Furthermore, consider that atmospheric pressure at the place where the engine is located is 14.7 lb. per sq. in., and that, with $r_a = 15$, it is fair to assume 3 per cent, by weight, of the mixture present during compression to be products of combustion which remained, from the preceding cycle, in the clearance.

For convenience, assume that the cycle is carried out with one pound of the products of combustion used as the working substance during the adiabatic expansion. Then the weight present during compression consists of 0.03 lb. of products of combustion and 0.9327 lb. of air. Further, let the specific heat of water vapor be

$$c_v = 0.353 - 1.533(10)^{-5}\,T + 2.35(10)^{-8}\,T^2,$$

and

$$c_p = 0.463 - 1.533(10)^{-5}\,T + 2.35(10)^{-8}\,T^2.$$

First, the constants M, B, C, M', and R, for the mixtures during expansion and compression must be obtained.

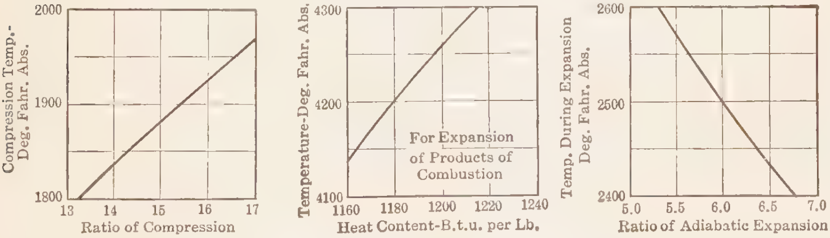
For the *expansion mixture*, from Eqs. 61(*a*), (*b*) and (*c*),

with $T < 2900°$,

$$M = 0.1743, \quad M' = 0.2432, \quad B = 0.9842(10)^{-5} \quad \text{and} \quad C = 2.884(10)^{-9},$$

with $T > 2900°$,

$$M = 0.1878, \quad M' = 0.2567, \quad B = 0.5137(10)^{-6} \quad \text{and} \quad C = 4.489(10)^{-9},$$



(*a*) (*b*) (*c*)

FIG. 88.

and from $R = (M' - M)/A$, the value of $R = 53.6$.

For the *compression mixture*, with $T < 2900°$,

$$M = 0.1706, \quad M' = 0.2391, \quad B = 3.067(10)^{-7}, \quad C = 4.099(10)^{-9},$$

and $R = 53.29$.

Then, starting at the point d, with $T_d = 660°$, from Eq. (163)

$$i_d = 113.1 \text{ B.t.u. per lb.}$$

For the compression mixture the proper values of the constants should be introduced in Eq. (112),* which can then be used for constructing the curve shown by Fig. 88 (*a*), from which, for $r_a = 15$, it is found that

$$T_a = 1881° \text{ F. abs.}$$

Corresponding to this temperature, from Eq. (173), the specific heat content at *a* of the mixture being compressed, is

$$h_a = 459.4 \text{ B.t.u. per lb.}$$

When combustion takes place, after compression is completed and the fuel injected, the energy available for producing a rise in temperature of the working substance is

$$Q_t = 0.0373 \times 18{,}650 = 695.0 \text{ B.t.u.}$$

The heat content of the products of combustion, at the absolute temperature $T_a = 1881°$, is, from Eq. (173),

$$h'_a = 481.3 \text{ B.t.u. per lb.}$$

Since the pressure remains constant during the heat addition, Q_t increases the *heat content* of the products of combustion. Hence, at *b*, at the end of combustion,

$$h_b = h'_a + Q_t = 481.3 + 695.0 = 1176.3 \text{ B.t.u. per lb.}$$

The corresponding temperature at *b* can then be obtained from Fig. 88 (*b*), constructed from Eq. (173). It is found to be

$$T_b = 4187° \text{ F. abs.},$$

which is the maximum temperature in this Real Mixture Ideal Cycle.

Starting the isentropic expansion at *b* with this temperature and using Eq. (112),* the curve shown in Fig. 88 (*c*) is constructed. From this curve the temperature T_c may be found as soon as the ratio of expansion V_c/V_b is determined. The value of this ratio will now be found. Since, for any gas, or mixture of gases, with $p_a = p_b$,

$$\frac{V_b}{V_a} = \frac{w_b R_b T_b}{w_a R_a T_a},$$

it follows that, for this case,

$$\frac{V_b}{V_a} = \frac{1 \times 53.6 \times 4187}{0.9627 \times 53.29 \times 1881} = 2.326;$$

hence, the *ratio of adiabatic expansion* is

$$r' = \frac{V_c}{V_b} = \frac{r_a V_a}{V_b} = \frac{15}{2.326} = 6.45.$$

For this value of r', from Fig. 88 (*c*),

$$T_c = 2438° \text{ F. abs.}$$

---

* Using proper constants above and below 2900°.

and from Eq. (163) the corresponding internal energy at c is

$$i_c = 468.2 \text{ B.t.u. per lb.}$$

Then, from Eq. (171), the *net work* of the cycle is, for the actual weights involved,

$$\textit{Net work} = I_d - H_a + H_b - I_c = w_d(i_d - h_a) + w_b(h_b - i_c)$$

$$= 0.9627(113.1 - 459.4) + 1176.3 - 468.2 = 374.7 \text{ B.t.u. per lb. of expanding mixture.}$$

The heat supplied per cycle based on the higher heating value is,

$$Q_1 = w_f(H.H.V.) = 0.0373 \times 19{,}150 = 714.3 \text{ B.t.u.};$$

hence, from Eq. (172), the efficiency is

$$e_{i(mix.)} = \frac{374.7}{714.3} = 0.525, \text{ or } 52.5 \text{ per cent.}$$

(e) Comparison of the above result with the efficiency given in the table of Sect. **131** (c) for the same compression ratio ($r_a = 15$), the same initial temperature ($T_d = 660$), and the same weight of air per pound of fuel ($w_a = 25$), shows that the efficiency there given for $\gamma = 1.4$ is 58.1 per cent, and for $\gamma = 1.35$ it is 53.0 per cent. This latter result agrees very closely with the value of 52.5 per cent obtained from the real mixture calculations, hence, for a mixture ratio of 25 to 1, using Eqs. (167) and (169) with $\gamma = 1.35$ will be very satisfactory. If still leaner mixtures are used, as is actually the case with real engines running at light load, a slightly higher value of γ should be selected.

The Cold Air Standard ($\gamma = 1.4$) has often been used, but such a value of γ for air corresponds to a temperature that is relatively very low. If it be desirable to choose some one value of γ that is to be used in all ideal air Diesel cycle calculations for full load, the value 1.35, which corresponds to air temperature of about 2000° F., is certainly more reasonable than 1.4.

133. The Dual Combustion Cycle.—(a) Many oil engines in common use have combustion begin at constant volume and then continue at constant pressure. They may be called either "Dual Combustion" or "Semi-Diesel" or "Mixed Combustion" engines. Usually they are not provided with auxiliary devices for igniting the fuel, but, generally, rely on the temperature produced by a moderately high compression, perhaps assisted by certain hot parts of the cylinder head, to start the combustion. The fuel is supplied in such manner that the burning of some of it is retarded or delayed. How this is accomplished will be discussed in a later chapter. The ideal cycle is shown in Fig. 89.

(b) From Eqs. (67), (75) and (148), with w pounds of ideal gas as the working substance, the *net work* per cycle is found as follows:

$$Q_1 = wc_v(T_b - T_a) + wc_p(T_c - T_b),\ \text{B.t.u.}$$

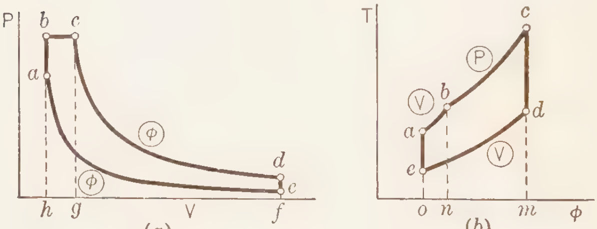
$$Q_2 = wc_v(T_d - T_e).$$

$$\textit{Net work}_{cycle} = Q_1 - Q_2 = wc_v[(T_b - T_a) - (T_d - T_e)] + wc_p(T_c - T_b),\ \text{B.t.u.} \quad . \quad (174)$$

Whence the *efficiency* of this ideal cycle is

$$e_t = 1 - (Q_2/Q_1) = 1 - \frac{wc_v(T_d - T_e)}{wc_v(T_b - T_a) + wc_p(T_c - T_b)}$$

$$= 1 - \frac{T_d - T_e}{T_b - T_a + \gamma(T_c - T_b)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (175)$$



(a) (b)

FIG. 89.—Ideal Dual Combustion Cycle.

(c) Equation (175) is not convenient to use in many cases, hence it will be transformed into one involving no temperatures but only certain ratios of volume and pressure, as well as γ. Thus,

let r_a = adiabatic compression ratio $= \dfrac{V_e}{V_a}$;

r_c = cut-off ratio $= \dfrac{V_c}{V_b}$; and

X = constant-volume pressure-ratio $= \dfrac{p_b}{p_a}$.

Hence,

$$T_b = XT_a;\ T_c = T_b\left(\frac{V_c}{V_b}\right) = Xr_cT_a;$$

$$T_d = T_c\left(\frac{V_c}{V_d}\right)^{\gamma-1} = T_c\left(\frac{r_c}{r_a}\right)^{\gamma-1} = Xr_c{}^{\gamma}\frac{T_a}{r_a{}^{\gamma-1}};\ \text{and}$$

$$T_e = \frac{T_a}{r_a{}^{\gamma-1}}.$$

Substituting these values in Eq. (175), there results

$$e_t = 1 - \frac{1}{r_a{}^{\gamma-1}}\left[\frac{Xr_c{}^{\gamma} - 1}{(X-1) + \gamma X(r_c - 1)}\right]. \quad . \quad . \quad . \qquad (176)$$

(d) Equation (176) gives the ideal cycle efficiency of the Dual combustion cycle using any ideal gas. If it is desired to analyze this case using a *real* mixture, the corresponding ideal cycle efficiency may be found by the methods given for the Otto and Diesel cycles. Since the Dual combustion cycle is a combination of the Otto and Diesel the proper value of γ to use in Eq. (176) for comparison with the performance of the actual engine operating under full-load conditions will probably lie some place between 1.3 and 1.36.

134. The Brayton Cycle.—(a) Antedating somewhat the Otto engine, the Brayton machine was introduced in this country, about 1873, with considerable success, and was an important step in the development of the internal combustion engine. However, in addition to the main cylinder, it required a second one for compressing the charge, and owing to the relatively large size and greater first cost of the engine, it was soon displaced when the smaller and cheaper Otto engine appeared. The Brayton cycle is of importance because, when, reversed, it can be used in refrigerating machines.

Figure 90 (*a*) shows the diagram for the compressor cylinder, which raises the pressure of the working substance and then discharges it into a receiver. Fig. 90(*b*) is the diagram for the motor cylinder which gets from the receiver the combustible mixture which while entering burns at constant pressure until the supply is cut off, after which the products of combustion continue their expansion until the exhaust pressure is reached. The cyclic diagram in Fig. 91 represents the same net work as that with the two cylinders combined and with the same heat supplied, and, hence, is the cycle used in the analysis of the Brayton engine.

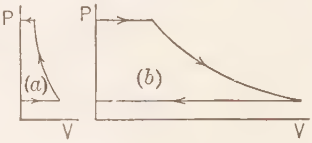


FIG. 90.

(b) From Eqs. (75) and (148), with w pounds of ideal gas as the working substance, the *net work* per cycle having *complete expansion* is found as follows:

$$Q_1 = wc_p(T_b - T_a), \text{ B.t.u.}$$

$$Q_2 = wc_p(T_c - T_d).$$

$$\textit{Net work} = Q_1 - Q_2 = wc_p(T_b - T_a - T_c + T_d), \text{ B.t.u.} \qquad (177)$$

Whence the cyclic *efficiency* is

$$e_t = 1 - \frac{Q_2}{Q_1} = 1 - \frac{wc_p(T_c - T_a)}{wc_p(T_b - T_a)} = 1 - \frac{T_c - T_d}{T_b - T_a}. \quad . \quad (178)$$

From Eq. (107),

$$T_c = T_b\left(\frac{P_c}{P_b}\right)^{\frac{\gamma-1}{\gamma}} = T_b\left(\frac{P_d}{P_a}\right)^{\frac{\gamma-1}{\gamma}}, \quad \text{and} \quad T_d = T_a\left(\frac{P_d}{P_a}\right)^{\frac{\gamma-1}{\gamma}},$$

or

$$e_t = 1 - \frac{T_b\left(\frac{P_d}{P_a}\right)^{\frac{\gamma-1}{\gamma}} - T_a\left(\frac{P_d}{P_a}\right)^{\frac{\gamma-1}{\gamma}}}{T_b - T_a} = 1 - \left(\frac{P_d}{P_a}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{V_a}{V_d}\right)^{\gamma-1}$$

$$= 1 - \left(\frac{1}{r_a}\right)^{\gamma-1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (179)$$

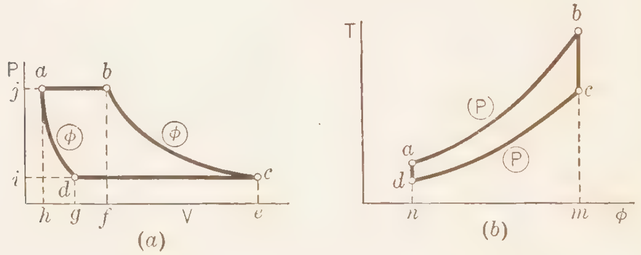


FIG. 91.—Ideal Brayton Cycle with Complete Expansion.

in which $r_a = V_d/V_a$ and is the adiabatic compression ratio, which is also equal to (V_c/V_b), the adiabatic expansion ratio.

(c) If the Brayton Cycle is carried out with *incomplete expansion*, as shown by *abc′kd* in Fig. 92, it reduces the ideal cycle efficiency slightly, but it has the great advantage of reducing the size of the engine per unit of power output. From Fig. 92 it is apparent that the maximum volume would be reduced from V_c to V_k, and yet the corresponding reduction in the net work, shown by the area *c′ck*, would be small.

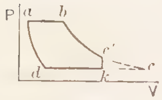


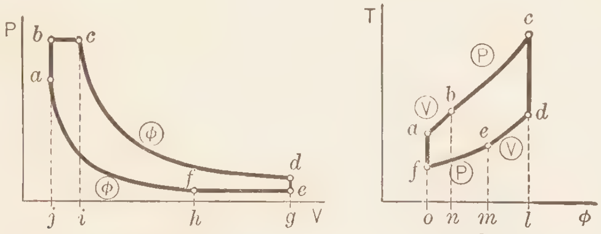
FIG. 92.

The expression for the ideal Brayton Cycle with incomplete expansion may be obtained from Eq. (182) in the next section by substituting $X = 1$, and the result is as follows:

$$e_t = 1 - \frac{1}{r_a^{\gamma-1}}\left[\frac{\frac{r_c^{\gamma}}{Z^{\gamma-1}} - Z + \gamma(Z-1)}{\gamma(r_c - 1)}\right], \quad . \quad . \quad (180)$$

where $Z = \dfrac{T_k}{T_d} = \dfrac{V_k}{V_d}$ in Fig. 92.

135.[22] **The Composite, or Walker, Cycle.**[23]—(a) This cycle, recently proposed, consists of two constant volume, two constant pressure and two constant entropy processes, as shown in Fig. 93. It has not been used commercially as yet, but because of the relatively high efficiency of its ideal cycle it is worth studying, although its application in practice may be handicapped by the complication of the mechanism that may be needed for carrying it out.



(a) (b)

FIG. 93.—Ideal Composite Cycle.

(b) Considering an ideal gas as the working substance, the efficiency of this cycle is

$$e_t = 1 - \frac{Q_2}{Q_1} = 1 - \frac{wc_v(T_d - T_e) + wc_p(T_e - T_f)}{wc_v(T_b - T_a) + wc_p(T_c - T_b)}$$

$$= 1 - \frac{T_d - \gamma T_f + (\gamma - 1)T_e}{(1 - \gamma)T_b - T_a + \gamma T_c}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (181)$$

Equation (181) may be expressed in more useful terms by using Eq. (106) and the following ratios:

$$r_a = \frac{V_f}{V_a} = \left(\frac{T_a}{T_f}\right)^{\frac{1}{\gamma - 1}} = \text{the adiabatic compression ratio;}$$

$$r_c = \frac{V_c}{V_b} = \frac{T_c}{T_b} = \text{the ``cut-off ratio'';}$$

$$X = \frac{P_b}{P_a} = \frac{T_b}{T_a} = \text{the pressure ratio for constant volume combustion;}$$

$$Z = \frac{V_e}{V_f} = \frac{T_e}{T_f} = \text{the volume ratio for constant pressure compression.}$$

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[22] This section may be omitted in brief courses.

[23] See "A New Thermodynamic Cycle," by W. J. Walker, in (London) Engineering, April 9, 1920, p. 467. Also Proceedings of the British Inst. of Mechanical Engineers, Vol. July to Dec., 1920, p. 1235.

Thus, from the foregoing, $T_b = XT_a$, $T_c = r_cT_b = Xr_cT_a$, $T_d = T_c\left(\frac{V_c}{V_d}\right)^{\gamma-1} = \frac{Xr_c{}^{\gamma}T_a}{(r_aZ)^{\gamma-1}}$, $T_e = ZT_f = \frac{ZT_a}{r_a{}^{\gamma-1}}$, and $T_f = \frac{T_a}{r_a{}^{\gamma-1}}$.

By substituting these values in Eq. (181) and simplifying, the efficiency of the Walker cycle becomes

$$e_t = 1 - \frac{1}{r_a{}^{\gamma-1}}\left[\frac{\frac{Xr_c{}^{\gamma}}{Z^{\gamma-1}} - Z + \gamma(Z-1)}{X - 1 + \gamma X(r_c - 1)}\right]. \quad . \quad . \quad (182)$$

(c) Since this cycle is made up of a combination of the Brayton, Diesel and Otto cycles the expression for its efficiency will reduce to the equations previously given for them, if the proper values of the several ratios involved are substituted in Eq. (182). These values are as follows: for the Brayton cycle with complete expansion, $X = 1$ and $Z = r_c$; for the Otto, $r_c = 1$ and $Z = 1$; for the Diesel, $X = 1$ and $Z = 1$; and for the Semi-Diesel, $Z = 1$.

136. Higher and Lower Heating Values, and Efficiencies.—(a) Most fuels used in internal combustion motors contain some hydrogen which during combustion, combines with oxygen to form water vapor, and as a result such fuels have two heating values. If the products of combustion have sufficient heat abstracted from them to condense all of the water vapor formed by combustion the amount of heat thus liberated per pound of fuel is known as the *higher heating value* (H.H.V.), and if none of this steam be condensed the so-called "*lower heating value*" (L.H.V.) would be obtained. This complication gives rise to a very considerable difference of opinion as to which value should be used in many kinds of engineering calculations. The Germans prefer to use the lower value, while in England and America the tendency is in favor of the higher one for determining thermal efficiencies.

(b) In the case of the internal combustion motor it may be said in favor of using the *lower* heating value to determine thermal efficiencies that there is no possible chance to cool the products of combustion down to such a low temperature that the condensation can take place within the engine. Even though the products be exhausted from the engine at a temperature equal to that at the beginning of compression little condensation can take place because the partial pressure of the water vapor is so low that the temperature necessary for condensation is in the neighborhood of 125° F., or even less.

(c) In favor of using *higher* heating values for determining thermal efficiencies of internal combustion motors and other apparatus it may be said that the machine itself is at fault if it cannot cool the products down to the temperature of the environment, say the atmosphere. Hence all thermal efficiencies should be based on the higher value.

(d) With internal combustion motors there is a third method that has the most merit, and is as follows: For the *thermal efficiency* of the actual and ideal engines, the engine should be charged with the *higher* heating value, but for all calculations of the *rise in temperature* due to combustion and for *the net work* of the ideal cycle use the *lower* value, since the temperatures thus found are actually obtained under ideal conditions. With real mixtures the specific heats are so dependent upon temperatures, that the ideal cycle calculated with the lower heating value gives the most accurate measure of the net work of the cycle for the given mixture of fuel and air that it is possible to calculate.

(e) Which one of the above schemes should be used by American engineers is a question that has not been answered completely as yet. The American Society of Mechanical Engineers specifies, in the Test Code for Internal Combustion Engines, the use of the higher heating value in determining the thermal efficiency of an internal combustion engine, but says nothing on the subject of finding the temperature rise from the combustion of a given weight of a real mixture of air and fuel.

(f) For the two numerical cases analyzed in Sects. **129** and **132** the third method, given in (d) of this section, is used because the authors believe that it is the most accurate and therefore the best one. Until there is an accepted standard for this class of work, various engineers may obtain slight differences in the numerical values of all efficiencies that are in any way dependent upon the heating value of the fuel.[24]

137. Applicability of Different Cycles.—In order that any ideal cycle may be commercially applicable, in the sense that it is good engineering to build actual engines to operate as nearly as possible on such a cycle, the following requirements must be considered:

(1) The *cycle efficiency* should be high.

(2) The *maximum pressure* should not be so great that the structural parts of the actual engine become unduly massive and therefore costly.

(3) The *mean effective pressure* should be as high a percentage of the maximum pressure as possible, thus obtaining the greatest output from an engine whose parts are designed to withstand the highest pressure.

(4) The *maximum temperature*, if long sustained, must not be so high that its close approach by the working substance in

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[24] The heating values of fuels will be discussed further in later chapters dealing with combustion and fuels.

the actual engine will cause detrimental effects in the cylinder, piston, valves or other mechanical parts.

(5) The *processes involved* should be such that they are approachable to a reasonable degree by the real engine without too much complication of the mechanism.

(6) The *processes* should be such that they may be carried out *rapidly*, so as to reduce to a minimum the size and cost of the engine for a given power output.

138. Comparison of Cycles.—(**a**) From the preceding sections it is apparent that the several ideal cycles may be compared on many different bases and the resultant conclusions may be very different. From the *thermodynamic standpoint the same temperature limits* should be chosen as the basis of comparison and it is in this sense that the Carnot stands highest. From the standpoint of any of the other requirements the Carnot is lowest in the list.

(**b**) The Otto, Diesel and Dual Combustion cycles are rather closely balanced when all of the requirements are considered, and it is for this reason that the three corresponding types of engines are in close competition to-day in certain classes of work. In other fields some one of them may be far superior to the others. All three are very important at the present time.

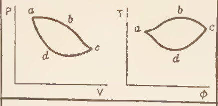
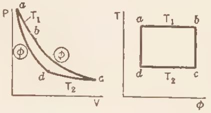
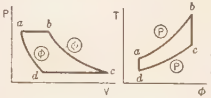
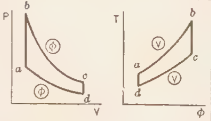
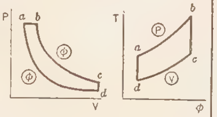
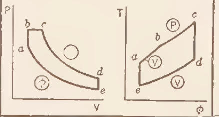
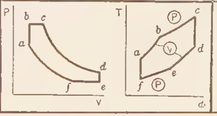
(**c**) The Carnot, Otto and Brayton cycles may be compared as to their processes of *reception* and *rejection* of heat. As has already been shown, in the Carnot cycle the working substance is at constant temperature during these processes. This cycle is therefore sometimes referred to as of the *constant-temperature* type. Similarly, since the working substance is at constant volume during the reception and rejection of heat in the Otto cycle, it may be called the *constant-volume* type. Likewise, since the heat transferring processes in the Brayton cycle occur only at constant pressure, this cycle may be considered to be of the *constant-pressure* type. The other processes making up each of these ideal cycles are reversible adiabatics.

(**d**) These same three ideal air standard cycles may be compared on the basis of having the *same adiabatic compression ratio*, r_a. From Eq. (151) the efficiency of the Carnot is $\frac{(T_1 - T_2)}{T_1}$, which becomes

$$e_{t\,Carnot} = 1 - \frac{T_2}{T_1} = 1 - \frac{T_d}{T_a} = 1 - \left(\frac{V_a}{V_d}\right)^{\gamma-1} = 1 - \left(\frac{1}{r_a}\right)^{\gamma-1}, \quad (183)$$

the last form of which is exactly the same as Eqs. (159*b*) and (179) for the Otto and Brayton cycles, respectively. Thus, for an ideal gas, the constant temperature, the constant volume and the constant pressure

TABLE VII.—IDEAL CYCLES WITH GASES HAVING CONSTANT SPECIFIC HEATS

Cycle		Ratios			Heat Supplied in B.t.u		
Name	Diagrams	Adiab. Compr.	Cut Off	Total	Q_1	Net Work in B.t.u.	Efficiency
		r_a	r_c	r			
General				$\frac{V_c}{V_a}$	Area under *abc* on the $T\phi$-diagram	Area enclosed Q_1-Q_2 Efficiency $\times Q_1$	Net Work ÷ Heat Supplied $\frac{Q_1-Q_2}{Q_1}$
Carnot		$\frac{V_d}{V_a}$		$\frac{V_c}{V_a}$	$wART_1 \log_e \frac{V_b}{V_a}$	$wAR(T_1-T_2) \log_e \frac{V_b}{V_a}$	$1-\left(\frac{T_2}{T_1}\right)$ $1-\left(\frac{1}{r_a}\right)^{\gamma-1}$
Brayton (Complete Exp.)		$\frac{V_d}{V_a}$ $\frac{V_c}{V_b}$		$\frac{V_c}{V_a}$	$wc_p(T_b-T_a)$	$wc_v(T_b-T_a-T_c+T_d)$	$1-\left(\frac{T_d}{T_a}\right)$ $1-\left(\frac{1}{r_a}\right)^{\gamma-1}$
Otto		$\frac{V_d}{V_a}$ $\frac{V_c}{V_b}$		$\frac{V_c}{V_b}$ $\frac{V_d}{V_a}$	$wc_v(T_b-T_a)$	$wc_v(T_d-T_a+T_b-T_c)$	$1-\left(\frac{T_d}{T_a}\right)$ $1-\left(\frac{1}{r_a}\right)^{\gamma-1}$
Diesel		$\frac{V_d}{V_a}$	$\frac{V_b}{V_a}$	$\frac{V_c}{V_a}$ $\frac{V_d}{V_a}$	$wc_p(T_b-T_a)$	$w[c_p(T_b-T_a)-c_v(T_c-T_d)]$	$1-\left(\frac{1}{r_a}\right)^{\gamma-1}\left[\frac{r_c^{\gamma}-1}{\gamma(r_c-1)}\right]$
Dual Combustion ("Semi-Diesel")		$\frac{V_e}{V_a}$	$\frac{V_c}{V_b}$	$\frac{V_d}{V_b}$ $\frac{V_c}{V_a}$	$w[c_v(T_b-T_a)+c_p(T_c-T_b)]$	$w[c_v(T_b-T_a-T_d+T_e)$ $+c_p(T_c-T_b)]$	$1-\left(\frac{1}{r_a}\right)^{\gamma-1}\left[\frac{Xr_c^{\gamma}-1}{(X-1)+\gamma X(r_c-1)}\right]$ where $X=P_b, P_a=T_b/T_a$
Composite or Walker		$\frac{V_f}{V_a}$	$\frac{V_c}{V_b}$	$\frac{V_d}{V_b}$ $\frac{V_e}{V_a}$	$wc_v(T_b-T_a)+wc_p(T_c-T_b)$	$w[c_v(T_b-T_a-T_d+T_a)$ $+c_p(T_c-T_b-T_e+T_f)]$	$1-\left(\frac{1}{r_a}\right)^{\gamma-1}\left[\frac{\frac{Xr_c^{\gamma}}{Z^{\gamma-1}}-Z+\gamma(Z-1)}{X-1+\gamma X(r_c-1)}\right]$ where $X=P_b/P_a=T_b/T_a$ $Z=V_c/V_f=T_e/T_f$

types of ideal cycles all have the same efficiency *provided they all have the same ratio of adiabatic compression.* Since the statement is sometimes made that these three cycles have the same efficiency, it is very important to note the *conditions* under which this is true, and to observe that they may not be feasible to adopt in commercial applications.

(e) Only the better-known and more important of the cycles have been considered and compared in this chapter. Many other cycles have been proposed, and some of these have been actually used.[25] Still others may yet be devised.

(f) The consideration of ideal cycles enables the engineer to draw very important conclusions regarding the relative performance, size and cost of the different engines. Such conclusions may be thus reached with only a small fraction of the cost in time, labor and material that would be involved in the process of manufacture of the various types for experimental purposes. The study of the ideal cycles also furnishes one of the best measures of the performance of any actual engine, as will be seen in the next chapter.

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[25] See Glazebrook's Dictionary of Applied Physics, Vol. I, pp. 279 to 311, and the more extensive treatises on Internal Combustion Engines.

CHAPTER XII

EFFICIENCIES AND PERFORMANCES OF ENGINES

139. Significance of Efficiencies.—(a) In this chapter will be given the definitions of the various terms that are used by engineers to express many important ratios, called efficiencies, that are involved in the design and operation of all types of heat engines. The use of a particular name for each efficiency permits engineers to express themselves concisely and accurately, provided only that there is no confusion in the definitions of these technical terms. In some cases several different names have been commonly used for the same efficiency, hence the more important of these will be given in addition to those that are preferred. Most of those given in this chapter have now been standardized by the American Society of Mechanical Engineers in its Code of Definitions and Values and in its various Power Test Codes. Whether the form of heat engine under consideration be a steam turbine, a reciprocating steam engine or an internal combustion motor, the various efficiencies defined in this chapter will be applicable to each case, unless otherwise specified.

(b) The power delivered by a prime mover is often determined by means of a brake-type of dynamometer, and hence is commonly referred to as the " brake " output, which may be expressed in terms of " brake-horsepower." The efficiencies that are based on it are termed " brake efficiencies."

In many cases the engine is " direct-connected " to the machine that it drives and cannot conveniently be tested separately. Also, when the prime mover is very large, there may not be available a dynamometer of sufficient capacity to measure its power, but the output of the driven machine usually can be readily determined. Further, interest often lies primarily in the performance of the combination of engine and driven machine. In these cases the complete unit is tested to determine its " combined performance " or its " combined efficiencies."

(c) The efficiencies of prime movers usually change when the loads on the machines are varied. Engines are usually " rated " at powers at which they give about their most economical performances, and at the same time allow some " overload capacity " to meet larger loads in emergencies.

140. Carnot Efficiency.—It has been shown in Sect. **119** that the Carnot efficiency is $(T_1-T_2)/T_1$, the symbols having the significance as defined in that section. When using the Carnot cycle efficiency as the standard of comparison for any real case in which an entire plant is considered, the values of T_1 and T_2 are to be taken respectively as the highest and lowest temperatures that occur *within the whole plant.* In case other apparatus besides the prime mover are involved with it, and it is desired to compare only the engine itself with the corresponding Carnot, the values of T_1 and T_2 are then to be taken as the highest and lowest temperatures, respectively, that occur in the working substance that is *within the engine* or in its immediate appurtenances.

141. Ideal Cycle Efficiency.—This term signifies the efficiency of an ideal engine, or plant, operating on a cycle that corresponds closely to the one followed in the actual case under consideration. The real engine, or plant, always has losses due to some or all of the following causes: leakage, throttling, turbulence, friction and loss of heat by radiation, conduction and convection; and furthermore, if the engine be of an internal combustion type, or if the plant includes a steam boiler, there will also be losses due to the imperfections of combustion. All such losses are considered as entirely absent in any ideal cycle. Since Q_1 and Q_2, as already defined, are respectively the heat supplied to the working substance and the heat rejected from it during an ideal cycle, their difference must represent the net work obtainable from this fluid as it passes through the particular ideal cycle involved. Hence, as given by Eq. (149), for the complete cycle of any working substance,

$$\text{The } \textit{Efficiency of any Ideal Cycle} = e_i = (Q_1 - Q_2)/Q_1.$$

142. Thermal Efficiencies.—(**a**) Of the heat supplied to an engine, the **Indicated Thermal Efficiency** (I. T. Eff.) represents the fraction, or percentage, that is transformed into mechanical energy *at the piston.* Since the latter energy is determined by an indicator, the reason for the name of this efficiency is apparent. Sometimes the phrase " piston thermal efficiency " is used instead.

(**b**) **The Delivered,** or **Brake, Thermal Efficiency** (B. T. Eff.) is the term used to show what fraction, or percentage, of the heat supplied to an engine is actually delivered by it. Sometimes the alternative terms " shaft efficiency " and " flywheel efficiency " are used.

(**c**) Similarly, the Thermal Efficiency of the Combined Unit, or **Combined Thermal Efficiency** (C. T. Eff.), shows what portion of the heat supplied to the engine has been delivered by the machine that is directly driven by it.

(d) In a steam plant, in addition to the prime mover and the machine it drives, there are boilers, with their furnaces, pipe lines, and various auxiliaries; hence, in determining the thermal efficiency of such a plant as a whole, the losses of all the equipment involved and the absorption of energy by the auxiliaries, must be considered. The **Plant Thermal Efficiency** (P. T. Eff.) is therefore the ratio of the energy delivered by the plant in a given time to the heat supplied by the fuel that it consumes during the same period.

In most internal combustion engine plants the prime movers and the machines directly driven by them constitute the complete plant; hence, in such cases, the Plant Thermal Efficiency is also the Combined Thermal Efficiency.

143. Mechanical Efficiency.—(a) The ratio of the power delivered by the engine shaft to that supplied to the piston is called the Mechanical Efficiency (M. Eff.), which may also be defined as the ratio of the brake horsepower to the indicated horsepower. The difference between these last two quantities represents the horsepower lost in the mechanism or, in other words, the "*friction horsepower*." If F is the ratio of the friction horsepower to the indicated horsepower, then $1-F$ is the Mechanical Efficiency.

(b) Since the steam turbine does not permit using an indicator, its i.hp. is not directly obtainable, and therefore its mechanical efficiency cannot be obtained by the ratio given in the first part of (a). In certain cases, however, it may be possible to calculate or estimate all of the mechanical losses in the turbine, and consequently its approximate mechanical efficiency may be determined.

144. Engine Efficiencies.—(a) The ratio of the thermal efficiency of the engine to its corresponding ideal cycle efficiency is called its *Engine Efficiency* (Eng. Eff.). In the past it has often been termed the "efficiency ratio," but this name has now been discarded by the American Society of Mechanical Engineers for the one given above, which is the more expressive because this efficiency represents a real measure of the performance of an engine. It applies to all heat engines whether of the reciprocating or turbine types. From the definitions of the three thermal efficiencies given in Sect. **142** (a), (b) and (c), it follows that these are the three corresponding engine efficiencies.

(b) **The Indicated Engine Efficiency** (I. Eng. Eff.) is given by the ratio (Indicated thermal efficiency)/(Ideal cycle efficiency), or by (Heat supplied per hp-hr. to the ideal engine)/(Heat supplied per i.hp-hr. to the actual engine). It shows how nearly the designer has succeeded in eliminating all leakage, turbulence, throttling and friction of the fluid, and, in the case of the internal combustion engine, it also indicates the perfection of combustion. It is evident that this efficiency shows how closely the actual engine has approached the corresponding ideal one, aside from the mechanical losses.

(c) The **Delivered Engine Efficiency**, or **Brake Engine Efficiency** (B. Eng. Eff.), is given by the ratio (Brake thermal efficiency)/(Ideal cycle efficiency), or by (Heat supplied per hp-hr. to the ideal engine)/(Heat supplied per b.hp-hr. to the actual engine). Unless otherwise specified the unqualified term "engine efficiency" is understood to mean the "*brake engine* efficiency," thus including the mechanical losses as well as the thermodynamic ones specified in (b).

(d) **The Engine Efficiency of the Combined Unit**, or **Combined Engine Efficiency** (C. Eng. Eff.), which includes the losses in the direct-driven machine as well as those in the prime mover itself, is found from the ratio (Combined thermal efficiency)/(Ideal cycle efficiency); or, if the engine drives an electric generator, this efficiency is determined from the ratio (Heat supplied per kw-hr. to the ideal engine)/(Heat supplied per kw-hr. delivered by the generator).

145. Heat Consumption.—(a) Since all heat engines are operated by supplying heat to them, their thermal performances, or those of the plants containing them, may be expressed in terms of the heat supplied to produce a unit of energy output, which is normally taken as the horsepower-hour, or the kilowatt-hour. The term "Heat Consumption," unless otherwise stated, is understood to mean the amount of heat required per *unit* of energy output. Obviously it can be obtained by dividing the total consumption (T. C.) in a given time by the simultaneous energy output. Evidently the Heat Consumptions (H. C.), expressed in B.t.u., for the different cases may be obtained in the following manner:

$$\text{H. C. per i. hp-hr.} = \text{T. C.}/(\text{Total i. hp-hr.}) = 2545/(\text{I. T. Eff.}),$$

$$\text{H. C. per b. hp-hr.} = \text{T. C.}/(\text{Total b. hp-hr.}) = 2545/(\text{B. T. Eff.}),$$

$$\text{H. C. per kw-hr.} = \text{T. C.}/(\text{Total kw-hr.}) = 3413/(\text{C. T. Eff.})$$

(b) The unit rate of heat supplied to an internal combustion engine, or to a steam plant, is the product of the heating value per pound of fuel and the amount of fuel used per unit of energy output. With steam prime movers, the determination of the unit rate of heat supply involves the use of the properties of steam and requires special definition which will be given later.

146. Comparison of Methods of Expressing Performances.—(a) To the purchaser of an engine, the definite assurance of either a certain *heat consumption* per unit of energy delivered, or a definite *thermal efficiency* to be obtained under the specified conditions of operation, is of the utmost importance because upon such a guarantee he may make correct estimates of the cost of fuel that will be needed to operate such an engine. These same measures of performance of engines and plants are of great value in analyzing the unit costs of operation, in B.t.u. or dollars. Also they are often used in making comparisons between different installations.

(b) However, just because one engine has a higher thermal efficiency, or lower heat consumption, than another, it should not always be concluded that the first necessarily represents better design or construction. The case may be somewhat analogous to that of two water wheels of equally good design and manufacture, but one may require twice as much *water* to furnish the same power as the other because the " hydraulic head " is half as great. Similarly, one heat engine may consume twice as much *heat* as another because its " temperature head," with the conditions under which it is operating, may be only about half as large as with the latter. Further, under certain conditions, the gain due to high thermal efficiency may be offset by additional costs of equipment, or of operation, required for its accomplishment.

(c) To the designer and builder of an engine the *engine efficiency* is of the greatest importance, because it shows the degree of perfection attained in reducing *all* of the losses that are under control, or, in other words, it indicates the success, under the particular limitations imposed by the conditions under which the engine has to operate, in utilizing the energy that is available. In case the designer or others are concerned only with certain changes in the engine, or in comparisons that involve only thermal losses, the *indicated* engine efficiency then becomes of primary interest.

147. Effect of Load on Economy.—(a) The friction horsepower is the amount that is needed to overcome the mechanical friction of the engine. Even if an engine is delivering no power it requires a certain continuous supply of energy to keep it running, and consequently the mechanical efficiency is zero under the " no load " conditions. In most engines the friction remains nearly constant regardless of the amount of power that is being developed, hence this efficiency is highest when the engine is fully loaded, and it becomes less when the power output is reduced. Similarly, the thermal losses in most engines are such that the indicated engine efficiency is reduced when the load is decreased. On the other hand, the cycle efficiency in most cases is nearly independent of the load. The Diesel engine, however, is one exception to this last statement, since its cycle efficiency usually improves with decreases in loads, as was shown in Chapter XI. The impairment of the "engine efficiency" with reduction of loads is usually sufficient to cause the delivered thermal efficiency to decrease also.

(b) The wide variations of the thermal economies with the changes in the loads on engines give rise to the use of "**performance curves,**" or "*economy load curves,*" in which the ordinates are the various efficiencies and heat consumptions, and the abscissas are either horsepower, kilowatts, or percentages of rated load. The heat consumptions and the various efficiencies are factors of very great importance. How some of these vary with changes in load in the usual cases is illustrated in (*a*) and (*b*) of Fig. 94, in which the abscissas are percentages of the rated load. In the first of these it is assumed that the total consumption (T. C.) of heat by the engine per unit of time (for example, in B.t.u. per hour) obeys the *straight-line* law, the y-intercept being the heat per hour required to overcome the losses of the engine when it is not loaded. By dividing the ordinates of the TC-curve by the corresponding b.hps., it being assumed that the rated load is known, the ordinates for the curve of the unit rate (U. R.) of heat consumption, in B.t.u. per b.hp-hr., are obtained. The UR-curve in this case continues downward as the load increases, even though the engine is loaded beyond its rated power. By making use of the curve of mechanical efficiency, the curve of B.t.u. per i.hp-hr. can be obtained from the UR-curve on the b.hp-hr. basis which has already been drawn.

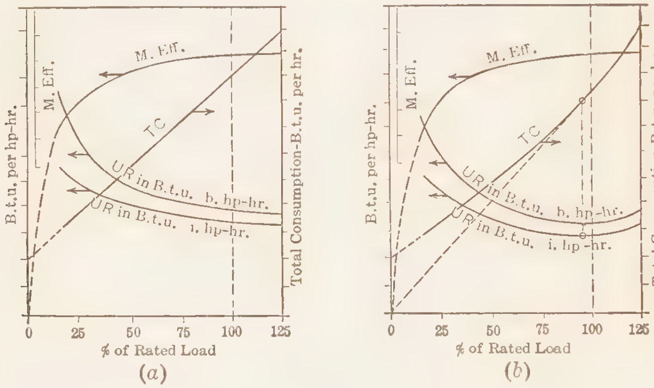

FIG. 94.

In (*b*) in Fig. 94 is shown a TC-graph that is curved, and the corresponding UR-curves have low points of inflection that are immediately below the point where a straight line from the origin is tangent to the TC-curve. It is usually preferable to operate the engine at, or near, the load that gives the minimum consumption, although this may not always be desirable when certain other economic factors or conditions of operation are considered, as will be seen later. Engines having UR-curves of this character are usually rated near this minimum point.

In charts like those in Fig. 94, the ordinates, instead of being on the basis of heat consumption, may represent the corresponding amounts or costs of fuel or working substance; or they may be on the basis of proportionalities or percentages.

How the other efficiencies vary with the loads will be considered later in connection with the discussion of each of the different kinds of prime movers.

148. Summary.—The definitions of the several efficiencies that are most often used in the analyses of the performance of heat engines and power plants, and the relations between them are summarized as follows, the B.t.u. being taken as the unit of heat:

$$\text{Mechanical Efficiency (M. Eff.)} = \frac{\text{b.hp.}}{\text{i.hp.}} = \frac{\text{B. T. Eff.}}{\text{I. T. Eff.}} = \frac{\text{Heat supplied per i.hp-hr.}}{\text{Heat supplied per b.hp-hr.}} \quad . \quad . \quad (184)$$

$$\text{Indicated Eng. Eff. (I. Eng. Eff.)} = \frac{\text{I. T. Eff.}}{\text{Cycle Eff.}} = \frac{\text{Heat supplied per hp-hr. to ideal engine}}{\text{Heat supplied per i.hp-hr. to actual engine}}. \quad (185)$$

$$\text{Engine Efficiency (B. Eng. Eff.)} = \frac{\text{B. T. Eff.}}{\text{Cycle Eff.}} = \frac{\text{Heat supplied per hp-hr. to ideal engine}}{\text{Heat supplied per b.hp-hr. to actual engine}}. \quad (186)$$

$$\text{Combined Engine Eff. (C. Eng. Eff.)} = \text{C. T.Eff.} \div \text{Cycle Eff.} = \frac{\text{Heat supplied per kw-hr. to ideal unit}}{\text{Heat supplied per kw-hr. output from actual unit}}. \quad (187)$$

$$\text{Indicated Thermal Eff. (I. T. Eff.)} = \text{Cycle Eff.} \times \text{I. Eng. Eff.} = \frac{2545}{\text{Heat supplied engine per i.hp-hr.}} \quad . \quad (188)$$

Brake Thermal Eff. (B. T. Eff.) = Cycle Eff. × B. Eng. Eff.

$$= \frac{2545}{\text{Heat supplied engine per b.hp-hr.}} \quad . \quad (189)$$

Combined Th. Eff. (C. T. Eff.) = Cycle Eff. × B. Eng. Eff. × Eff. of driven mach. (190*a*)

$$= \frac{3413}{\text{Heat supplied eng. per kw-hr. of elec. generator output}} \quad . \quad (190b)$$

Plant Th. Eff. (P. T. Eff.)

$$= \frac{2545}{\text{Heat supplied plant per hp-hr. deliv. by plant}} \quad . \quad (191a)$$

$$= \frac{3413}{\text{Heat supplied plant per kw-hr. deliv. to switchboard.}} \quad . \quad (191b)$$

CHAPTER XIII

INTERNAL COMBUSTION ENGINES

TYPES AND MECHANICAL FEATURES

149. Introductory.—(a) In one type of engine the combustion of the fuel takes place within the working cylinder and the products of combustion constitute the working substance. These are called *Internal Combustion Engines*, and to them this chapter is devoted. In other types the working substance is heated before it enters the cylinder and it is not composed of the burnt gases. Such prime movers are *External Combustion Engines.* Steam engines and turbines are in this class, as the steam receives its heat in the boiler, which is separate from the power unit. *Hot-air Engines* also are of this type, the air being heated externally to the working cylinders. These engines are of interest principally because they generally use a regenerative cycle such as those of Stirling or Ericsson. However, relative to the power output, they are bulky and costly; and their actual efficiencies are quite poor. Because their application is very limited they will not be considered further in this text.[1]

(b) It is the purpose of this and the following chapter to show, very briefly, how a few typical actual internal combustion engines operate to carry out their cycles, and to discuss, also briefly, the method of determining the sizes needed for a given power generation, and the performances and efficiencies that are obtainable. The properties of the fuels used and their combustion will be considered later.

150. Some General Types of Internal Combustion Engines.—(a) The longitudinal section of a medium size, Otto *four-stroke* cycle engine is illustrated in Fig. 95, in which the more important parts are labeled. During the suction stroke of such an engine the inlet valve is held open so as to allow the combustible mixture to flow into the cylinder. This valve then closes and the compression stroke occurs. After ignition and combustion, the expansion stroke takes place, and is followed by the exhaust stroke, during which the exhaust valve is kept open by the valve gear. Thus there are the two strokes, suction and exhaust, which do not produce power, but, instead, absorb it. However, they insure a fairly complete and pure charge of combustible mixture for the combustion process, and this may more than offset their disadvantage. The four-stroke cycle requires two revolutions of the crank, hence the valve-operating mechanism is driven at half speed from the main shaft. An automatic governor controls the supply of working substance so as to maintain substantially constant speed under all loads. Only one side of the piston is acted on by the working substance, hence the engine is said to be *single-acting*. The cylinder and head are water jacketed, and the piston is of the trunk or bucket type, with no special provision for cooling it.

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[1] For detailed information regarding hot-air engines, see Carpenter and Diederich's "Experimental Engineering," Rankine's "Steam Engine," Roentgen's "Thermodynamics," and Encyclopedias.

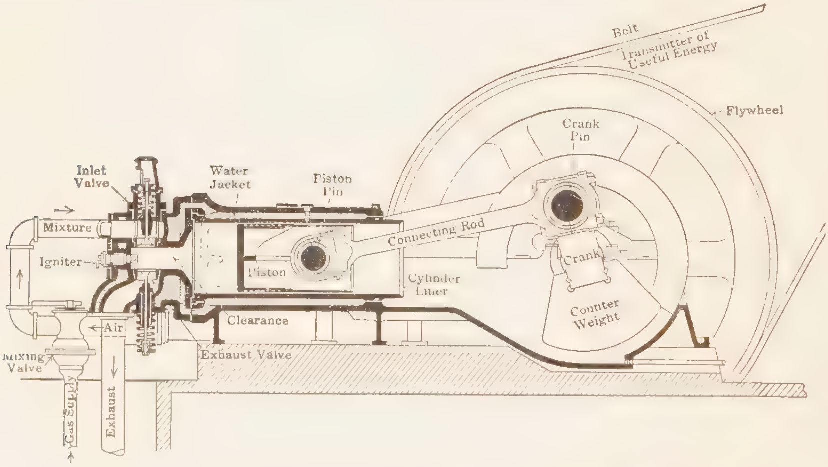

Fig. 95.—Longitudinal Section of an Otto Four-stroke Cycle Engine.

(b) Figure 96 shows diagrammatically a large engine of the Koerting type that uses the *two-stroke* Otto cycle and, further, is *double-acting*.

This engine has separate pumps for supplying the air and gas under slight pressure. With the piston in the position shown, the working stroke has just been completed at the left end of the cylinder, the spent gases are leaving through the central exhaust ports, which have been uncovered by the piston, and these gases are being replaced by the combustible mixture which is entering through the inlet valve I. The piston, in addition to its usual function, acts also as an exhaust valve. When it moves to the left it closes the exhaust ports, and compresses the fresh charge, the inlet valve now being closed. Upon reaching the extreme left position ignition occurs and the power stroke begins. This stroke continues until the exhaust ports are again uncovered by the piston, thus completing the two-stroke cycle. The right-hand end of the cylinder performs in a similar manner. Each end of the cylinder has one valve, the inlet, and has a working stroke at each revolution of the engine. The valve gear would be driven at the same speed as the engine shaft.

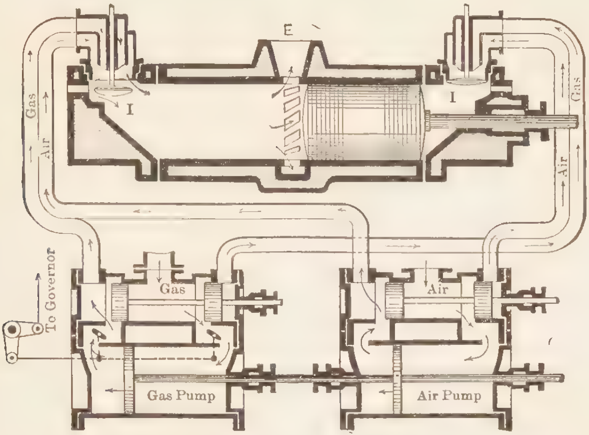

Fig. 96.—Diagram of Koerting Double-acting Two-stroke Cycle Engine.

(c) Figure 97 shows a small *single-acting*, vertical two-stroke cycle engine in which the mixture is admitted to the crank case, where it is compressed by the underside of the piston, which acts as a pump during the down stroke. The opening of the inlet and exhaust valves are replaced by the piston uncovering the inlet and exhaust ports when it is near the end of its stroke. The fresh charge, under slight pressure, enters the cylinder through the inlet port and is so baffled as to assist in driving out the burnt gases. There are many other arrangements for compressing and admitting the charge.

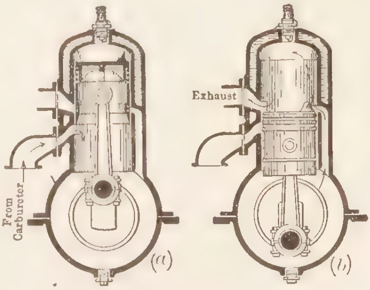

Fig. 97.—Two-stroke Cycle Engine with Crank-case Compression.

(d) As compared with the four-stroke cycle engine having the same cylinder size and speed, the two-stroke cycle engine might be expected to develop double the power. Actually, however, the power output is not doubled, but is increased in the ratio ranging from 1.7 to 1.9.

This may be partly due to imperfect charging of the cylinder and to waste of unburnt combustible mixture through the exhaust ports, and partly to the expenditure of energy in pumping. For these same reasons the usual two-stroke cycle engine is not in general as economical in the utilization of fuel as the four-stroke cycle one. Also, it may be limited to lower rotative speed in order to provide time for the substantial completion of the exhaust and charging processes. Extra "scavaging" air may be introduced to clear the cylinder of spent gases before admitting the new charge, but this may be done at the expense of additional pumping and consequent energy absorption, and may involve considerable complication of mechanism.

The two-stroke cycle engine will have a more uniform turning effort on the crank shaft, and hence require a smaller flywheel, and it can dispense with the inlet and exhaust valves, thus discarding their valve gear mechanism, and making the cylinder head less complicated.

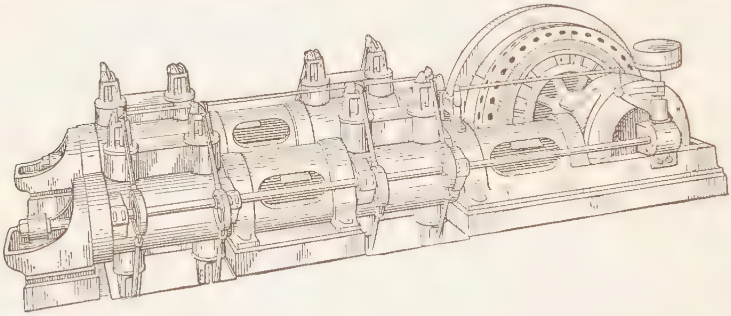

FIG. 98.—Twin, Tandem, Double-acting, Four-stroke Cycle Engine.

Hence, per horsepower of capacity, the two-stroke cycle engine has lighter weight, costs less, and occupies a smaller space than the other.

(e) The *single-acting* four-stroke cycle engine has one impulse in two revolutions, i.e., in four strokes; the corresponding two-stroke cycle engine has a cycle each revolution, or every two strokes; the *double-acting* engine of the latter type develops power at each stroke with consequent saving of weight and space, but involves some added complications.[2]

If two cylinders are arranged in *tandem,* so that one is behind the other on the same piston rod, they will give double the power of a single cylinder, while using the same frame, connecting rod and crosshead that the latter would have. If two horizontal engines are side by side and drive the same shaft, they are called *twin engines.* Fig. 98 shows a twin-tandem, double-acting, four-stroke cycle engine. A similar arrangement may, of course, be used with the two-stroke cycle, in which case there would be as many impulses per revolution in each tandem element as occur in a single-cylinder double-acting steam engine.

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[2] See C. E. Lucke's "Large Oil Engines with Special Reference to the Double-Acting Two-Cycle Type." Trans. A. S. M. E., 1924.

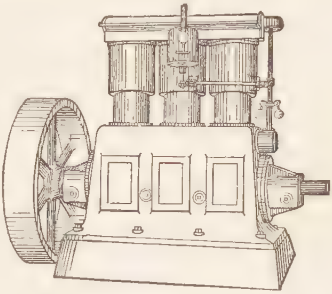

Fig. 99.—Three-cylinder Gas Engine.

(f) Vertical engines may be single or multi-cylinder ones. In the latter, the cylinders are arranged in line with as little space as possible between them, and they are termed *two-cylinder vertical*, *three-cylinder vertical*, etc., depending on the number of cylinders. A three-cylinder vertical engine is shown in Figs. 99 and 100. As many as six or eight cylinders in line are sometimes used. The turning effort on the shaft is improved by having a multi-cylinder arrangement, and a lighter weight of flywheel may be used than with a single cylinder engine of the same power. Also, the engines can be better "balanced," so as to prevent vibrations. Because of the intense heat to which it is subjected, there is a limit to the size of the uncooled piston that can be used, hence the multi-cylinder arrangement is often adopted for large powers for this reason as well as for the others already given.

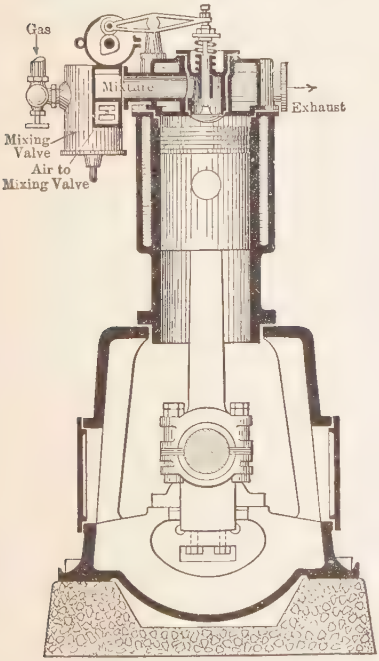

Fig. 100.—Gas Engine.

(g) In the engines thus far shown the cylinders and heads were *water jacketed*, the cooling medium being constantly supplied to keep the parts from becoming overheated. In double-acting engines the pistons and piston rods are made hollow, and cooling water or oil is circulated through them. Large trunk pistons are also sometimes similarly cooled. Likewise, the larger exhaust valves and stems are usually made hollow and water is forced through them.

The cylinders and heads of small engines may be *air-cooled*, extra surface for this purpose being obtained by ribbing, as in Fig. 101, and special means for forcing air over the surfaces often being provided. This system of cooling, however, is not adequate for large cylinders because, as the size is increased, the volume, which is an important factor in determining the amount of heat generated, and which is a cubic function of the cylinder diameter, increases faster than does the surface, which is a second power function, and hence there is a limit to its applicability.

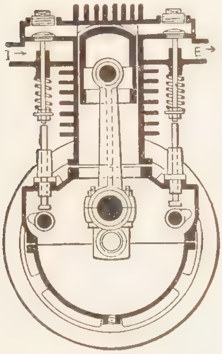

FIG. 101.—Air-cooled Engine.

(**h**) When the main shaft has a crank pin that is supported by crank webs on both sides and that is located between two main bearings it is called a *center-crank* arrangement. A *side-crank* engine is one which has a crank pin that is overhung from a single crank arm and that has a main bearing on one side only.

151. Gas Engines.—In Sect. **32** there was given, in connection with Fig. 12, a brief description of the method of using in large engines the gases from a blast-furnace plant. Fig. 102 shows the cross-section of such an engine, the largest of which so far built has double-acting cylinders 59 inches in diameter and a stroke of 59 inches.[3] The inlet and exhaust valves are operated by a valve gear which receives its motion from a "lay shaft," or "half-time shaft." Fig. 98 illustrates a large twin-tandem blast-furnace gas engine. The longitudinal section of a small single-acting gas engine has already been shown in Fig. 95, and other gas engines are shown in Figs. 96, 98, 99 and 100. Some of these engines operate on the two-stroke and others on the four-stroke Otto cycle.

The supply of gas, or of mixture of gas and air, furnished during each cycle to such engines, is usually controlled by an automatic governor, and can also be adjusted by hand. The methods of governing, the various kinds of mixing valves and ignition devices, and the many possible arrangements of engines will not be considered at this point.

152. The Gasoline Engine.—(**a**) Gasoline is relatively an expensive fuel and hence is adopted for the production of power only when a small output is needed or when other considerations, such as convenience or pleasure, offset this cost. It is used in farm, contractors' and other kinds of small *stationary* engines and in automobile, motorboat, airplane, and other automotive engines. As gasoline volatilizes readily at ordinary temperatures, a *carburetor* is commonly employed on the latter engines to atomize and mix it with the amount of air needed for supporting its combustion. Fig. 103 illustrates a simple form of a carburetor. In this device the float maintains the gasoline in the chamber at a level just below the tip of the central nozzle. The rapid passing of air past this nozzle causes the gasoline to overflow and atomize, and thus to become entrained in the air which is being drawn into the cylinders by the movement of the pistons during the suction strokes. On automotive engines, which must operate over a

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[3] "The Gas Engine in the Steel Industry," by A. C. Danks, in Mechanical Engineering, 1924.

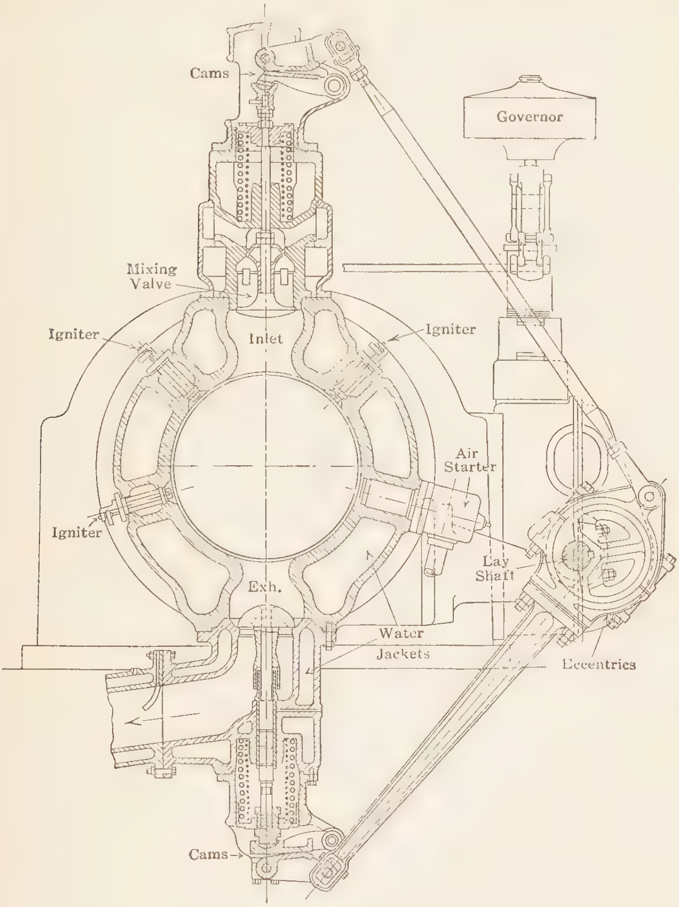

Fig. 102.—Large Gas Engine.

wide range of speeds, the main throttle valve is controlled by hand, and at high speeds of the engine the mixture ratio tends to become too rich, hence the carburetor is often provided with an auxiliary valve which is so arranged that the increased suction will cause it to open and admit additional air to dilute the mixture. With engines which operate at constant speeds the auxiliary air valve is not needed.

In Sect. **30**, in connection with Fig. 10, was given a brief description of an automobile engine and its method of functioning.

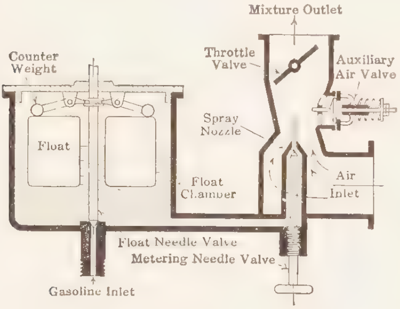

Fig. 103.—Simple Carburetor.

(**b**) In gasoline engines of the *stationary* type, "carbureting valves," or other simple devices, are often substituted for the carburetor; and the supply of fuel or of the mixture is automatically regulated by some kind of a "governor."

Gasoline engines are nearly always single acting and have electric ignition of some kind. Those for automobiles and airplanes commonly use the Otto four-stroke cycle. For other service the two-stroke Otto cycle is often used. The cylinders of these small engines may be either water or air cooled, and the pistons are not provided with any special cooling medium.

In automobile engines the compression ratio is usually limited to about 5, and often is less, to avoid trouble from detonation. The corresponding pressures at the end of compression usually range from 60 to 95 lb. per sq. in. The largest diameter of uncooled piston found feasible to use in automotive engines is not much over 5 inches. Alcohol, benzol, and other volatile fuels, may be used in a carburetor either singly, or in combination with each other, or blended with gasoline, the engine being substantially the same as for the latter fuel, except for a possible change in the clearance volume, so as to use a more suitable compression ratio.[4]

153. Kerosene Engines.—Kerosene gives off little vapor at ordinary temperatures and therefore it is more difficult to form a proper mixture of fuel and air with this fuel than with gasoline. Although cheaper than the latter, it is nevertheless costly relative to some of the other liquid fuels that are available, hence its application is limited usually to small engines of the types used on farms and by contractors. Under very favorable conditions a carburetor may be used, as with gasoline, except that it is heated, usually either by the jacket water or the exhaust gases; but under ordinary conditions considerable tar and carbon will be deposited in the cylinder and spontaneous ignition may occur under heavy load. The injection of water, in the form of vapor or fine spray, into the combustible mixture helps to overcome these difficulties, hence a water " carburetor " is often added, or is combined with the one for the kerosene.

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[4] For supplementary reading see any of the numerous texts on automobile, airplane and other small high-speed engines.

The expedients usually employed, either singly or in combination, to use kerosene successfully, are (1) using a much finer spray than with gasoline, (2) heating the liquid to a much higher temperature, (3) heating the air supply, (4) passing the mixture over a hot surface, (5) using a mixture of kerosene and gasoline with proportions varied to suit the load, and (6) using gasoline alone for starting. The use of two extra strokes after suction, to thoroughly vaporize and mix the kerosene, has also been proposed but involves a larger engine for the same power and speed.

Kerosene engines generally follow the Otto cycle and are most satisfactory when the load does not change rapidly. They require especially careful cooling of the cylinder head, and the avoidance of hot spots within the combustion chamber, to prevent preignition. The charge is usually electrically ignited. The compression pressure is usually less than 65 lb. per sq. in., or a compression ratio below $3\frac{1}{2}$, to avoid detonation and consequent " knocking." Principally because of the low compression, poor vaporization and the considerable choking action needed through the throat of the vaporizer, and fuel mixer, which lowers the volumetric efficiency, these engines are generally less economical in the consumption of fuel than are those using gasoline. For the various means of using kerosene, and for a description of the special mechanical features involved, see reference books specially devoted to small engines of this type.[5]

154. Oil Engines.—In this connection the term " oil " is limited to those liquid fuels that have little volatility and are relatively viscous, or " heavy." The oils generally used are comparatively cheap. Thus kerosene, gasoline, benzol, alcohol, and other of the lighter and more expensive liquid fuels, are not included.

Oil-using engines may be classified as low-compression hot-vaporizer ones, Semi-Diesel medium-compression ones, and Diesel high-compression ones. However, it should be noted that it is difficult to classify some engines, and that a change in the character of the fuel, or in the operating conditions, may modify the method of functioning of a given engine to such an extent as to shift it into another class.

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[5] "Farm Gas Engines," by Hirshfeld and Ulbricht and "The Gasoline Automobile," Vol. IV, by P. M. Heldt.

155. Low-compression Oil Engines.—(a) These engines are also said to be of the "surface ignition" or "hot vaporizer" type. Commercial engines of this kind nearly always follow the Otto *two-stroke* cycle and use a compression that terminates with a pressure varying from 70 lb. per sq. in. for light oils up to 140 with those that are heavy. The temperatures at the end of compression are not high enough to cause ignition, hence reliance is placed on assistance from some hot part of the metal in the clearance space.

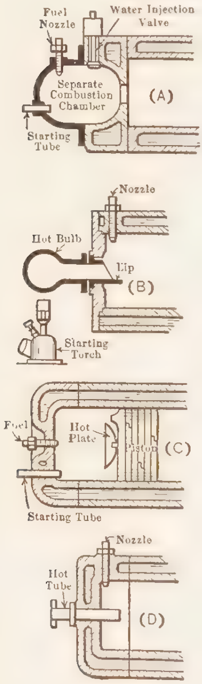

Fig. 104.

(b) In one design, (*A*) in Fig. 104, most of the clearance volume is in a **separate combustion chamber,** which is in the cylinder head and which has only a small neck connecting it to the cylinder. Part of this chamber is not water jacketed and hence becomes very hot. After a cycle is completed, this cavity remains filled with spent gases. Into it the fuel is injected and is supposed first to become "cracked,"—i.e., the heavier hydro-carbon constituents break up into lighter and less complex ones,—and then vaporization occurs,—ignition, however, cannot take place because of the absence of oxygen. As is usual in the two-stroke cycle engine, the air for combustion is compressed in the crank case, or in other space, by the crank side of the piston, and enters the cylinder through the inlet ports when they are uncovered by the piston, just after it has exposed the exhaust ports. On the return stroke the piston compresses this air and finally forces it through the inert gases in the combustion chamber and brings it into contact with the fuel vapor therein. Ignition then results and expansion takes place, driving out the piston until the exhaust and air inlet ports are again uncovered, thus completing the cycle.

(c) The **hot-bulb** arrangement, (*B*) in Fig. 104, is somewhat similar to the one just described except that the chamber is smaller and its shell has a lip that projects into the cylinder. The oil is dropped onto this heated lip and the resulting vapor and surrounding burnt gases are driven into the bulb by the air during compression. Near the end of the stroke the air is forced into contact with the heated vapors and ignition occurs. This and the foregoing type are especially suitable when the fuel consists of the heavier oils that are not easily volatilized

(d) Another type, see (*C*) in Fig. 104, uses a dished **hot-plate** which is attached to the end of the piston and which retains so much heat and is at a sufficiently high temperature that when the oil is injected toward it, as the piston is approaching the end of the compression stroke, the fuel is vaporized and ignition follows.

(e) In a fourth type, (*D*) in Fig. 104, a sealed **hot-tube** projects through the water-jacketed cylinder head into the clearance space where it becomes heated to a high temperature The oil is dropped onto this tube which vaporizes it, and aids in causing ignition to occur near the end of compression. For starting the engine a torch is used to heat a part of the tube that projects outside of the cylinder for that purpose, or the tube may be removed for heating. These last two engines are best suited to the use of the lighter oils.

(f) For *starting*, engines of the separate-combustion-chamber and hot-plate types may be provided with small hot-tubes which may be heated by a torch. In the hot-bulb engine the bulb projects through the cylinder head and can be directly heated by external means.

(g) These two-stroke cycle engines do not become completely cleared of exhaust gases, and in the separate-combustion-chamber and hot-bulb types reliance is placed on a stratified layer, or blanket, of these remaining burnt gases to prevent the compressed air from reaching the vaporized fuel before the proper time. In the other two cases, the design is such that the temperature needed to cause ignition is not reached until compression is about completed. Under heavy loads it is often necessary to use water injection in all these engines in order to prevent preignition from occurring. The processes of cracking and vaporization take a relatively long time, hence to provide for their completion the fuel must be injected when the crank is at a considerable angle, 20 to 40 degrees, preceding the dead center position for starting the working stroke, and the engines must not be operated at rotative speeds that are too high.

Since these low-compression engines commonly use the two-stroke cycle and have low maximum pressures, they are relatively small and cheap for a given power output. As they are simple and reliable and can be operated by unskilled attendants, they have been used in great numbers in medium sizes, say, from 10 hp. to 200 hp.,[6] although they are not as economical in the use of fuel as are the engines using higher compressions.

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[6] For a further study of these engines see Morrison's "Oil Engines."

156. Semi-Diesel Engines.—(a) To fall strictly into this class an engine must have a compression sufficient to produce a temperature that will cause ignition to occur without other assistance, and the fuel must be burned with dual combustion. Many engines are now built to approach these conditions, although in some of them a hot-bulb vaporizer is included as an extra precaution to meet certain emergencies that may develop. They are also called Dual-Combustion Engines, Medium Compression Engines, and Mixed Cycle Engines; they use either the two-stroke or four-stroke dual-combustion cycle, the ideal of which was considered in Sect. **133**. In all cases the air contains no fuel while it is being compressed.

(b) In these engines the oil is introduced in such manner that part of it burns substantially at constant volume and the remainder at constant pressure. This dual process is accomplished by injecting one portion of the fuel after the other, or by delaying or prolonging the ignition or by other methods, only a few of which will be discussed. As this type of engine is comparatively recent, many of the elements are still covered by patents.

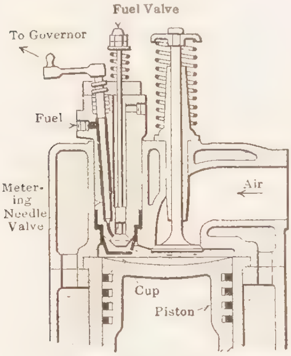

Fig. 105.—*Hvid* Pre-combustion Arrangement.

(c) The **Hvid, or Brons, pre-combustion** device, Fig. 105, has an uncooled fuel cup with fuel admission and metering valves leading thereto. During the air admission, in a four-stroke cycle, the fuel charge is deposited in the cup which has very small perforations around its base. At the end of compression the hot air forces its way through these openings and the more volatile portion of the fuel-charge explodes at approximately constant volume and sprays the remainder into the cylinder in which much of it is burned substantially at constant pressure. The small size and the limitation of the number of holes possible to use make this method usually inapplicable to engines of over 30 hp. per cylinder end.

(d) **Divided combustion** can be partly accomplished by employing a more ample chamber with larger holes than the foregoing and by injecting the fuel in an atomized form into this chamber just before the end of compression. Under these conditions part of the combustion occurs at constant volume within the chamber and the remainder at constant pressure outside of it, after the piston has started on its power stroke.

In other engines, which use fuels that have light constituents, the more volatile parts burn first, perhaps assisted by a heated vaporizer, followed by the **delayed combustion** of the heavier portions, after the piston has started its power stroke. Delayed combustion may also be brought about by compressing into a separate clearance pocket part of the air which does not come in contact with the fuel until combustion has progressed somewhat.

(e) In the **dual injection** arrangement a primary charge is followed by a secondary one. In Fig. 106 is shown an engine having **super-compression,** the air entrapped in the annular space *a* becomes highly compressed and, issuing through the small space around the projecting plug on the piston, enters at high velocity the combustion space where it meets the spray of fuel. In other arrangements the air from the annular space may be forced through a small passage in such a manner as to blow into the combustion space the fuel charge which has been previously deposited in a suitable recess. In the Price arrangement two jets are made to impinge on each other in the middle of a combustion chamber which is approximately spherical.

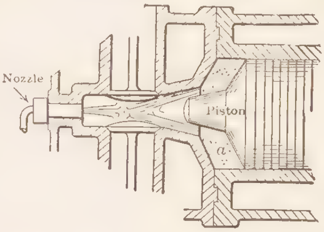

Fig. 106.—Super-compression Arrangement.

(f) The fuel may be sprayed into the cylinder by compressed air, in which case a multi-stage air compressor is included as part of the engine, but generally **solid injection** is used, the fuel being pumped at high pressure, often over 4000 lb. per sq. in., through suitable nozzles which cause it to break up into minute particles and mingle with the compressed air in the cylinder.[7]

(g) Semi-Diesel engines use either the two- or four-stroke cycle, and generally use compression pressures of from 275 to 350 lb. per sq. in., the amount depending on the method of preparing the fuel mixture—in one case as low as 200 lb. per sq. in. being used. The maximum pressures within the cylinder are usually from 400 to 600 lb. per sq. in. These engines have mechanisms that are somewhat less complicated than in Diesels. They are less costly, give substantially as good fuel economy, and usually require less-skilled attendants than with the latter, hence are serious competitors in sizes, say, below 150 hp.

157. Diesel Engines.—(a) Of all types of commercial internal combustion engines now built, the Diesel usually gives the best performance, and hence it is quite commonly used in land and marine applications, especially in the larger sizes. It may follow either the two- or four-stroke Diesel cycle, the ideal form of which was discussed in Sects. **130** and **131.** It compresses the charge of air to about 500 lb. per sq. in. (practice usually ranges from 450 to 550) which results in a temperature sufficiently high to ignite the fuel when it is injected in the form of a fine spray. Until comparatively recently an air blast has been used to introduce this oil. Now, some engines use other means.

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[7] See R. Matthew's "Atomization Obtained in Fuel Nozzles of Solid-Injection Oil Engines," in *Power,* Oct. 13, 1925. Also "Spray Penetration with a Simple Fuel-Injection Nozzle," Technical Note No. 238, National Advisory Committee for Aeronautics.

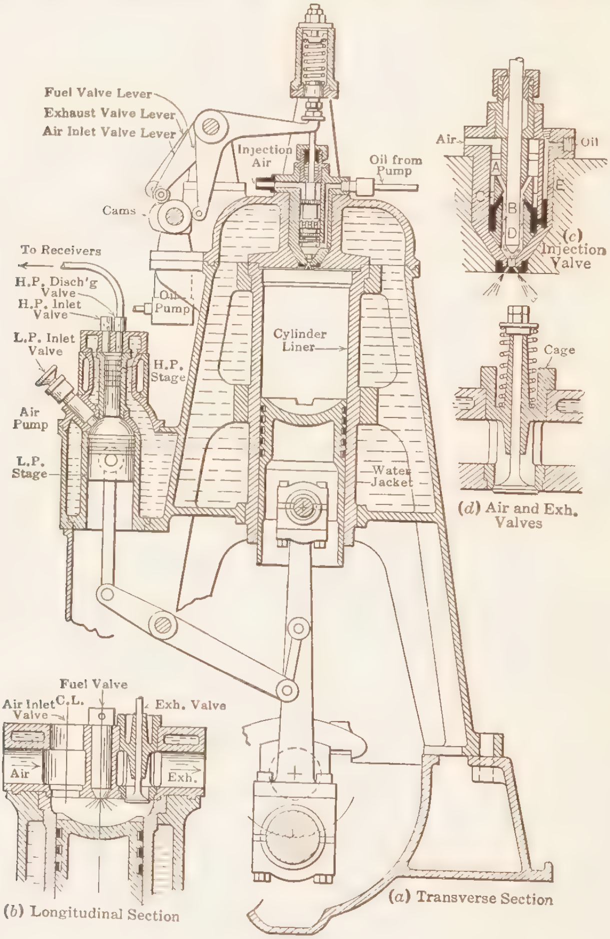

(a) Transverse Section
(b) Longitudinal Section
(c) Injection Valve
(d) Air and Exh. Valves

FIG. 107.—Small Diesel Engine with Air Injection.

(b) When **air-injection** of the fuel is used the pressure of the air for the blast must be very much higher than that at the end of compression so as to propel the fuel with such high velocity that it is "atomized" or " pulverized " and is distributed through the main mass of compressed air in the cylinder. As the injection air is relatively very cold and encases the atomized fuel more or less perfectly, the compression of the main air must be carried to a higher degree than would otherwise be necessary to cause ignition. The injection pressure is varied to suit the load and ranges from 700 lb. per sq. in. for light loads to as high as 1200 for maximum demands. This auxiliary air is generally compressed in a two- or three-stage compressor, which is usually driven directly by some part of the main engine and uses from 7 to 15 per cent of the power developed.

(c) The elements of a power plant having an air-injection Diesel engine were illustrated in Fig. 11 and briefly described in Sect. **31**. Fig. 107 shows diagrammatically the principal parts of such an engine which is of small size and uses the four-stroke cycle. In this case the water-jacketed, two-stage air compressor receives its motion from the main connecting rod. Usually, however, especially if the engine is a large one, the compressor is driven by a crank on the main shaft, as in Fig. 108. The fuel injection nozzle is quite complicated and is relatively small in diameter. The one in views (*a*), (*b*) and (*c*) of Fig. 107 is shown disproportionately large for clearness. The type of nozzle illustrated and others will be discussed later, in Sect. **162**(**b**).

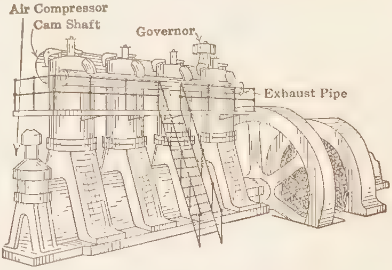


FIG. 108.—Four-cylinder Diesel.

The injection, air inlet, and exhaust valves shown in the longitudinal section (*b*) are actuated by cams on the half-time lay shaft illustrated in the main figure (*a*). In addition to these valves, one or more cylinders of an engine are also provided with "starting valves," which can be used to supply air under pressure to those cylinders for starting the engine.

(d) In an attempt to save the power used by the injection-air compressor, and to simplify and reduce the cost of the engine, many schemes for using what has been variously termed the *mechanical injection*, *airless injection*, or **solid injection,** of the fuel have been developed. In these, at the proper time, oil is supplied, by one or more pumps, to the working cylinders under a pressure of 4000 lb. per sq. in. or more, and is forced through specially designed nozzles that produce the necessary fineness of spray.

(e) In order to reduce the weight, size and cost for a given power, Diesel engines may be made to follow the **two-stroke cycle**; and many of the recent machines of this kind approximate the fuel economies obtained with the four-stroke cycle quite satisfactorily, especially in the larger sizes. Fig. 109 illustrates one arrangement of such an engine, the water jackets not being shown. This has exhaust ports which are uncovered by the piston when at the end of its power stroke, and in the cylinder head has inlet valves *A*, *A* (sometimes as many as four are used) through which air under a slight pressure is admitted from an air pump *P*, to scavenge the cylinder while the exhaust ports are open, and also to provide a charge of air for the compression process, which begins as soon as the exhaust ports have been covered by the piston on its up stroke. With this "*valve scavenging*" arrangement, the cylinder head is complicated and somewhat difficult to construct with adequate provision for cooling. In order to avoid these undesirable features and to eliminate the necessary valve gear, some engines use "*port scavenging*," in which, for the valves *A*, *A*, are substituted *air ports* which the piston uncovers when at the bottom of its stroke, as shown in Fig. 109(*b*). In the arrangement illustrated, valve *C* is kept closed while the piston descends, and the top of the lower port *D* is below that of the exhaust port *E* so that it will not be uncovered until after the pressure within the cylinder has fallen below that of the scavenging air. The piston top is curved in order to deflect the entering air so that it will clear the cylinder as completely as possible. On the up stroke the piston closes this lower air port while the exhaust is still open, hence the air entrapped would be at atmospheric pressure at the beginning of compression, if no other air port than *D* were used. However by opening valve *C* at the proper time, the compression can be made to begin with a pressure equal to that of the scavenging air, the resultant "surcharging" making it possible to obtain an increased mean effective pressure. There are various other schemes possible for double port, single port, and valve scavenging; also the latter two may be combined.

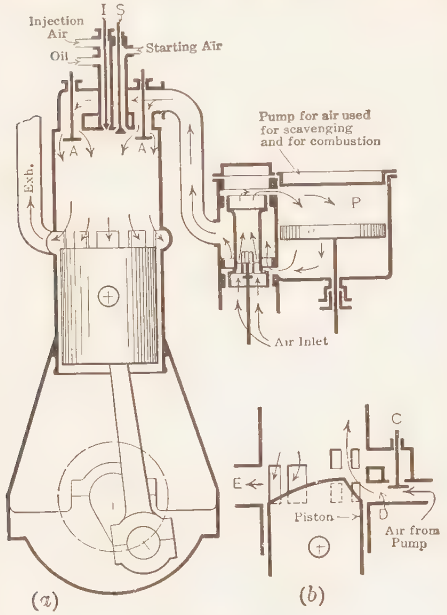


FIG. 109.

(f) The so called "**scavenging" pump,** or compressor, or blower, required with a two-stroke cycle engine, usually is designed to develop an air pressure of from 2 to 7 lb. per sq. in., gage. It may be of the older, *direct coupled*, reciprocating type, in which case it is quite large; or it may be an *independent unit*, or possibly a geared unit, which operates at high speed and hence is relatively small. In some cases the *crank end of the cylinder* is used for this compression, the piston having a rod which, passing through a stuffing box in a cylinder head, is attached to a crosshead to which the connecting rod is coupled. By having sufficient clearance volume and receiver space any desired blast pressure may be maintained. Sometimes the crosshead and piston rod are also used on four-stroke cycle engines, but, as with the type under discussion, this results in a higher engine and a more costly one than when the usual trunk type of piston is used. Another arrangement for furnishing scavenging air has a *stepped piston*, the shoulder being used for the compression.

(g) To reduce still further the size, weight and cost of the engine, it may be made **double acting.** The more usual arrangements have piston rods and crossheads, the piston and rod being made hollow and being cooled by water, or oil, which is circulated through them. In such engines the bottom head is very complicated and difficult to design. In another arrangement the upper and lower ends of a water-cooled piston reciprocate in the two ends of a *double cylinder.* At its middle the piston has two connecting rod pins which are located diametrically opposite each other, and the two sections of the cylinder are spaced far enough apart to provide for the motion of these parts.

In order to eliminate entirely the cylinder heads and the inlet and exhaust valves, some engines are made with **opposed pistons,** such as were first used by Oechelhauser for two-stroke-cycle gas engines, and later applied to Diesels by Junkers and others. The Junkers arrangement is shown in Fig. 110 with air and exhaust ports respectively at *A* and *E*. In multi-cylinder engines of this type the lower piston of one cylinder may be cross-tied to the upper one of the next, thus eliminating the extra pairs of connecting rods that otherwise would be necessary.

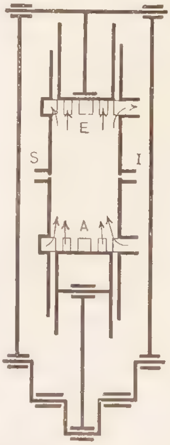


FIG. 110.—Diesel Engine with Opposed Pistons.

(h) The intense heat to which the pistons in Diesel engines are subjected limits their diameter to about 20″ if not specially cooled. Engines with water-cooled pistons have been built to develop

1000 hp. per working end of cylinder, and such pistons as large as 34.45 in. in diameter have been used in double acting two-stroke cycle engines.[8]

In developing the Diesel and semi Diesel engines for automotive applications there are many serious difficulties to be overcome, among which are the relatively large weight per hp., the metering of the minute quantities of oil used per stroke, the proper atomizing, distributing and burning in the almost infinitesimal time available, and the obtaining of flexible and definite control.[9]

While the foregoing discussion has been with respect to vertical Diesel engines, it applies also to horizontal ones, and much of it likewise pertains to engines using other cycles. There are a great many different types of engine parts and many arrangements other than these considered here. For further study see references at end of the next chapter.

158. Classifications of Engines.—The general type of an engine is described by stating the various classes into which it falls. It may be classified in the following ways: (1) As to cycle (Otto, Diesel, etc.); (2) as to strokes per cycle (two or four strokes); (3) as to the kind of fuel used; (4) as to use,—(*a*) stationary, (*b*) portable, (*c*) automotive (automobile, marine, aircraft); (5) as to position of cylinder axis,—(*a*) vertical, (*b*) horizontal, (*c*) inclined, (*d*) "V," (*e*) star; (6) as to action,—single- or double-acting; (7) as to cylinder arrangement,—(*a*) twin, (*b*) multi-cylinder, (*c*) tandem, (*d*) opposed; (8) as to governing, — hit-and-miss, throttling, quality, etc.; and in many other groupings. Thus an engine may be described, for example, as a marine, vertical, single-acting, two-stroke cycle, six cylinder, solid injection, low speed, Diesel engine.

159. Methods of Governing Otto Engines.—(a) Stationary engines are generally mechanically regulated to maintain *approximately constant speed* of rotation. Automobile and marine engines are commonly hand-governed, although they are sometimes fitted with a limit governor to prevent overspeeding, or "*racing*."

(b) In order to govern or regulate an engine, the i.hp. must be varied to suit the demand. There are *three* available methods of doing this: (1) The amount of energy made available per cycle may remain constant, but the number of cycles per unit of time may be changed; (2) the number of cycles may remain constant and the amount of energy made available per cycle may be varied; and (3) a combination of the two preceding may be used.

[8] "Recent Diesel Engine Developments," by P. L. Scott, London Soc. of Automotive Engineers, Oct., 1925. Recent Oil Engine Developements, Diesel Engine Users Association, Apr. 17, 1925, or Mechanical Engineering, Oct., 1925.

[9] See Diesel Automotive Engines, P. M. Heldt, Automotive Industries, June, 1926.

(c) In general, there are four different ways of applying these methods. They are called: (1) hit-and-miss governing, (2) quantity governing, (3) quality governing, and (4) combination systems. These are each considered in detail in the following paragraphs.

(d) In **hit-and-miss governing** the number of working cycles per unit of time is varied so as to adjust the average i.hp. to the demand for power. With this system, some part of the mechanism for opening the inlet valve is under the control of the governor, so that when a "*working cycle*" is to occur it *hits* another part and opens the valve, but when the cycle is to be omitted it *misses* engagement and the valve remains closed. When a "miss" occurs, not only does the inlet valve remain closed, but the exhaust valve is usually held open, so that, during the strokes corresponding to the ordinary cycle, the piston pumps exhaust gas into and out of the exhaust pipe without waste of energy, except for the slight friction and heat loss.

In some engines, when the working cycle is to be omitted, a fuel valve, which is separate, is held closed while the inlet and exhaust valves act as usual; thus the piston draws in a charge of pure air, which it compresses, expands, and exhausts. This method is generally considered less satisfactory than the former, because of the cooling effect on the cylinder walls.

With hit-and-miss governing all working cycles are supposedly exactly alike, and are equal to the maximum for the particular engine. As all types of internal-combustion engines using the Otto cycle show greatest thermal efficiency when developing normal cycles of about maximum power, this method of governing has the theoretical advantage of giving *high thermal efficiencies at all loads.* The cycles actually produced, however, are not all alike, because of irregular cooling and heating effects, the varying mixtures resulting from intermittent operation, etc. The variations become more marked with increase of the number of "misses," and the method therefore gives lower efficiencies at light loads than would be expected. In general, however, it is the most economical method of governing yet devised. As considerable intervals of time may intervene between "working" cycles, a very heavy flywheel is needed on engines governed by this method.

Hit-and-miss governing is very satisfactory for engines in which close speed regulation is not necessary, and is commonly used on the smaller sizes, say up to 25 or 50 hp. Where close regulation is required, it is never used.

(e) In **quantity governing,** the number of cycles and the proportions of the mixture are maintained constant, but the *amount* of mixture admitted per cycle is varied to suit the power demand. This is generally done in one of two ways,—by "cut-off governing" or by "throttling governing."

In **cut-off governing,** after the amount of mixture necessary to produce the required power has been taken in, the inlet valve is closed, and the charge expanded as the out stroke, or suction stroke, continues. The cycle is then completed as usual, producing under low load a diagram like that of Fig. 111, in which the lower loop is exaggerated for clearness.

In **throttling governing,** except at the maximum load, the charge is throttled during the entire suction stroke to reduce the amount of mixture entering the cylinder. This gives a diagram like Fig. 112, in which the lower loop is again exaggerated.

In both of these methods of governing, the reduction in quantity of mixture with decrease in load is accompanied by a *lowering of the compression* curve, although the compression ratio, and therefore the cycle efficiency, is not changed. If not carried too far, this is desirable from a mechanical standpoint, as it tends to produce a more uniform turning effort, and thus reduces the necessary weight of flywheel.

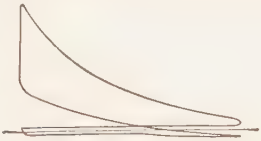


FIG. 111.—Cut-off Governing.

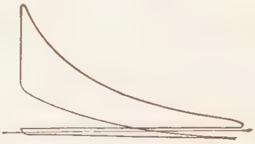


FIG. 112.—Throttling Governing.

Of the two methods the cut-off is the better for large stationary engines because it gives a smaller lower loop and less lost work. It also has the advantage that the governor action is delayed to the latest possible instant in the cycle, and hence each working cycle more nearly meets the power demand. For automotive types the throttling method is far superior because of its extreme simplicity.

(f) In **quality governing** the number of cycles and quantity of material per cycle are maintained constant, but the proportion of fuel to air, or quality of the mixture, is varied, so that the power developed in the cylinder just meets the power demand.

Since the same volume of mixture is drawn in during each cycle and is compressed to the same fuel volume, the "air standard" efficiency is constant at all loads, but the real mixture standard gives cycle efficiencies that increase slightly as the values of γ become larger, which is generally the case for the leaner mixtures. Furthermore each fuel has an air-to-gas ratio that gives the best engine efficiency. Since the brake engine efficiency is affected appreciably more by the mixture ratio than is the cycle efficiency, it follows that this method of regulation gives maximum brake thermal efficiency only at one particular load. With some fuels it is exceedingly difficult to obtain satisfactory ignition of the very "weak" mixtures introduced at low loads, and such mixtures also burn very slowly, the combustion continuing in extreme cases throughout the entire expansion stroke.

A group of indicator diagrams from a quality-governed engine is given in Fig. 113. The slow burning of the weak charges is shown by the gradual tilting of the combustion line as the load decreases.

The *constant compression pressure* has an undesirable effect on the crank effort (see (e) of this section), as the m.e.p. of the compression line does not change with the m.e.p. of the expansion line.

(g) **Combined systems** are sometimes used in an effort to obtain the advantages, with as few as possible of the disadvantages, of the different methods previously described. Thus, hit-and-miss governing may be used at low loads and quality governing at the higher loads which call for sufficient gas to make a readily ignitable mixture. Or, quality governing may be used at the higher loads, gradually merging into quantity governing as the load decreases.

All these combinations tend to complicate the valve gear and call for more or less sensitive and intricate adjustments. They are, therefore, commercially handicapped, though otherwise desirable.

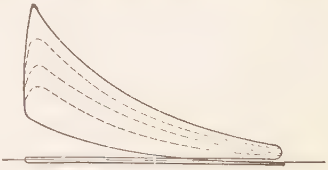


Fig. 113.—Quality Governing.

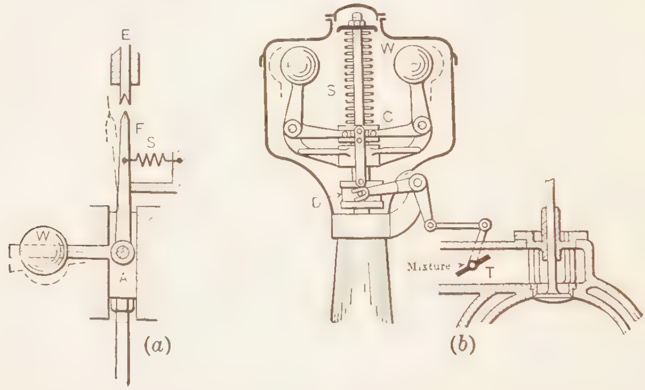
(h) As the form and area of the diagram may be changed by *altering the time of ignition*, this might be used for governing. It is actually used for that purpose to a certain extent in marine and auto engines. Since there is some best time of ignition for each mixture in each engine running at each speed, it is generally better to change the time of ignition to suit the conditions brought about by governing rather than govern by changing the time of ignition.

In some combination systems an ignition timing device under control of the governor has been incorporated.

(i) In all oil engines, whether of the low-compression, Diesel, or semi-Diesel types, and regardless of whether " solid injection " or " air injection " of the fuel takes place, the output of the engine is determined by the amount of fuel injected per cycle. Since the same amount of air is drawn in during each cycle, the ratio of air to fuel may vary enormously with these types of engines when operating under a wide range of loads, hence this case falls under the so-called "quality governing."

(j) Figure 114 (*a*) shows one form of automatic *governor for hit-and-miss* regulation. In it the block A is constantly reciprocated and the finger F moves E, which controls the fuel supply. If reciprocated too rapidly, the inertia of weight W will overcome the spring S, and F will not then engage with E, thus causing a miss.

Figure 114 (*b*) shows a *flyball type* of governor in which the weights W occupy the inner position when the engine is not running, and also when it is heavily loaded, and take the outer position when no power is being delivered. When these weights change position, due to a variation in the load, they move collars C and D, which, in turn, through suitable mechanism, shift the throttle valve, or some other device that controls the supply of fuel or of mixture, thus causing the power that is being developed to become equal to the demand.



Hit-and-Miss Governor. Flyball Type of Governor.

Fig. 114.

There are a great many other arrangements of governing devices used, but they cannot be considered here. The types of governors and their methods of functioning will be treated in a later chapter.

160. Gas Valves, Mixing Valves, etc.—(a) When gas is supplied under pressure to an engine a *gas valve* of some sort is necessary to shut off the gas supply during all but the suction stroke of the engine.

(b) This valve may be combined with the inlet valve of the engine, giving the arrangement shown diagrammatically in Fig. 115. The *air* and *gas cocks* shown are used for proportioning the mixture by hand, and the gas cock is also used as a permanent shut-off valve. Such an arrangement can be used with hit-and-miss or with quantity governing, but is obviously unsuited for quality governing because of the hand regulation of the proportions.

(c) The gas valve is more commonly a separate valve, although it may be carried loosely on the same stem as the inlet valve, as *a* in Fig. 116. When thus made separate from the inlet valve, it can be put under governor control, so that any kind of governing can be adopted, at the option of the designer. In all cases it is common practice to supply gas and air cocks or their equivalent so that the proportions of the mixture can be roughly regulated by hand and so that the gas can be permanently shut off from the engine.

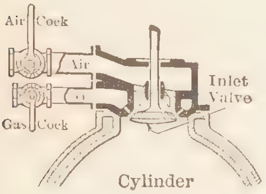


FIG. 115.

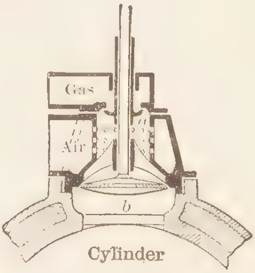


FIG. 116.

In Fig. 116 the small gas valve *a* is guided by the sleeve sliding on the stem of the inlet valve *b*. It is operated by separate linkage under governor control, so that the time, or extent, of its opening can be varied to suit the load. In operation, the inlet valve opens first, allowing fresh air to enter the cylinder and blow away hot burned gases. The gas valve *a* then opens, admitting gas, which, traveling downward, is thoroughly mixed with the air as it issues from the small holes shown. The valve *a* closes before the inlet valve *b*, so that the mixing chamber becomes filled with pure air before being shut off from the cylinder.

Such a device is commonly known as a *combined mixing and inlet valve*, although the gas valve is occasionally designated as a mixing valve or a proportioning valve.

(d) The elements of another type of mixing valve are shown in Fig. 117. The inner cylinder is supposed to be under governor control, so that it can be rotated more or less as the load varies, thus changing the effective openings of the gas and air ports to suit the demand for power. By properly proportioning the gas and air ports, their areas may be made to change at the same rate under the action of the governor, thus giving throttling regulation; or the areas may be made to change differentially, giving quality governing, or mixed quality and quantity regulation.

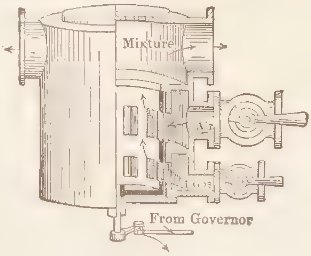


FIG. 117.

(e) Experience has shown that proportioning valves of the type shown in Fig. 117, and others using sliding surfaces, are perfectly satisfactory when used with such fuels as natural gas and illuminating gas. Producer gas and blast-furnace gases, however, carry impurities which quickly foul such sliding surfaces and impair the action of the valve. For such gases, mixing and proportioning valves made without sliding surfaces, such as that shown in Fig. 116, must be used. Even the valve shown in this figure might give trouble because of deposits on the stem of the main valve, and a design eliminating this possibility would probably give better results.

161. Carburetion.—(a) In Sect. **30,** in connection with Fig. 10, was shown the use of a carburetor and intake manifold on an automobile engine, and in Fig. 103 was illustrated a carburetor, which was briefly described in Sect. **152.** The carburetor, which is used with the more volatile liquid fuels, is a device that is intended to give the correct mixture of air and fuel at all speeds and loads on the engine. With multi-cylinder engines it is essential that the carburetor shall "*atomize*" the fuel and thoroughly mix it with the air passing through this device,—because, if the resultant mixture is not uniform by the time it reaches a branch of the manifold leading to some particular cylinder, then that cylinder will receive a different charge, both as to quality and quantity, from that going to the others. This difficulty is increased when the multi-cylinder engine operates over wide ranges of speeds and loads, such as occur in many types of automotive engines. In such case there is a complex carburetor problem as compared with the single-cylinder constant-speed motor, such as is used in stationary practice and in small motor boats.

(b) In connection with the simple form of carburetor illustrated in Fig. 103 it was shown that by means of a suitable chamber with a needle valve controlled by a float, the fuel can be maintained at constant level near the top of the nozzle. As the engine is turned over, the air will be sucked, by the pistons, through the contraction of the "Venturi tube" that surrounds the nozzle, and at the narrowest section, or "throat," the velocity will be highest and the pressure will be appreciably less than atmospheric. This difference in the pressure of the air causes the liquid fuel to flow from the nozzle, and it might be thought that if the size of nozzle be adjusted to the size of Venturi tube, that correct mixtures would be obtained at all times. Unfortunately such is not the case as it has been found by experiment that as the velocity of the air through the Venturi becomes greater the flow of fuel increases at a faster rate than does the air. Hence if the nozzle be adjusted to give correct proportions at any specified velocity of air past the Venturi, the mixture will be too "rich" at higher air velocities and too "lean" at the lower ones. It is this fact regarding the mixture ratio of air to fuel in the simple jet carburetor that has given rise to many thousands of inventions, each one for the purpose of offering some relatively simple scheme to give nearly a uniform mixture at all engine loads and speeds.

(c) To compensate for the tendency of the mixture to change with variation in power outputs it is apparent that (1) additional air may be admitted above the Venturi tube for large power outputs, (2) extra fuel may be supplied at the lower loads, (3) the throat area may be made to vary, or (4) a combination of these may be used. The first scheme is generally carried out by some sort of auxiliary air valve, as shown in Fig. 103. The second sometimes involves the use of an additional fuel jet, called the "compensating jet," which has constant flow maintained through it by gravity regardless of the "suction" in the Venturi tube. Fig. 118 shows the essentials of one type of double-jet carburetor. When the engine is not running, the fuel stands at the same level in the main jet *M*, the compensating jet *C*, the well *W*, and the float chamber *F*. When starting the engine, the fuel is supplied from both jets and the result is an over-rich mixture which is needed at such a time. The extra fuel in *C* and *W* will soon be used up completely. After that, the only fuel obtained from *C* will be the constant flow supplied by gravity through the small orifice *O*, which has atmospheric pressure maintained upon its delivery side by means of the well *W*, which also admits a small stream of air after the fuel has been drained from it. The "Zenith" carburetor is a well-known make that uses the double-jet principle.

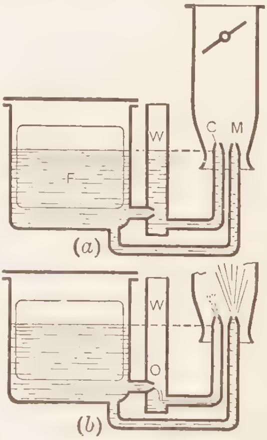


FIG. 118.

(d) *In starting* an Otto engine with liquid fuel, or in idling at very low speeds, it is desirable to have an over-rich mixture, and many devices are in use for this purpose. The most common one is some sort of a "choking" or throttling device, placed in the air intake, so that the pressure within the entire Venturi tube, and in intake manifold, is reduced very much below atmospheric pressure, thus forcing considerable liquid from the nozzle, and at the same time having only a small amount of air passing.

(e) The *throttle valve* above the carburetor may be opened or closed any desired amount by the operator or by a governor. The partial closing reduces the amount of the mixture going to the engine per cycle and consequently the speed will be reduced unless the load on the engine has also been decreased. It is the relationship between the power developed in the engine, the friction horsepower, and the external resistance that determines the speed of the engine. The throttle may be wide open and yet the engine may be turning over very slowly because the forces acting in opposition to the engine are so large, as in the case of an automobile climbing a steep grade on high gear. On the other hand, the throttle may be only partially open and the engine will run at a very high rotative speed if the only forces opposing its motion are those due its own friction. Irrespective of the throttle opening, it is the amount of air flowing past the Venturi tube that determines its velocity, and therefore the pressure, at the throat. Clearly then the greatest flow of air will occur at wide-open throttle with the engine running at full speed, a condition to give an over-rich mixture if no compensating device is used. For idling purposes, however, the flow of air is so reduced by the throttle that the mixture may be too lean even though the engine is running at fairly high speed.

(f) Since the carburetor is such a vital part of the engine it is essential that it shall be *reliable*. It is desirable that it be *simple* in its construction and have its parts *accessible* for cleaning and adjusting. It is not easy to design a simple mechanism that will give absolutely correct carburetion. Many modern types are very satisfactory, yet not complex, while many others are extremely simple, but have little else to recommend them.

(g) The carbureting devices so far considered are of the *float-feed jet type*. There are many other kinds. Thus, there are *bubbling carburetors*, in which some or all of the air is made to pass or bubble through the volatile liquid on its way to the engine. There are *surface carburetors* in which the volatile liquid is spread over screens, marbles, or anything else which will give a large wetted surface over which the air may be drawn. *Wick carburetors* have also been used. In these the liquid is drawn up into wicks by capillary action, and the air passing over the surface of the wicks vaporizes part of the exposed liquid. However, the type now most generally used is the *jet carburetor*.

(h) In the case of stationary engines a *carbureting valve*, as shown in Fig. 119, is sometimes used. The valve may be the inlet valve of the cylinder, but more often it is a separate valve through which air is admitted to the mixture pipe leading to the main inlet valve. A small hole is drilled in the seat of the carbureting valve in such a position that it is closed when the latter is seated. When the valve rises (automatically) to admit air to the engine, the liquid under slight pressure issues from the hole in a very small stream, which mixes with the air and is partly or wholly vaporized before the mixture enters the cylinder.

In Fig. 119 the supply tank receives from a small pump a surplus of fuel, the proper level being maintained by the overflow. This arrangement is one that is quite commonly used with stationary engines.

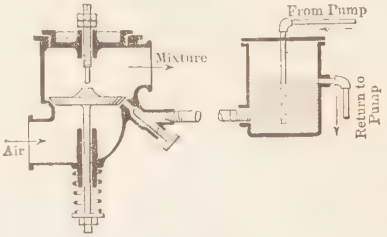
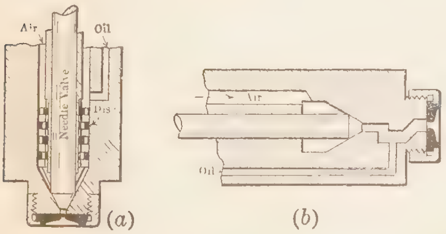


FIG. 119.

162. Treatment of Heavy Oils.—(**a**) The principal methods of producing combustible mixtures of the heavier oils and air were given briefly in connection with the discussion of Kerosene Engines, Low Compression Oil Engines, Semi-Diesels, and Diesels, in Sections **153** to **157,** inclusive. It was seen that the pulverization, vaporization, or other preparation of the fuel is accomplished usually by producing a fine spray, or by thermal means, or by a combination of these, perhaps with the addition of water vapor.

(**b**) The pulverization by means of high-pressure air can be satisfactorily accomplished only by using very carefully proportioned and highly developed types of nozzles, one of which was shown at (*c*) in Fig. 107. In this, before the time of injection, a measured quantity of oil is forced by the fuel pump into annular pocket *C* where it is subjected to the pressure of the injection air. When the needle valve *D* is raised, the rush of air through the annular passage *B* around it causes a decrease in pressure at that point with little effect at *C*. In consequence the oil is forced into the air blast which atomizes it and projects it through a distributing, or "flame," plate into the cylinder.



(*a*) (*b*)

FIG. 120.

In Fig. 120 (*a*) is shown another form of nozzle, in which the oil is broken up by being blown through the numerous small staggered holes in the rings surrounding the injection valve. Fig. 120 (*b*) shows an "open type" of nozzle in which the oil is deposited in an unsealed pocket before it is needed, and is blown therefrom at the proper time. This last nozzle is applicable only to horizontal engines. There are many other arrangements of air-injection nozzles.

(c) When "solid injection" is used, to produce the desired fineness of spray the nozzles generally either rely on very small discharge orifices, of the order of 0.015 in., or on a whirling action that is given to the oil by having helical or tangential passages leading to the discharge chamber which has a central orifice. With the former case oil pressures as high as 10,000 lb. per sq. in. have been used, but commonly they are only about one-half of this value. In the latter case still lower pressures may be used.

(d) Fuel pumps are usually eccentric-operated, single-acting, direct-driven ones with plungers which are merely small reciprocating rods. The amount of oil discharged may be automatically regulated by the governor, by changing the throw of the eccentric, by holding open the suction valve during part of the discharge stroke, by by-passing some of the oil discharged, or in other ways. On multi-cylinder engines there may be a separate pump for each cylinder, or a single pump may be used with some kind of a *distributor* to direct the flow at the proper time to the cylinder that is to function next.

163. Methods of Ignition.—(a) In the early development of gas engines the charge was ignited by opening communication at the proper time between the compression space of the engine and a small chamber containing an open flame. This method was complicated mechanically, and had so many objectionable features that it did not survive. Other methods used are: (*a*) hot-tube ignition; (*b*) spontaneous ignition by the heat of compression (assisted, or not assisted, by the action of a hot chamber, such as a vaporizer or hot bulb); (*c*) electric ignition.

(b) A simple type of hot-tube ignition is shown schematically in Fig. 121. The burner *b* produces a hot zone, which is at about red heat, and it can be located anywhere along the tube *a*, which at the end of the exhaust stroke is filled with burned gases. During the compression stroke these gases are compressed into the tube by the combustible mixture until the latter finally reaches the hot zone, when it is ignited. By moving the hot zone along the tube, the time in the compression stroke at which the mixture is ignited can be varied. By this method ignition is generally certain, but its timing is untrustworthy because of variations in the condition of the tube or of the mixture. Hence, despite its simplicity and lack of moving parts, hot-tube ignition is not now very widely used. "Timing valves" have been used to close the cylinder end of the tube and thus to control ignition, but few have survived. Hot-tube ignition involves a constant supply of gas to the burner, and this of course adds to the fuel consumption of the engine.

(c) Spontaneous ignition in many engines using liquid fuels heavier than gasoline is produced by the temperature attained during compression, as has already been seen, but this method of igniting has not proved applicable to the more volatile liquid fuels and to the gaseous fuels because of the difficulty of timing. The most satisfactory method of igniting these latter fuels is by an electric spark, and this method will be discussed in the next section.

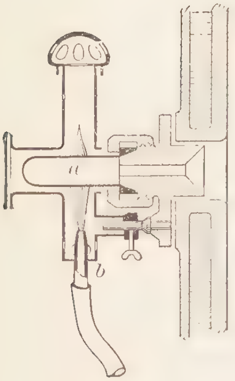


FIG. 121.

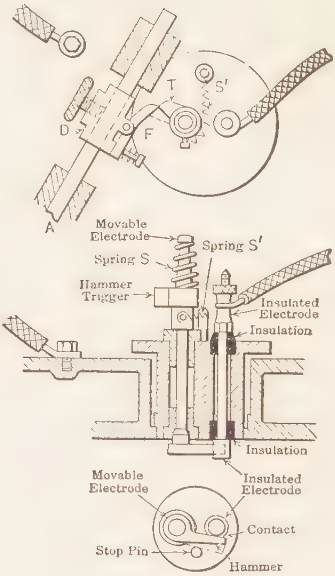


FIG. 122.

164. Electric Ignition.—(a) All electrical ignition systems in use, with few exceptions, fall under either "*make-and-break*" or "*jump-spark*" ignition. The less descriptive terms, "low-tension" ignition and "high-tension" ignition, are often used in place of these.

(b) In the **make-and-break ignition system** two "*electrodes*" are brought together within the combustion space to "*make*," or close, the circuit, and are separated suddenly to "*break*" the circuit and produce a spark.

One arrangement of such a system is shown in Fig. 122, the "*igniter block*," or "*plug*," entering the combustion space through the cylinder head or wall. The *stationary electrode*, or "insulated electrode," and the "*movable electrode*" are designated in the figure.

(c) The operation is as follows: A cam or eccentric pushes the rod *A* toward the igniter, and the finger *F* on the strike block *D*, engages the trigger *T* which, moving toward the right, rotates the movable electrode, by means of the spring *S*, until the hammer inside of the cylinder is brought into contact with the stationary electrode. The circuit is then made and current flows until the finger F travels past the edge of the trigger T, when the spring S' then pulls the hammer out of contact and the circuit is broken. A spark then results from the action of the induction coil, because the rapid change in the number of lines of force through the core causes sufficient self-induction to generate an electromotive force of such intensity as to bridge the gap between the separating electrodes. The timing of the spark is effected by moving the strike block d on the rod a.

(**d**) The wiring diagram is shown in Fig. 123. In this figure, B represents a battery, or other low-voltage generator, C an "induction" or "*intensifier coil,*" E the stationary electrode, which is insulated from the igniter block and engine frame, and S a stud or other convenient screw fastening on the engine. The movable electrode is in electrical contact with the igniter block and metal of the engine frame, as shown in Fig. 122.

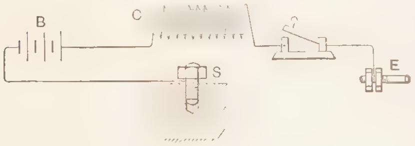


Fig. 123.

(**e**) The type of igniter just described is known as a "*hammer make-and-break igniter*" to distinguish it from another known as a "*wipe-spark*" or "*wipe make-and-break igniter,*" in which a movable electrode periodically wipes or slides across a stationary electrode. The wipe spark automatically cleans the contact surfaces within the cylinder, which is in a way advantageous, but it is not so extensively used as the hammer type. There are also make-and-break plugs that are operated by electromagnets, the magnet also acting as a "kick" or reaction coil.

(**f**) The make-and-break system has the advantages of being *electrically simple* and operating with *low e.m.f.*, so that short circuits are not so likely to occur as in the systems described in following sections.

(**g**) In the **jump-spark system,** or high tension, there are within the cylinder two *fixed terminals*, with a short intervening gap, across which a spark jumps when sufficient difference of potential has been developed. In its simplest form the apparatus has two circuits, as shown in Fig. 124, with heavy lines representing the "*low-tension circuit*" and the light lines the "*high-tension circuit.*"

In the figure, B is the source of electromotive force, T is a rotating "timer," C a "condenser," K a coil," and S a "spark plug," several examples of which are shown in Fig. 125.

(**h**) In *operation,* the primary circuit is closed by the timer T and then suddenly opened by it with the result that a spark jumps between the terminals of the plug. The action of the coil is as follows: When the primary circuit is closed by rotation of the timer, the magnetic field induces an electromotive force in the secondary circuit. This is not great enough, however, to cause a spark to pass between the plug terminals. But when the primary circuit is quickly broken, the sudden collapse of the magnetic field about the core of the coil induces for the instant in the secondary circuit a very high potential difference, which may be made sufficient to cause the passage of a spark, with resultant ignition.

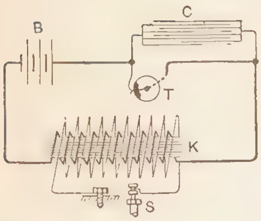


FIG. 124.

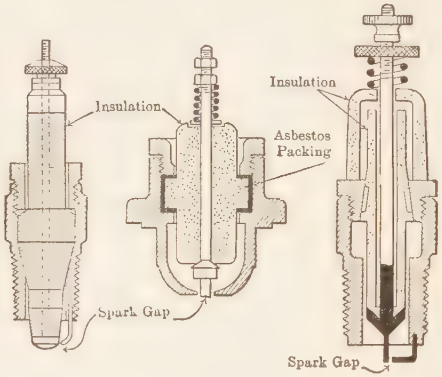


FIG. 125.

The *function of the condenser*, which bridges the timer in the primary circuit, is to prevent sparking at the contact points of that apparatus. Such sparking would cause rapid deterioration of the contact surfaces and is therefore undesirable.

(i) One type of jump-spark apparatus uses a "*trembler coil*" instead of the plain induction coil shown in Fig. 124. This apparatus is so arranged that the trembler forms part of the primary circuit, and is in such position that it is attracted to the core of the coil when this is magnetized, and thus breaks the primary circuit. This in turn demagnetizes the core, hence the trembler flies back and makes the circuit once more; thus the core is again magnetized and attracts the trembler, breaks the circuit, and so on, as long as the timer is in position to close the primary circuit. This intermittent making and breaking of the primary circuit causes a succession of sparks at the spark plug in the secondary circuit, which action is generally supposed to insure more certain ignition. The great advantage achieved is really quick action and accurate timing, though these are often counterbalanced by considerable trouble with the trembler which may call for almost constant adjustment.

(j) Both of these high-tension or jump-spark systems are *easily timed* by shifting the phase relation of timer, or commutator, and engine crank, and they are particularly *satisfactory for high speeds.* Recently there has been a tendency to adopt these systems for ordinary slow-speed stationary work; but as the spark does not seem to have the same igniting power as that of the make-and-break system, most applications have been limited to the more easily ignitable fuels like natural and illuminating gas and gasoline.

(k) The source of current may be dry cells, storage batteries, electric generators, or magnetos. *Magnetos* are really dynamos, in which the armature coils move in the field of a *permanent* magnet. They may be made to deliver either direct or alternating current, and the coils may be rapidly rotated or merely quickly oscillated across the lines of force. They may be used in either low-tension or high-tension systems with motion properly synchronized with the timer.

(l) Multi-cylinder engines could have separate ignition systems for each cylinder, but it is simpler to have one general system with a *distributor* to connect it to the various igniting devices at the proper times. Engines with cylinders above 40 inches in diameter usually have three spark plugs in each clearance space, in order that the combustion may proceed more nearly instantaneously throughout the entire charge.

(m) Automobiles almost invariably use the high-tension jump-spark system. Starting with (1) *the source of current,* there is (2) a *switch* for opening or closing the circuit, (3) an *induction coil,* for producing the high voltage when the primary circuit is interrupted, (4) an "*interrupter,*" which may be a magnetically operated "*magnetic vibrator,*" or a mechanically actuated "*breaker,*" (5) a *condenser* for preventing the contact points from deteriorating rapidly, (6) a *distributor* for directing the current to the proper spark plug, and (7) the *spark plug.* Duplex systems are commonly used in which there is an engine-driven generator and a storage battery. The generator furnishes current for ignition and for charging the battery. For ignition at low engine speeds and for driving the starting motor, the current is supplied by the battery. If a magneto is used it is often supplemented by a battery, which is used for emergencies, and also generally for starting.

165. Internal-Combustion Engine Valve Gear.—(a) The plain *slide valve,* so common in steam-engine practice, is never used in its simple form on internal-combustion engines for admission or exhaust because the high temperatures to which these valves are subjected make lubrication difficult and cause warping of the valve and seat, and the high pressures make it difficult to keep the valve on its seat to prevent leakage. When the fuel used contains sulphur, which is not an uncommon occurrence, the valve and seat are often quickly pitted and corroded.

(b) Some highly specialized forms of slide valves are, however, in use in "sleeve-valve" automobile engines, one example of which is shown semi-diagrammatically in Fig. 126. The two sleeves, reciprocating vertically under the action of eccentrics or cranks on a side shaft, act in conjunction with the cylinder head and external cylinder to control admission and exhaust by means of the ports shown. The advantages of this type are rapid and wide opening and quick closing of valves, long period of approximately maximum opening, and silent operation.

(c) With the exception of the cases cited above, the *poppet* or *mushroom valve* is in practically universal use for internal-combustion engines. It maintains its correct shape under changing temperatures more perfectly than other types; it requires a minimum of contact surface between valve and seat; it opens inward and is therefore forced to its seat by the high pressures in such engines; it requires no lubrication; and it and its seat are easily kept comparatively true by grinding.

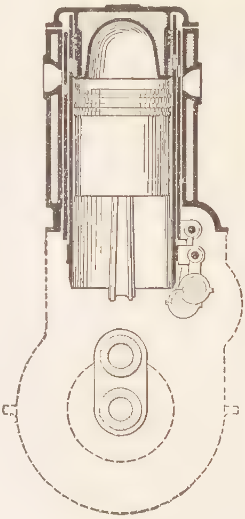


FIG. 126.

(d) In modern designs, inlet valves are practically never *water-cooled*, as the ingoing charge cools them sufficiently during each suction stroke. Exhaust valves, and their seats, on the other hand, are usually water-cooled when larger than five inches in diameter, and often in smaller sizes. This is deemed necessary not only because the very hot exhaust gases may have a detrimental effect on the valve, but also because the high temperature retained by the latter may cause pre-ignition in certain cases. In large engines the valve heads are usually of cast iron and the stems of steel. Automotive engines use cast-iron heads with steel stems welded to them, or various alloy steels, the head of the valve often being of different material from the stem.

(e) Two types of inlet valve are in use,—the *automatic valve* and the *positively actuated valve*. The automatic valve is held to its seat by a weak spring, and is raised by the difference between atmospheric and suction pressures during the suction stroke. The positively actuated valve is opened mechanically and generally is closed by spring pressure.

Automatic valves are uncertain in their action, opening only after a considerable pressure difference has been created, and then more or less slowly. After opening they do not remain wide open during the remainder of the suction stroke, but "chatter" more or less, thus materially decreasing the volumetric efficiency of the engine. For these reasons they are seldom used except in very small engines.

Positively actuated valves, on the other hand, can be made to open at the time desired, can be given an amount of opening approximately equal to that calculated for each piston position, and can be made to close very nearly at the right time.

(f) On the smaller engines the valves are usually driven by cams, which, especially in multi-cylinder engines, are mounted on side, or overhead "cam shafts" that are driven from the main shafts by spur, helical or chain gearing. On large horizontal engines the layshaft may have eccentrics which, through suitable rods and levers, operate cams at the valves, as was shown in Fig. 102. To overcome the inertia of the rapidly moving mechanism and keep the followers at all times in contact with the cam surfaces, it is necessary to use springs, which often must be of great strength.

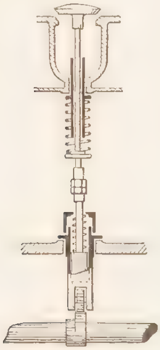


FIG. 127.

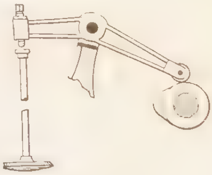


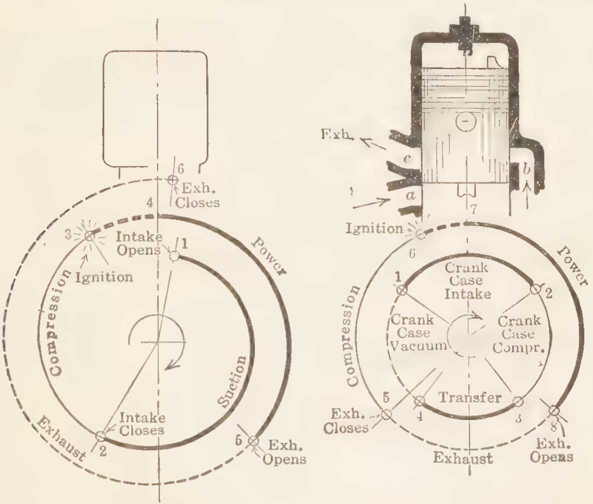
FIG. 128.

Valves may be operated by cams in direct contact with the valve stem (Fig. 127); or by contact with one end of a pivoted lever, the other end of which contacts with the cam (Fig. 128); or through eccentric-driven *rolling, rocking, or floating levers*, one arrangement of which is shown in Fig. 102.

(g) In the ideal Otto cycle the opening and closing of the valves and the ignition and combustion were assumed to occur instantaneously with the piston at the respective ends of its stroke. In actual engines it is necessary to have these events take place at other times, some before the crank reaches a dead center position and others afterward.

Diagrams which show the phase relationship between these events and the crank positions are called *Timing Diagrams.*

(h) Figure 129 shows illustrative Timing Diagrams for Otto four- and two-stroke cycle vertical engines. Since two revolutions of the crank are required for a four-stroke cycle, a 720° spiral is used to show the occurrence of events and the periods during which the various processes take place during such cycle. Referring to Fig. 129 (*a*) for this case, it is seen that the Intake Valve Opens (I. O.) when the crank is at 1, that the suction period then follows until the Intake Closure (I. C.) occurs at 2; that Compression then starts and continues until Ignition takes place at 3, at a crank position somewhat before the dead center 4; that the power period is from 4 to 5, when Exhaust Opens (E. O.); and that the exhaust period continues until Exhaust Closure (E. C.) occurs at 6, thus completing the cycle.



(*a*) Four-stroke Cycle (*b*) Two-stroke Cycle

FIG. 129.—Timing Diagrams for Four-Stroke, and Two-Stroke Otto Cycles.

(i) In Fig. 129 (*b*) is shown a Timing Diagram for a three-ported two-stroke cycle engine, such as was shown in Fig. 97. In this, the outer circle is for the space above the piston, and the smaller circle is for the under side. The combustible mixture flows into the crank case while the crank rotates from 1 to 2, the piston being near the top of its stroke and hence uncovering port *a*. With crank at 2 this port is closed and the confined mixture is compressed in the crank case until the crank reaches position 3, when the top of the piston uncovers the transfer port *b*. The mixture under slight pressure then flows into the cylinder, being guided by the deflection lip on the piston, and helps to expel the burnt gases, the exhaust port having already been uncovered when the crank was at 8. The exhaust period occurs while the crank rotates from 8 to 5, and the compression of the new charge, which begins at 5, completes the cycle. Between 4 and 1 on the upstroke the movement of the under side of the piston produces a decrease of pressure in the crank case, hence when port *a* is uncovered the mixture rushes in rapidly to fill this space.

(j) The time (with reference to crank and piston positions) as which valves open and close varies widely, depending on such factors at the location of the valve, the type of engine and its rotative speed. The exhaust valve universally opens early, generally when the piston is at about 0.9 stroke. It may close before the end of the return stroke, or on dead center, or it may remain open until after the suction stroke has started. The object of leaving it open after dead center has been passed is to take advantage of the inertia of the moving exhaust gases and thus get more perfect discharge. The inlet valve very commonly opens after the beginning of the suction stroke, though it is sometimes opened just before, or on dead center, in order to obtain a wider opening by the time suction actually starts. It is very generally closed after the end of the suction stroke in order to take advantage of the inertia of the moving column of gas, thus increasing the volumetric efficiency.

In general, the higher the speed of an engine the later may the valves close, and the greater may be the overlap of exhaust closure and inlet opening if the valves are widely separated.

To allow for the time element involved, combustion is started somewhat before dead center, both in the Otto and Diesel types. Handbooks on technical engineering and treatises on internal combustion engines give data on the timing commonly used with different types of engines.

CHAPTER XIV

INTERNAL COMBUSTION ENGINES

ACTUAL OPERATION, POWER AND PERFORMANCE

166. Real Indicator Diagrams.—(a) In the upper part of Fig. 130 is shown a water-jacketed cylinder with its head, inlet and exhaust valves, igniter, piston, and connecting rod. With such an engine operating on a **four-stroke Otto cycle** the indicator diagram would appear something like that shown at (*A*) in this figure. At (*B*) this is shown by 0123456, but somewhat distorted for clearness, and on it ideal air diagrams have been superposed. For the case in which the specific heat is considered constant at its cold value the amount of heat represented by the fuel used per cycle would cause the pressure rise *ab*, and the corresponding ideal "Cold Air Standard" cycle would be *abcda*. With variable specific heat during heat addition the ideal "Real Mixture Standard" cycle would be *a′b′c′da′*. From a comparison of these ideal cycles it is apparent that the mere change from constant to variable specific heat causes a big difference in the ideal diagrams, as was also shown by the numerical case analyzed in Sect. **129** (f).

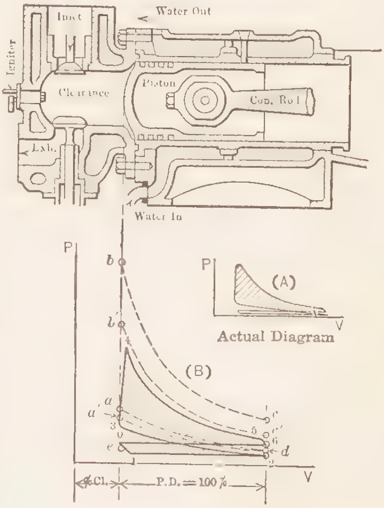


FIG. 130.

It still is necessary, however, to account for the appreciable difference that exists between the actual diagram and the ideal Real Mixture Standard. The reasons for this departure are due to the (1) chemical and physical properties of the real working substances; (2) thermal properties of the metallic parts of the engine; and (3) mechanical faults, such as leaking piston and valves. These items will now be considered in detail.

(b) Starting at *c'* at the end of the ideal expansion line, the *exhaust* valve would be opened to allow the charge of the preceding cycle to escape into the exhaust pipe. In the real case, however, this valve must start to open before the end of the stroke, say at *5*, which is usually at from 85 to 90 per cent of the out-stroke. This is necessary so that the valve may have time to open fully before the end of the stroke is reached, and because the gas in the cylinder, due to its inertia and fluid friction, takes an appreciable time to pass through the exhaust opening despite the fact that the gas pressure of from 15 to 35 lbs. or more above the atmosphere is available to accelerate it.

From *5* the expansion line drops rapidly to the end of the stroke, both because additional space is vacated by the piston as it continues outward and because of the exit of gas from the cylinder. By opening the exhaust valve early less area at the toe of the diagram is lost than if the opening is at the end of the stroke, and, as less hot gas remains in the cylinder, the tendency to overheat the metallic walls is reduced.

(c) The exhaust line *6–0* is from 1 to 3 lbs. per sq. in. higher than the ideal exhaust line *de*. As the area opened by the exhaust valve is limited by practical considerations, this pressure difference is necessary to cause the high average velocity of gas flow through the valve and pipe to the atmosphere and is needed in order to empty the cylinder in the short time available.

(d) At *0*, with the piston at the end of the stroke, the clearance is filled with products of combustion at a pressure slightly above atmospheric and at a temperature probably 700° to 900° F. As the piston starts on the "*suction stroke*," these gases expand to some pressure *1*, from 1 to 6 lbs. per sq. in. below that of the mixture supply (which is usually at atmospheric pressure), before the new charge begins to flow through the open inlet valve into the cylinder. This flow continues, as the piston moves out, until the end of the stroke is reached at *2*, when the cylinder is filled with a mixture of the new charge and the burnt gas previously left in the clearance. The "suction line," *1–2*, is only approximately horizontal.

Evidently with an exhaust pressure higher than the intake pressure, the piston must do a small amount of net work on the gas, as shown by the area of the "loop" *0–1–2–0*, and this decreases the power that the engine can deliver.

(e) As shown exaggerated in the figure, the *compression* line *2–3* is generally below *da'* (1) because compression begins at *2* with pressure below atmospheric, (2) because the process is not adiabatic,—for there is heat interchange between the gas and the walls of the piston, cylinder, and head,—and (3) because of leakage past piston and valves.

(f) At or near *3 ignition* occurs, and as it actually takes an appreciable time for the flame to spread throughout the mixture, and as the piston does not remain stationary at the end of the stroke during the complete process of combustion, the sloping combustion line *3 4* results, instead of the vertical line $a'b'$ of the ideal process. Combustion is seldom complete even when the highest pressure is reached, hence heat is still being generated when expansion starts.

The combustion line represents the most complicated process in the cycle, and is the most difficult to investigate, as the phenomena take place with comparative rapidity and vary with the mixture, method of ignition, the surface form of the combustion space, piston speed, and other factors.

In considering the slope of the combustion line, it is a case of balancing gains and losses. The piston movement reduces the maximum pressure and temperature, thus decreasing the heat lost to the cylinder walls, but this is offset more or less completely by the larger surfaces exposed while the temperature is high. The slope which will give the highest efficiency cannot be predicted, but usually an inclination which will bring the top of the combustion line at about 2 per cent of the stroke seems to give the best results. This form of the actual diagram also improves the mechanical operation of the engine because the pressure changes are less sudden and less intense.

The pressure does not rise as high as the ideal value b' calculated for the real mixture, presumably because (1) the initial pressure *3* is less than the ideal at a'; (2) the movement of the piston increases the volume during combustion; (3) the average specific heat of the actual mixture may be slightly higher than that assumed for it; (4) the surrounding metallic walls absorb some of the heat generated; (5) there may be a certain amount of dissociation at the higher temperatures; and (6) there may be leakage past the piston and through the valves.

(g) The chemical reactions accompanying combustion may result in products occupying less volume than the original mixture. For example, when two volumes of CO unite with one volume of O_2, but two volumes of CO_2 result when measured at the original temperature and pressure. In calculating the ideal cycle for the real mixture[1] the value of R for the mixture before combustion is taken as different from the value of R for the products of combustion, thus, the calculations involving these constants take care of any possible volume shrinkage due to combustion of the real mixture. The subject of combustion will be treated in a later chapter.

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[1] As was done in Sect. **129**(e).

(h) The *expansion* line is at first generally above an isentropic curve through *4* because of "*after burning*," or the continuation of combustion, which usually generates heat in excess of that absorbed by the walls. Later, as the motion of the piston continues, the relatively cooler cylinder walls are uncovered and they rapidly absorb heat from the gas, causing the expansion line to drop below this isentropic. The expansion line generally incloses slightly more area than the isentropic, unless the engine is of such proportions as to expose excessive wall area.

(i) During part of the compression, and all of the combustion and expansion, *heat is absorbed by the inclosing metallic walls* and is carried away by the water jacket, or air. This is a direct loss, but it is necessary in order to prevent overheating the metal.

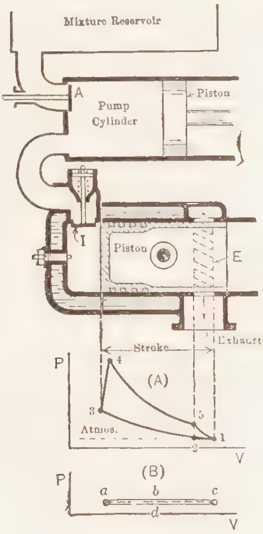

Fig. 131.

(j) Figure 131 illustrates diagrammatically a **two-stroke Otto cycle** engine with separate pump for supplying the combustible mixture to the working cylinder, the pump being driven by a crank on the engine shaft. At (*A*) and (*B*) in this figure are also shown respectively the indicator diagrams for the power cylinder and for the pump. The differences between the actual work diagram and the "Real Mixture Standard" cycle are quite similar to those occurring in the four-stroke cycle engine, and arise largely from the same causes. The work corresponding to the pump diagram is, of course, subtractive and reduces the power output of the engine. When the crank-case compression, or other kind, is used, a similar negative pump-diagram would be obtained for the gas undergoing the process. The pump does not actually operate in the ideal manner, and it must produce a gas pressure of from 0.5 to 7 lb. per sq. in. above atmospheric in order to fill the power cylinder in the short time available after the inlet valve opens. Energy is lost in overcoming the pump friction and the resistance to flow.

Because of its great velocity, the entering charge generally mixes more or less with the burnt gases, and some portion usually escapes through the exhaust ports before they are covered. Hence, instead of developing twice the power of a four-stroke cycle engine of the same size and r.p.m., the actual ratio is from 1.7 to 1.9, owing to losses due to the method of operation. In some two-stroke cycle engines the power cylinder is first scavenged by admitting air under pressure ahead of the mixture so that none of the fresh charge escapes with the exhaust. This, however, is done at the expense of greater complexity of mechanism and a larger expenditure of energy in pumping.

(k) Figure 132 (*a*) and (*b*), respectively, show indicator diagrams for an air-injection **Diesel** and for a solid-injection **dual-combustion** engine, for full and part loads. The departures from the ideal "Real Gas Standard" cycle are due principally to the same causes that have already been considered with the Otto cycle. In the case of the Diesel, the combustion does not occur strictly at constant pressure, and does not instantaneously cease when the fuel injection is cut off, but continues somewhat after expansion has started. This has some influence on the maximum temperature attained and on the character of the expansion line. Although it results in a reduced area of indicator diagram, it is easier to keep cylinder and piston properly cooled. With late injection, or with slow-burning fuels, the combustion line will show an appreciable drop from the horizontal, as at (*A*) in Fig. 132.

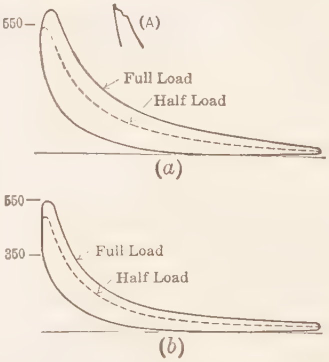

Fig. 132.—Diesel and Semi-Diesel Diagrams.

(l) The actual expansion and compression lines of internal combustion engines can be roughly *approximated* by curves of the polytropic type—i.e., with PV^n = const. The values of the exponent vary widely, however, depending on the kind of fuel, the type of engine, leakage, heat transfer to or from the cylinder walls, and on many other factors. For compression, n ranges from 1.22 to 1.38 for large engines, and even goes as high as 2.0 for small high-speed Diesels; and for expansion the values are from 1.10 to 1.7.[2]

167. Indicated Work and Power of the Four-stroke Cycle Engine.—(a) In the diagram shown in Fig. 133, with the "lower loop" *fghef* considerably exaggerated, the arrows indicate the directions in which the various lines are traced.

(b) If areas on a *PV*-diagram surrounded by lines generated in one direction (here clockwise) represent work done *upon* the piston, or positive work, then areas inclosed by lines of reverse direction (here counter-clockwise) indicate work done *by* the piston upon the working substance, or negative work.

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[2] See E. B. Norris, "Gas Engine Compressions," Lefax Data Sheet, 5–184; Clayton's "Analysis of Cylinder Performance," A. S. M. E. Trans., Vol. 34, or Univ. of Ill. Exp. Sta. Bul. 58. Also "Power," 1923, pages 250 and 555; Mechanical Engineering Handbook, by Marks.

Thus the work represented by the upper area *abcdea* is positive; the work corresponding to the "lower loop," *fghef*, is negative; and the *net useful work on the piston* would be represented by the *difference* between these areas.

(c) The exact interpretation of "**indicated power**" in the case of a four-stroke cycle gas engine is still unsettled. It is now somewhat common practice in this country to integrate both the upper and lower loops by tracing the complete diagram in the order of its generation, as shown by the arrows in Fig. 133, thus giving an area that represents the net work delivered to the piston during the four strokes. This method gives an i.h.p. that includes only the *friction* of the *mechanism* and not the fluid friction, because the latter is automatically subtracted by the method of integration.

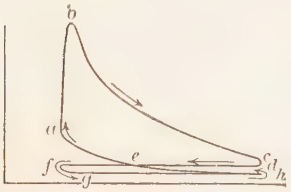

Fig. 133.

On the other hand, if the i.h.p. is determined from the area *abcha* that does not involve any fluid friction, the difference between such an i.h.p. and the b.h.p. will represent the horsepower lost by both fluid and machine friction.

(d) The **mechanical efficiency** is always the ratio d.hp. i.hp. As applied to gas engines it will include either the mechanical loss alone, or both kinds of friction loss, depending on how the i.hp. is computed.

168. Volumetric Efficiency. (a) The weight of the charge that is drawn into an Otto engine during the suction stroke is the chief factor in determining the amount of work done during the cycle. Hence, for a given piston displacement and a specified engine speed the indicated power of such an engine will depend chiefly on how well the given cylinder volume is filled with the fresh charge. The term *volumetric efficiency* is used to express the ratio of the volume of the actual weight of charge drawn in per cycle, measured at atmospheric temperature and pressure, to the piston displacement; or it may be defined on the basis of weights as follows:

Vol. Eff. = (wt. of charge[3] drawn in per cycle) ÷ (wt. of charge[3] to fill a volume equal to the piston displacement, when at atmospheric temperature and pressure).

(b) The factors that reduce the volumetric efficiency of an Otto engine are:

(1) *Throttling* of the incoming charge by reason of restricted open-

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[3] These charges are assumed to be of the same composition.

ings for the passage of the gas,—such as the intake valve, carburetor, intake pipe and manifold, and air cleaners; and also the partially closed throttle valve, in case of low speeds or part load.

(2) *Engine clearance*, which permits a certain weight of burnt gases to remain within the cylinder at the end of the exhaust stroke. These gases expand to suction pressure before the fresh charge can enter.

(3) *Increasing the temperature* of the entering charge by any means, such as passing through a hot inlet valve and port, or through an inlet manifold that has been purposely heated, say, by the exhaust gases, in order to assist in the distribution of equal charges to each cylinder.

(c) The volumetric efficiency of an Otto engine is very materially increased by using a **supercharger,** which is usually some simple form of centrifugal or rotary air compressor that increases the pressure of the air going to the carburetor. The device is especially useful on airplane engines operating at high altitudes.

(d) The actual heat interchanges during exhaust are problematical. The enormous rush of gas through the exhaust port takes away from the cylinder parts heat which is lost to the atmosphere, but there is a somewhat compensating gain due to increased volumetric efficiency resulting from contact of the new charge with cooler walls.

Quite remarkable success has been achieved by engines in which cold air is blown through the cylinder during part of the exhaust period. This operation cools the walls and tends to remove the burnt gases from the clearance space, and hence the charge drawn in is cooler and purer than in the ordinary type of engine. Such engines are known as "*scavenging*" or "*positive scavenging*" engines.

169. Methods of Determining the Size of Cylinder of an Engine.—(a) In Sect. **95** it was shown that, *per cylinder end, i.hp.* = $(p_mLan) \div 33{,}000$, in which the stroke L is in *feet*. Then since $a = \pi d^2/4$, d being the cylinder diameter in inches, it follows that the "piston displacement" in cubic inches per stroke of l *inches* is $\pi d^2 l/4$, and that

$$d^2 l = \frac{504{,}200 \times i.hp.}{p_m n}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (192a)$$

in which p_m is the m.e.p. in pounds per square inch, n is the number of *cycles* per cylinder end per minute, and the numerical constant is $4 \times 12 \times 33{,}000/\pi$. For given or assumed values of $i.hp.$, n, and p_m, the corresponding product d^2l can be determined. Then, assuming d, or l, or l/d, the effective dimensions of the cylinder become fixed. Here p_m is the mean pressure *within the cylinder* and is called the **indicated m.e.p.,** as it may be obtained from an indicator diagram. The delivered output of the engine thus found would, of course, be $b.hp. = i.hp. \times M.\ Eff.$

(**b**) Often it is difficult to obtain an indicator diagram from an engine, and this is especially the case when the rotative speeds are high; also, interest frequently lies primarily in the delivered output only, since the mechanical efficiencies of all engines in a class under consideration are substantially the same. In such case, the cylinder size for a desired $b.hp.$ and n may be found from

$$d^2l = \frac{504{,}200 \times b.hp.}{p_b n}, \qquad (192b)$$

where p_b is the "**brake m.e.p.**" For given engines, whose piston diameters and strokes are known, the values of p_b may be easily determined by obtaining by test the brake hps. when completing n cycles per minute, and substituting in Eq. (192*b*) in which p_b then becomes the only unknown. The relation between the brake and indicated m.e.ps. is the same as between the b.hps. and the i.hps., thus, $p_b = p_m \times M.\ Eff.$

The foregoing may be called the **m.e.p. methods of determining** the cylinder size.

(**c**) Values of the m.e.ps. depend on many things, some of which are the size and type of engine, character and amount of load, kind of fuel, mixture ratio, timing of ignition and valve events, completeness of scavenging, perfection of combustion, shape of clearance space, effectiveness of cooling, speed, and intake pressure as affected by throttling or by altitude of engine, whether stationary or airplane. In Table VIII are given a few illustrative values such as may be readily obtained under continuous operation. They may be exceeded in short special tests and under favorable conditions, and they may not be reached by poorly designed or improperly adjusted engines. This table also gives the values of the maximum pressures that occur within the cylinders of different types of engines.

(**d**) How the brake m.e.p. is affected by the greater relative cooling and lower M. Eff. in small sizes of *automobile engines* is indicated by the following empirical equation,

$$brake\ m.e\ p. = 130\left(1 - \frac{1.18}{d}\right) \text{lb. per square inch}, \qquad (193)$$

which gives values that may be reached in the best practice by such engines.[4]

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[4] Judge's "Automobile and Aircraft Engines," pp. 368.

Also, for the same kind of engines, when operating at a given speed, the effect of the ratio of air-to-gasoline on the m.e.p. and also on the thermal efficiency is shown in Fig. 134.[5] For such engines it is seen that, for a ratio which gives the highest efficiency, the brake m.e.p., or delivered power, is below the maximum possible; and, conversely, that the ratio giving greatest power or m.e.p. results in poor economy.

(e) The **mechanical efficiencies** of internal combustion engines range from 65 to 90 per cent or more, some typical values being given in Table VIII. The **piston speeds** ($2L \times$ r.p.m.) of stationary engines range from 500 ft. per minute with small sizes to 900 with large engines; and are from 700 to 1500 for automobiles, and from 1000 to 2400 for airplanes. The ratio **l/d** for automobile and airplane engines is from 1.1 to 1.5, and for stationary engines it is from 1.25 to 2, but usually 1.33 for large gas engines.

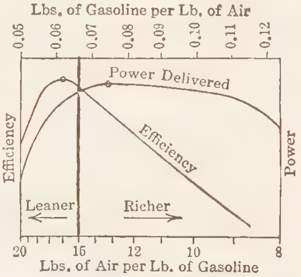

FIG. 134.

(f) The size of cylinder, or piston displacement per working stroke, can also be determined by the **charge-volume method.** Thus, remembering that the B.t.u. equivalent of the work done per minute is $42.42 \times$ b.hp., and that 1728 cu. in. = 1 cu. ft., then, since

Volume displaced per stroke per cylinder end =
(Volume[6] of fuel and air drawn in per suction stroke) ÷ Vol Eff.,

it follows that

$$d^2 l = \frac{4 \times 1728}{\pi \times n} \times \frac{42.42 \text{ b.hp.}_1}{B.T.Eff. \times H.H.V.} \times \frac{M \times X}{Volumetric\ Efficiency}, \qquad (194)$$

in which

b.hp._1 = the brake horsepower per cylinder end;
n = number of cycles per minute per cylinder end;
H.H.V. = the higher heating value of the fuel, in B.t.u., taken per pound for liquid fuels, and per cubic foot for gaseous fuels;
M = the volume[6] of theoretically correct mixture, in cubic feet, taken per pound for liquid fuels, or per cubic foot for gases;
= (1 + theoretical volume[6] of air per cubic foot of gas), or

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[5] See Berry, Trans. A. S. M. E., 1919, p. 333, or Jour. Soc. Automotive Engineers, Nov., 1919, p. 364. Also, Tice, S. A. E. Jour., Aug., 1920, p. 190; also, Report No. 49, National Advisory Committee for Aeronautics; and Berry and Kegerreis, Bulletin No. 5, Purdue Univ. Engineering Experiment Station.

[6] At atmospheric pressure and temperature.

M = (theoretical volume of air per pound of liquid fuel), the volume of the oil being negligible; and

X = excess air coefficient = actual air ÷ theoretical air.

A few illustrative values of B. T. Eff., H. H. V., X, and Theoretical Air are given in Table VIII. The volumetric efficiencies range from 60 to 93 per cent, depending on the type of engine and the various conditions of operation.

(**g**) The size of engines may also be determined roughly by using **empirical formulas.** Thus, for automobile engines having N cylinders, the rated b.hp., based on a brake-m.e.p. of 67.2 lbs. per square inch and a piston speed of 1000 ft. per minute, is

$$\text{Rated b.hp.} = d^2 \times N \div 2.5. \quad . \quad . \quad . \quad . \quad (195)$$

This is known as the A. L. A. M. or S. A. E. standard in this country, and as the R. A. C., or Treasury Rating in Great Britain. There are several other empirical formulas in use for such motors.[7] It should be particularly noted, however, that with other m.e.ps. and speeds, the power would, of course, be different from that given by such equations.

For stationary engines, equations of the form

$$d^2 ln = A \times dhp + B$$

may be used, the constants A and B having values depending on the type of engine, fuel used, etc.,[8] and B being a function of the friction horsepower.

(**h**) In Table VIII is given a small amount of data on the interrelated performances as to the power and the economy of internal combustion engines operating under full-load conditions and using some of the many different kinds of fuel. In many cases there is a lack of data available, and often the reports of tests are incomplete and are not all given on the same basis, or are otherwise unsatisfactory. The performances are dependent on so many factors, such as those mentioned in (**c**), that without complete information comparisons may be at fault. In view of these facts, this table should be considered as merely illustrative, or suggestive. The table is made up with the higher heating values, but many of the data available on the thermal economies are based on the lower heating values. The ratio of higher values to the lower ones is about 1.05 for heavy and light oils, and is 1.10 or more for gases, depending on the percentage of available hydrogen present, and these relations may be used in changing from one heat value to the other if more definite information is not available.

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[7] See Judge's "Automobile and Aircraft Engines."

[8] See Hirshfeld and Ulbricht's "Gas Power."

TABLE VIII.—PERFORMANCE DATA OF INTERNAL COMBUSTION ENGINES. (Approximate)

b = brake i = indicated r = compression ratio Volumes are taken under standard conditions (t = 62° F. and p = 14.7)

Kind of Engine	M.e.p. Lb./Sq. In.	Thermal Efficiency, Per Cent		Mech'l Eff.	r	Compr. Pres.	Max. Pres.	Theor. Air	Higher Heating Value		Excess Coef. $X = \frac{\text{Act. Air}}{\text{Theor. Air}}$	Consumption per B. Hp.-Hr.	
									Of Fuel	Of Mix.		Fuel	
		I. T. Eff.	B. T. Eff.	M. Eff.		$\frac{\text{Lb.}}{\text{Sq. In.}}$	$\frac{\text{Lb.}}{\text{Sq. In.}}$	Per Cu. Ft. of Gas	$\frac{\text{B.t.u.}}{\text{Cu. Ft.}}$	$\frac{\text{B.t.u.}}{\text{Cu. Ft.}}$		Cu. Ft.	B.t.u.
	1	2	3	4	5	6	7	8	9	10	11	12	13
Using Gaseous Fuel:													
Blast Furnace Gas	(*i*) 60±	27–32	20–27	75–85	7±	130–200	275–375	0.73	105	60	1.22	90–122	9500–13000
Producer Gas	(*i*) 65±	27–32	20–27	75–85	6.5±	140–170	275–350	1.00	150	74	1.25–1.3	63–86	9500–13000
Coke Oven Gas	(*i*) 80±	27–32	20–27	75–85	5±	120–130	275–350	5.40	600	95	1.25–1.67	15–21	9500–13000
Natural Gas	(*i*) 85±	27–32	20–27	75–85	4±	90–150	350–400	9.40	1000	95	1.3	9.5–13	9500–13000
								Per lb. of Fuel lb. cu. ft.	B.t.u. per lb.			Lbs.	
Using Liquid Fuel:													
Gasoline, Automobile	(*b*) 60–100	25–28	20–26	80–92	$3\frac{1}{2}$–$5\frac{1}{2}$	70–135	200–400	15 (189)	19400	103	1.10	.5–.65	9700–12600
Airplane	(*b*) 95–135	27–29	22–27	82–94	4–$6\frac{1}{4}$	100–135	300–400	15 (189)	19400	103	1.10	.48–.6	9300–11600
Kerosene, Tractor	(*b*) 60–65	19–18	14–16	75–90	$<3\frac{1}{2}$	40–70	150–200	15 (189)	19400	103	1.10	.8–.95	15500–18400
Oil, Low Compression	(*i*) 45–75	20–28	16–24	80–87	4±	50–160	200–400	.. (176)	19400	110	1.+	.55–.8	10700–15500
Semi-Diesel	(*i*) 22–65	26–36	21–32	80–90	8±	200–350	450–600	.. ....	19400	......	....	.41–.63	8000–12200
Diesel, 4-stroke Cycle	(*i*) 90–125	35–41	26–36	75–82	13±	450–550	450–600 (1000)	25 (316)	19400	62	1.67	.37–.5	7250–9700
2-stroke Cycle	(*b*) 60–70	31–39	22–33	70–84	13±	450–550	450–600	25 (316)	19400	62	1.67	.4–.6	7800–11600

170. Economy.—(a) The thermal economies of the several types of internal combustion engines, operating under full load, vary appreciably one from another, as may be seen from Table VIII. Furthermore, the same engine gives very different fuel consumptions, and, hence, thermal efficiencies, at various loads, as shown by Fig. 135.

(b) Some typical performance curves of a modern Diesel engine,[9] operating under various loads, are shown in Fig. 136. Additional curves representing both the indicated and brake thermal efficiency may be drawn from the mechanical efficiency and the fuel consumption per b.hp.-hr. Other types of engine exhibit curves of somewhat similar character.

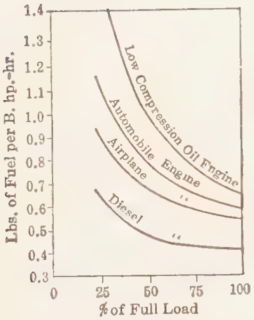

Fig. 135.—Variation of Fuel Consumption with Load.

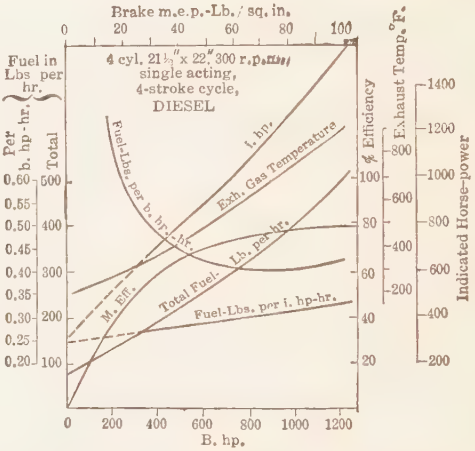

Fig. 136.—Performance Curves for a Diesel Engine.

(c) In Fig. 137 are shown the various efficiencies of a 1250 hp. marine Diesel engine. The curves were plotted from the data[10] obtained by a representative body of British engineers, known as the "Marine Oil Engine Trials Committee." These tests were very complete and made with great care. The engine being a marine one, the tests were carried out at various engine speeds corresponding to service conditions. It should be noted that the engine efficiencies as shown in Fig. 137 are based upon the ideal cycle efficiencies that were calculated by the authors of this text for the different mixture ratios found in the tests, with γ assumed to remain constant at 1.37. They realize that for the various cut-off ratios the average value of γ will change slightly, but the law governing this variation has not yet been determined, so far as they are aware. In the results given by the British engineers the engine efficiencies were based on a constant value of the ideal cycle efficiency of 52 per cent.

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[9] Engineering, June 6, 1924, p. 732.

[10] Reported in "Engineering," Nov. 28, 1924, p. 750.

It is important to note how the variation in engine speed and power affect the several efficiencies. At the rated load the ideal cycle efficiency is appreciably lower than it is at one-quarter load, but the mechanical efficiency more than offsets this difference so that the "engine efficiency" is very much better at the full load.

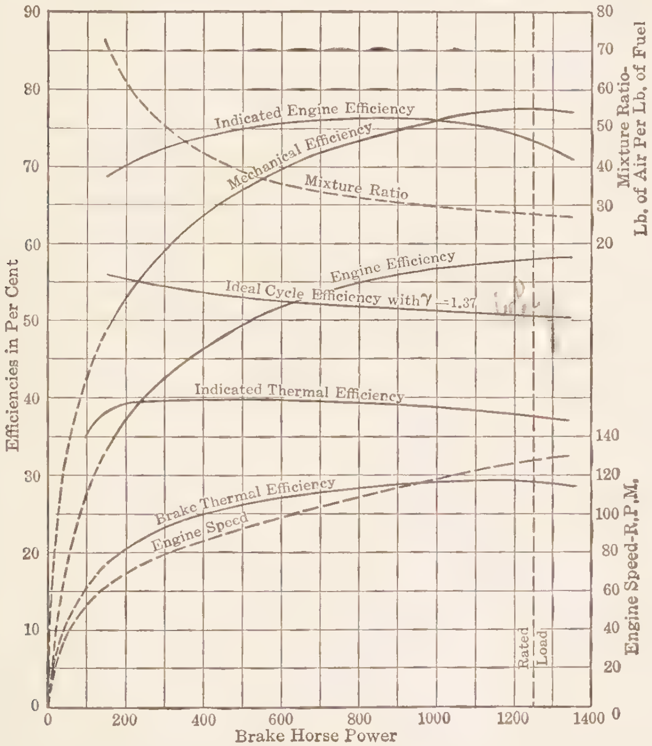

Fig. 137.—Tests of a Marine Diesel Engine, 6 Cylinders 24.4″×38.4″, Rated at 1250 hp. at 125 r.p.m.

(d) How the efficiencies vary with the speed of an engine is shown in Fig. 138, which presents the results of a test[11] on a Liberty Airplane engine of 12 cylinders 5″ × 7″.

These results are for a carburetor adjusted to give greatest power at each speed for an elevation approximately at sea level, and at the same time the attempt was made to use the least fuel possible without reducing the maximum power. For this figure the thermal efficiencies have been calculated on the basis of the higher heating value. whereas in the report they were on the lower one. These thermal efficiency curves represent test conditions in which a considerable waste of fuel takes place in order to secure the most power possible at each speed. In the heat balance an item of unaccounted for loss amounting to from 16 to 26 per cent appears to indicate that this waste was due to unburned fuel because of the rich mixture.

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[11] See Report No. 102, "Performance of a Liberty 12 Airplane Engine." National Advisory Committee for Aeronautics.

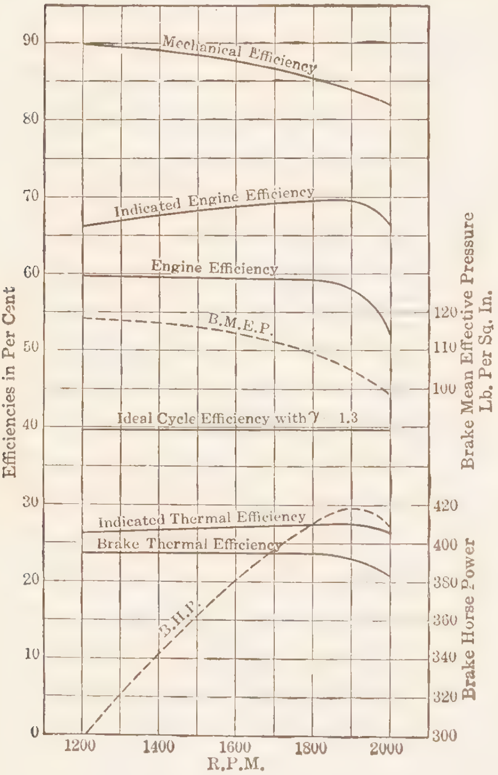

Fig. 138.—Test of 5″×7″ Twelve-cylinder Liberty Engine.

171. Heat Balance.—(**a**) Whenever a complete test is made on any kind of heat engine or plant it is possible to make a tabulation showing how all of the heat energy supplied has been accounted for. Such a table is often called the "heat balance" because it includes the item of "unaccounted for" energy in order to balance the account.

(**b**) Table IX represents a typical heat balance of a modern Diesel engine,[12] that has a high thermal efficiency. These same results are shown graphically in Plate V of the Appendix. One great advantage in studying such a heat balance is that it shows at a glance just where the large losses are, and how they compare with similar losses in other engines.

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[12] See "Further Experimental Work on Diesel Engines," by Engineer-Commander R. Beeman, "Engineering," May 9, 1924, p. 624.

TABLE IX

Heat Balance of Marine Diesel Engine

Mean Pressure = 142 lb. per sq. in., r.p.m. = 390, i.hp. = 442, Higher H.V. = 19,300, Lower H.V. = 18,100, B.t.u./lb.

Heat Supplied, in B.t.u. per Min.	Heat Accounted for, in B.t.u. per Min.	Percentage
By fuel. 38,480	Brake horsepower 14,255	36.7
By air. 330	Mechanical friction. 4,450	11.5
	Jacket and cover 4,850	12.5*
	Exhaust valve 545	1.4*
	Piston cooling oil. 1,050	2.7*
	Exhaust gases 12,225	31.5
	Radiation, etc. . . . . 1,435 (By difference)	3.7
Total. 38,810	Total. 38,810	100.0

* 16.6 per cent total loss to cooling media at engine.

REFERENCES

In addition to the references given in the preceding chapters on internal combustion engines, the following are also recommended from a very large list:

Bird—Oil Engines.
Clerk—The Gas, Petrol and Oil Engine.
Ewing—Thermodynamics for Engineers.
Goodenough—Principles of Thermodynamics.
Heldt—The Gasoline Automobile, Vols. I and IV.
Judge—Automobile and Aircraft Engines.
Marks—The Airplane Engine.
Morrison—Oil Engines.
Morrison—Diesel Engines.
Petrie—Elements of Internal Combustion Engineering.
Purday—Diesel Engine Design.
Ricardo—The Internal Combustion Engine; also, Engines of High Output.
Streeter—Internal Combustion Engines.
Wimperis—The Internal Combustion Engine.
Bulletin 139—Univ. of Ill. Eng'g Experiment Station Investigation of the Maximum Temperatures and Pressures Attainable in the Combustion of Gaseous Fuels—Goodenough and Felbeck.
Bulletin 150—Univ. of Ill. Eng. Exp. Sta. A. Thermodynamic Analysis of Gas Engine Tests—Rosencrans and Felbeck.
Bulletin 156—U. S. Bureau of Mines—The Diesel Engine—Haas.
Mechanical Engineers' Handbooks.

CHAPTER XV

PROPERTIES OF VAPORS

172. Vapors and Gases.—(a) In Sect. **13** it was stated that the engineer finds it convenient to speak of a vapor as one of the four phases of a substance. So important are certain vapors, such as steam, ammonia, and carbon dioxide, that he must consider them in great detail. He commonly designates the several stages of vapors as *saturated*, *superheated* and *supersaturated* ones—terms to be considered presently. In general, however, a **vapor** may be defined as the condition of a substance that is intermediate between the liquid and gaseous states and is such that it fills the entire container and thereby exerts a pressure, as does a gas, but it does not follow the laws of ideal gases even approximately.

(*b*) So important in heat-power engineering are certain vapors, and so complex are the relations between their properties, that much effort has been expended to determine them experimentally as accurately as possible and to give them in a form convenient for use. Very elaborate tables of these properties, known as **Vapor Tables,** have been prepared for such substances as steam, ammonia and carbon dioxide. But such tables cannot be used properly by anyone until he understands fully the technical meanings of all the terms used. These definitions will now be given.

173. Formation of Vapor at Constant Pressure.—(a) When a liquid is heated under *constant pressure*[1] its temperature will first rise until it reaches a certain *value which is dependent upon the pressure under which it exists*, after which further addition of heat will cause some of the material to *change physical state at the constant temperature* already fixed by this pressure. The amount of material that has changed state will increase as this further addition of heat progresses, and if sufficient heat is added *all* of the liquid present will thus change its phase into what is called a **Vapor.**

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[1] This pressure is assumed to be less than the *critical pressure*, a term to be defined later in this chapter.

(b) Consider a vertical cylinder fitted with a frictionless, loaded piston, and provided at its bottom with a valve, as shown in Fig. 139. The combined weights of the piston and its load, together with the atmospheric pressure produce a constant force acting downward on the piston. Now, suppose a *pound of liquid at 32 deg. fahr., and volume V_b cu. ft. be forced into the cylinder* below the piston and let P_b lb. per square foot be the pressure that is just sufficient to accomplish this. The *work done* during this process is therefore P_bV_b ft-lb., or AP_bV_b B.t.u.

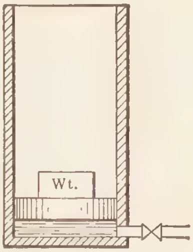

Fig. 139.

(c) The energy expended thus in forcing the liquid into this cylinder may be represented only on the PV-diagram and is shown in Fig. 140(a) by the rectangle $abio$. At low pressures this energy is very small in comparison with the other energy quantities involved with vapors; however, at high pressures it is large enough to demand consideration. Thus, if the liquid be water at 32 deg., its specific volume is very close to .016 cu. ft. per pound; hence, for a pressure of 1000 lb. per square inch, $AP_bV_b = 144 \times 1000 \times .016/778 = 2.96$ B.t.u. per pound.

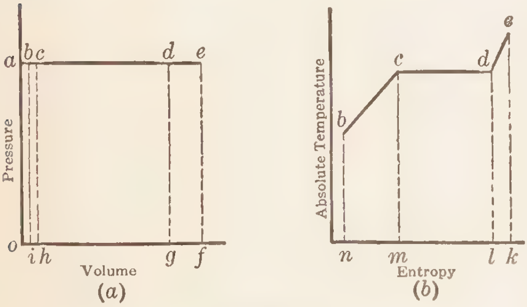

Fig. 140.

(d) Next, with the pound of liquid all in the cylinder, consider the valve to be closed and assume that there is no leakage, and *let heat be transferred to the liquid.* Then two results will be apparent: the liquid will rise in temperature appreciably, and its volume will increase. However, there will soon be reached a state c such that the temperature does not increase any more, but if heat is still added some of the liquid will form into vapor, and consequently the volume will increase very rapidly. This vapor is now at the temperature that is called the **Saturation Temperature,** or the **Temperature of Vaporization** (t_v) for this particular pressure. The material when in this state c may very properly be referred to as **Saturated Liquid,** because no more heat can be added to it at this pressure and still have it all remain liquid. The volume of a saturated liquid will be represented in general by the symbol

V_l. For water under a pressure of 1000 lb. per square inch the value of t_v would be 545 deg. fahr. and $\overline{V}_l$ would be 0.0217 cu. ft. per lb., or an increase of about 30 per cent over its volume at 32 deg. Had the pressure been higher, the saturation temperature would have been correspondingly greater, and vice versa.

(e) The *amount of heat added* to the unit weight of liquid during the constant pressure process *bc* is represented by the area *bcmn* in Fig. 140(*b*) and is called the **Heat of the Liquid,** which will be represented by the symbol q. Thus, since $t_c = t_v$

$$q = \int_{t_b}^{t_v} c_p dt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (196)$$

(f) The *gain in entropy* during the process *bc* is equal to $\int_{T_b}^{T_c} \frac{dQ}{T}$. Hence, for the one pound,

$$\Delta\phi_{bc} = \int_{T_b}^{T_v} \frac{c_p dT}{T} = \text{(very nearly) } \bar{c}_p \log_e \frac{T_v}{T_b}. \quad . \quad . \quad (197)$$

in which $\bar{c}_p$ represents the mean specific heat of the liquid for the particular pressure and temperatures involved. This gain in entropy is represented by the distance *nm* in Fig. 140(*b*).

(g) Now if *heat be added to the saturated liquid* in the cylinder, *vaporization* will take place at constant pressure and temperature. If a sufficient amount of heat be added the entire weight of liquid will become transformed into vapor whose state may now be represented by *d* in Figs. 140(*a*) and (*b*). During the progress of the process *cd* the vapor and its liquid are at substantially the same temperature,[2] and the space between the surface of the liquid and the piston is filled as full as it can be with this vapor,—or in other words, this space is *saturated with vapor* at the particular pressure used. As long as any liquid exists in the chamber the density of the vapor remains constant at its saturation value, which is also the maximum for the given pressure. In this condition the material is said to be **Saturated Vapor,** and its corresponding values of pressure and temperature are called the **Saturation Pressure** and **Saturation Temperature,** respectively.

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[2] The liquid in this case is at a slightly higher temperature than the vapor due to the hydrostatic pressure of the liquid and also to the fact that, in most actual cases, the heat is not applied exactly alike to all parts of the liquid. This small difference, however, is of no great importance except in the accurate calibration of thermometers, or in cases in which the vapor pressure is very low.

It is apparent from Fig. 140(*b*) that the temperatures at *c* and *d* are equal, and by the original hypothesis the pressures are equal; hence, it follows that the values of the *saturation pressure and temperature* for the saturated liquid are respectively the same as the values for the saturated vapor. It is for this reason that tables giving the properties of both the saturated liquid and saturated vapor have but one column for temperatures and one for pressures.

(**h**) The *amount of heat added* at constant pressure to the *unit weight* of liquid in order to vaporize it completely is called the **Latent Heat of Vaporization,** and is represented by r. Evidently r is shown by the area *cdlm* in Fig. 140(*b*).

(**i**) The *gain in entropy during vaporization* at constant pressure is known as the **Entropy of Vaporization** and for the unit weight is represented by ϕ_v. Evidently, since the temperature is constant,

$$\phi_v = \frac{r}{T_v}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (198)$$

and it is shown by the distance *ml* in Fig. 140(*b*).

(**j**) The *increase in volume* of one pound of substance during vaporization at constant pressure is designated by u. Hence, if $\overline{V}_d$ represents the volume of one pound of saturated vapor and $\overline{V}_l$ the volume of an equal weight of the saturated liquid at the same pressure, it follows that $u = \overline{V}_d - \overline{V}_l$, and that APu must represent the amount of external work done, in B.t.u. per pound, by reason of the increase in volume during this process of forming a vapor at constant pressure. The quantity APu is called the **External Latent Heat.**

(**k**) If from the latent heat, r, there is subtracted the external latent heat, APu, it will be found that there is an extremely large portion left. Since this remainder represents the gain in internal energy during the constant pressure process of vaporization it is called the **Internal Latent Heat,** and will be represented by ρ. Evidently,

$$\rho = r - APu. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (199)$$

It is interesting, although not essential to engineers, to speculate on the reasons for the large value of the internal latent heat. It appears that in order to separate the molecules of a liquid far enough from each other to form a vapor an enormous amount of energy is expended, and it is sometimes spoken of as the " disgregation work."

Recent investigations have led to the conclusion that liquid water is a more complex material than was originally supposed. It seems probable that instead of being simply a collection of molecules with the formula H_2O, it is really a mixture of at least three different kinds of molecules, H_2O, $(H_2O)_2$ and $(H_2O)_3$. It also seems probable that during the formation of vapor some of the more complex molecules break up into the simpler form, since steam is merely H_2O. If this is so, a possible use of at least part of the internal latent heat in the case of water becomes evident since it would be used for breaking up the complex molecules.

(l) Now, starting at d in Fig. 140, suppose that still more heat is added at constant pressure to the vapor just after the last drop of liquid has been vaporized. The result will be an increase in the temperature and volume of the vapor, which is now spoken of as a **Superheated Vapor**[3] because it has a higher temperature than saturated vapor of the same pressure. The superheated state is represented by e in Fig. 140. The **Degree of Superheat,**[3] which is represented by D, is defined as the difference between the temperatures of the superheated and the saturated vapors for the same pressure. Evidently for the case represented in Fig. 140, for the pressure p_e,

$$D_e = t_e - t_d = (t_{sup} - t_{sat})_{p_e}. \quad . \quad . \quad . \quad . \quad . \quad (200)$$

(m) The *amount of heat* required to carry out the process de for a unit weight is called the **Heat of Superheating** and is equal to $\bar{c}_p D$, where $\bar{c}_p$ represents the mean specific heat of the superheated vapor for the particular pressure and the temperatures involved. The values of $\bar{c}_p$ vary over a wide field, and they will be considered in detail later for different vapors under various conditions.

(n) The *gain in entropy* between d and e is called the **Entropy of Superheating,** and will be represented by ϕ_D. Evidently, for one pound

$$\phi_D = \int_{T_d}^{T_e} \frac{dQ}{T} = \int_{T_d}^{T_e} \frac{c_p dT}{T} = (\text{very nearly})\ \bar{c}_p \log_e \left(\frac{T_{sup}}{T_{sat}}\right)_{p_e}, \quad . \quad . \quad (201)$$

= distance lk, in Fig. 140(b), to scale.

174. The Saturation Lines.—(a) Consider the one pound of the same liquid used in the preceding section to be heated in the same cylinder as before, but this time with a greater weight on the piston; or, in other words, the heating is still to be done at constant pressure but at a higher value than in the previous case. Let this process be repeated again and again, always starting with the liquid at 32 deg. fahr. and under the successively higher pressures p_1, p_2, p_3, p_4 at which the vapor is to be formed. Let b_1, b_2, b_3, and b_4 in Figs. 141(a) and (b) represent the several initial states of the liquid at 32 deg. It will be noticed, in Fig. 141(*a*), that the volumes of the liquid at this constant temperature are almost the same, the line b_0b_4 being nearly vertical. There is, however, a slight degree of compression in all liquids at constant temperature. From Fig. 141(*b*), it may be seen, by referring to points b_0, b_1, b_2, etc., that, in general, the entropies are also slightly less for each successively higher pressure of the liquid at 32 deg.

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[3] For pressures higher than the critical, see Sect. **183**(f).

(*b*) After the *liquid*, in each of the above cases, has been heated at constant pressure to the various saturation temperatures corresponding to the several pressures, p_1, p_2, p_3, and p_4, the respective states of the liquid are those represented by c_1, c_2, c_3, and c_4. Similarly, the several states of the *saturated vapor* at the pressures p_1, p_2, p_3, and p_4 are shown by d_1, d_2, d_3, and d_4, respectively.

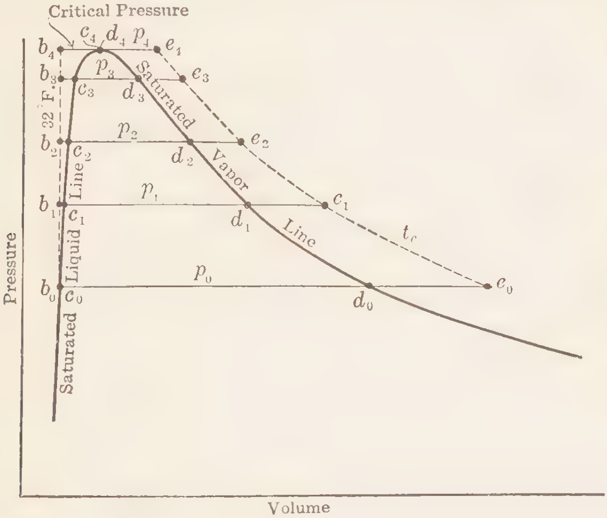

FIG. 141(*a*).

(c) If, for each pressure, the vapor is now *superheated* until it has the same final temperature t_e, the several superheated vapor states are represented by e_1, e_2, e_3, and e_4, respectively. It should now be apparent that each line $b_1c_1d_1e_1$, $b_2c_2d_2e_2$, etc., in this figure corresponds to *bcde* in Fig. 140, but each is for a different pressure, although the initial states b_1, b_2, etc., are each for liquid at 32 deg.

(d) Let the states represented by d_1, d_2, d_3, and d_4 be joined by smooth curves on both the pressure-volume and temperature-entropy diagrams of Fig. 141, as shown. In each diagram such a curve is called the **Saturated Vapor Line,** or sometimes, although less accurately, simply the **Saturation Curve** Similarly, if the states represented by

c_1, c_2, c_3, and c_4 be joined by a smooth curve on each diagram, the resultant line is properly designated as the **Saturated Liquid Curve.** Often this curve is called simply the *Liquid Line,* but this term is somewhat indefinite, because there are other liquid lines, such as b_1c_1, b_2c_2, etc., shown in Fig. 141. The *saturated* liquid line on any diagram is the locus of states of the liquid such that further addition of heat at each existing pressure will cause vaporization to begin, just as the *saturated vapor* line is the locus of states such that further addition of heat at constant pressure will cause superheating to begin.

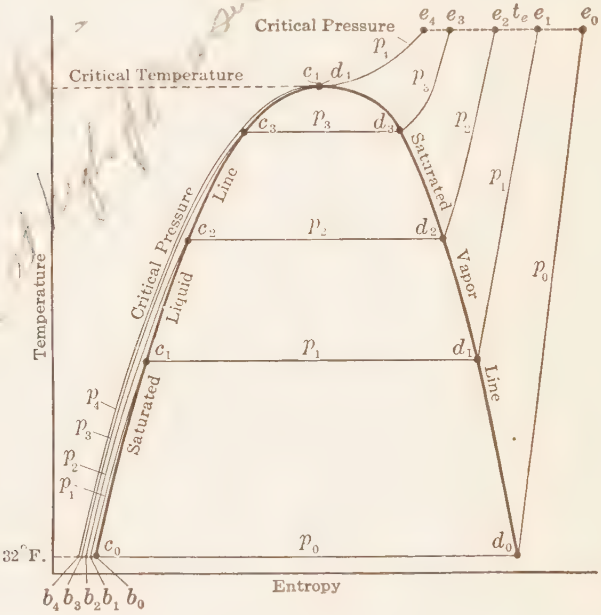

FIG. 141(*b*).

(c) The actual curvatures of the two saturation lines defined above depend upon the kind of substance used and on the range of pressures and temperature involved. However, for every material there is some state at which these two saturation lines merge, as shown by c_4 and d_4 coinciding in Fig. 141. This state is known as the **Critical State,** corresponding to which are a *critical pressure,* a *critical volume,* a *critical temperature,* and, similarly, critical values for the other properties, entropy, heat content, and internal energy. It should be noted that for the critical state the latent heat of vaporization is zero. For pressures higher than the critical, the discussion will be found in Sect. **183.**

175. The Zero of Internal Energy and Entropy of Vapors.—(a) In Sects. **16** and **58** it was pointed out that internal energy and entropy cannot be measured above an *absolute* zero. However, each of these properties may be measured above some conveniently, but *arbitrarily*, chosen zero, and then, for different states of any vapor for which certain properties have been carefully determined, a large number of values may be calculated and tabulated in the Vapor Tables. In making up such a list of tabulated values it is usually customary to take for the *zero of internal energy and entropy* that state in which the substance is *a liquid at the temperature of 32 deg. fahr. and under an absolute pressure of its saturated vapor*, or in other words the *datum is the saturated liquid at 32 deg. fahr.* At this temperature the saturation pressure is very low in the case of water, viz.: 0.0886 lb. per square inch; but for ammonia it is 62.29, and for carbon dioxide it is about 505 lb. per square inch. Since the two latter vapors are often used in refrigeration at temperatures much below 32 deg. their vapor tables are often based on a different zero. Thus in the recent Ammonia Tables of the U. S. Bureau of Standards [4] the zero of entropy and heat content was artibrarily chosen as saturated liquid ammonia at a temperature of −40 deg. fahr., corresponding to which the vapor pressure is 10.41 lb. per square inch abs.

(b) Supposing that the *saturated liquid* at 32 deg. fahr. is chosen arbitrarily as the zero of internal energy and entropy, it is now desirable to examine the results that follow from such an assumption. In Fig. 141 the zero state of the substance may be represented by b_0, and it is apparent from this figure that, for the liquid at 32 deg. fahr. but at the higher pressures p_1, p_2, etc., the corresponding entropies are appreciably less. This figure represents the case of carbon dioxide drawn to scale and the region to the left of the saturated liquid line is one in which actual properties of the liquid may sometimes be needed in refrigeration. Having the zero of entropy established at the state b_0, the entropy at the state such as b_4 having a higher pressure than p_0, although at the temperature t_0, must have a negative value.

(c) For the case of *water* the values of the entropy at the states b_0, b_1, b_2, b_3, and b_4 are so very close together that their differences are too small to be considered of significance in most engineering applications. Hence on the $T\phi$-diagram for this material the constant pressure lines of the liquid almost coincide with the saturated liquid line and generally therefore only the latter curve $b_0c_1c_2c_3c_4$ is drawn, and no negative values of the entropy of water usually need be considered.

[4] See Circular No. 142, Tables of Thermodynamic Properties of Ammonia, 1923.

176. The Heat Content of Liquids and Vapors.—(a) In Sect. **17**, which should now be reviewed thoroughly, the general definition of the **heat content** of any substance in any state was given as the sum of its internal energy in that state and its pressure-volume product. Thus,

$$H = I + APV, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (202a)$$

or, for one pound

$$h = i + AP\overline{V}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (202b)$$

(b) The last terms of the above equations can never be entirely equal to zero, because neither the pressure nor the volume of any substance can be reduced absolutely to zero. Hence, for a substance in the form of a liquid at the temperature and pressure corresponding to the state in which the internal energy, i_0, is arbitrarily chosen as zero the value of H_0 would be AP_0V_0, which may very properly be thought of as negligible in many cases, and in others it must be considered. For example, if the saturated liquid at 32 deg. fahr. is chosen as the zero of internal energy, the following values of h_0 are obtained:

For ammonia,

$$AP_0\overline{V}_0 = 144 \times 62.3 \times .025/778 = 0.29 \text{ B.t.u./lb.}$$

For carbon dioxide,

$$AP_0\overline{V}_0 = 144 \times 505 \times .018/778 = 1.68 \text{ B.t.u./lb.}$$

For water,

$$AP_0\overline{V}_0 = 144 \times .0886 \times .016/778 = 0.003 \text{ B.t.u./lb.}$$

(c) The numerical values of the heat content of a saturated liquid, a saturated vapor, or a superheated vapor may usually be obtained directly from a modern vapor table, or chart, in which the important properties of the particular fluid under consideration are arranged in convenient form for use by engineers. Such charts or tables always give *specific* quantities, and in this country the pound is the unit of weight used.

(d) The numerical value of the **internal energy** of a *liquid or vapor in any state* is commonly not given in the vapor tables, but it may be readily determined as soon as the three quantities, heat content, pressure and volume, are known. Thus, for example, if the pressure and temperature of a superheated vapor be specified, reference to the vapor table will give the corresponding specific values of heat content and volume. Hence APV may be found, and consequently

$$I = H - APV, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (203a)$$

or, for one pound

$$i = h - AP\overline{V}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (203b)$$

(e) *The difference in heat content of any substance in two different states, but at the same pressure, is equal to the amount of heat that must be added to the substance to carry out a constant pressure process between the two given states.*[5] Hence, it follows from the definition of the **heat of the liquid** (q), as given in Sect. **173**(e), that $q_c = h_c - h_b = h_c - AP_b\overline{V}_b$, it being assumed that the liquid at b is at the temperature t_b for which the internal energy, i_b, is taken as zero. Hence, $h_c = q_c + h_b = q_c + AP\overline{V}$. In other words, for a unit weight, the **heat content of a saturated liquid** (h_c) *is greater than the heat of the liquid* (q_c) (as defined in Sect. **173** (e)) *in the same state, by the quantity h_b—which in turn is equal to $AP_b\overline{V}_b$, if the internal energy of the liquid (i_b), at 32 deg.* and *whatever pressure it may have, is assumed to be zero.* This may be stated in another way by saying the *heat content* (h_c) *of the liquid* includes the energy expended in forcing the liquid into the vessel against the constant pressure maintained therein, whereas the *heat of the liquid* (q_c) does not include this energy. Since the specific volumes of liquids are small the term AP_bV_b is not very large except at high pressures, and consequently h_c and q_c are sometimes considered equal. In general, however, the term *heat content of the liquid* is the better one to use and is the one given in many of the modern tables. To illustrate the values of $AP_b\overline{V}_b$ the following cases are given:

Liquid	t_b	p_b	$\overline{V}_b$	$AP_b\overline{V}_b$
Ammonia	32	1000	0.025	4.63
CO_2	32	1000	0.018	3.33
Water	32	10	0.016	0.03
		1000	0.016	2.96

(f) The great *advantage* of using the *heat content of the liquid* is that in most cases of engineering applications the working substance is made to flow through some apparatus under steady flow conditions, and the *equations for this case, as given in Sect.* **44**, *involve the heat content of the fluid* entering and leaving the apparatus, regardless of what transformations take place within it. The flow of fluid through a steam boiler furnishes a good example. By means of a feed pump, the fluid is forced into the boiler feed pipe, at point 1, when in the form of a liquid under high pressure, but at a temperature that is usually considerably less than its saturation value corresponding to the entering pressure. In the boiler the fluid receives a large amount of heat from the furnace, and usually leaves, at point 2, as superheated steam which is at a pressure appreciably less than that in the feed line. It is desired to find the amount of heat that has been added to the fluid when passing through the boiler. Calling the amount of heat supplied by the boiler Q_B, it is apparent from the discussion of the General Energy Equation given in Sect. **44**, if it be assumed that $v_1 = v_2$, $z_1 = z_2$, and $R_c = 0$, that

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[5] The proof of this follows directly from Eqs. (5), (8) and (9).

$$Q_B = H_2 - H_1 \quad . \; . \; . \; . \; . \; . \; . \quad (204a)$$

$$= w(h_2 - h_1), \quad . \; . \; . \; . \; . \; . \; . \quad (204b)$$

in which w represents the weight of the fluid flowing per unit of time. The value of h_2, which is the specific heat content of the vapor leaving the boiler, is determined directly from a steam table for the particular pressure p_2 and temperature t_2 observed at section 2. The approximate value of h_1, the specific heat content of the liquid entering the boiler, is found by the following equation:

$$h_1 = h_a + A(P_1 - P_a)\overline{V}_a, \quad . \; . \; . \; . \; . \; . \quad (205)$$

in which the subscript a refers to the saturated liquid at the temperature t_1 at which the water is pumped into the boiler. Eq. (205) is derived as follows:

By definition,

$$h_1 = i_1 + AP_1\overline{V}_1,$$

and

$$h_a = i_a + AP_a\overline{V}_a.$$

Hence,

$$h_1 - h_a = i_1 - i_a + AP_1\overline{V}_1 - AP_a\overline{V}_a.$$

But since the temperature of the water is not changed appreciably by pumping it into the feed line, or in other words since $t_a = t_1$, it follows that i_a and i_1 are substantially equal; also $\overline{V}_1$ and $\overline{V}_a$ have practically the same value, hence Eq. (205) results. Since a is the state of the saturated liquid at the temperature $t_a = t_1$, the values of h_a, p_a, and $\overline{V}_a$, may be obtained directly from the saturated liquid table, and hence h_1 may then be calculated from Eq. (205). Then, with h_1, h_2, and w known, Q_B may be obtained by using Eq. (204).

(g) A numerical case will now be given to show the significance of Eq. (205): Suppose water at a temperature of 300 deg. fahr. is pumped into the boiler feed line in which the pressure is 1224 lb. per square inch abs., and that the steam from this boiler leaves at a pressure of 1200 lb. per square inch abs. and at a temperature of 700 deg. fahr. It is desired to find the heat added to each pound of fluid passing through the boiler, it being assumed that this working substance does not loose any heat by radiation, conduction, or convection between entering section 1 and the leaving section 2 at which the observations are made.

(Note that this statement specifies nothing regarding the losses that may take place from the hot furnace gases by radiation or otherwise.)

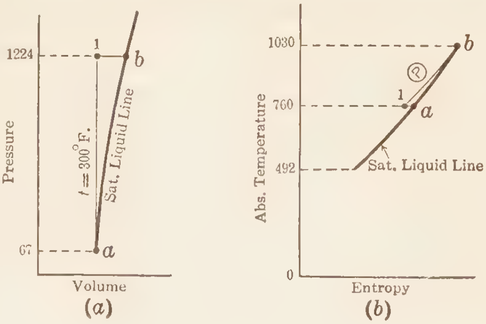

FIG. 142.

The saturation pressure corresponding to 300 deg. is found from the table to be 67 lbs. per sq. in. absolute; or, in other words, with $t_a = 300°$, then $p_a = 67$. Figure 142 may be used to show in exaggerated form the distinction between states a and 1, when $p_1 = 1224$, and $t_1 = 300$ deg. fahr. From the steam tables giving the properties of the saturated liquid, $h_a = 269.2$ B.t.u. per pound and $\overline{V}_a = 0.0175$ cu. ft. per pound; hence, from Eq. (205),

$$h_1 = 269.2 + \frac{144}{778}(1224 - 67)(.0175)$$

$$= 269.2 + 3.8 = 273 \text{ B.t.u. per lb.}$$

From the tables for superheated steam, with $p_2 = 1200$ and $t_2 = 700$, it is found that $h_2 = 1309$; hence the heat added to each pound of fluid flowing is $h_2 - h_1 = 1309 - 273 = 1036$ B.t.u. per pound.

The value of the term $A(P_1 - P_a)\overline{V}_a$ for this case is nearly 4 B.t.u. per pound, and this is an amount of energy that should not be ignored.[6]

(h) In general, then, it may be said that whether the substance is a liquid or a vapor (saturated or superheated), its heat content becomes a property of the utmost importance. If the problem is one of steady flow, as is the case in most power plant and refrigeration problems, the heat content is the term that usually is the largest factor appearing in the general energy equation (Eq. (21)) which would be used. Furthermore, in case a definite amount of working substance is enclosed in a cylinder, or similar apparatus, so that the Simple Energy Equation, Eq. (9), applies, the gain in heat content between the initial and final states is equal to the heat added to the substance during a constant pressure process between these states. For other processes the heat added is not equal to the difference of the heat contents and the method of treatment will be given in the next chapter. In all cases, however, it must be remembered that the heat content is a function of state alone, as is shown by Eq. (202).

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[6] It is true that the values of h_2 obtained from the various steam tables in use to-day may differ in extreme instances by amounts greater than 4 B.t.u. per pound when the pressures involved are high, but when more accurate tables are available, as they undoubtedly soon will be, commercial cases should be analyzed by the method given here.

177. Quality.—(**a**) The engineer often deals with enormous masses of vapor and liquids as they flow at high velocities through pipes and various pieces of apparatus. Sometimes the vapor is saturated but carries *entrained* moisture as it rushes along. Such a substance is called by various terms, such as "*wet vapor*,"[7] "*a mixture of a vapor and its liquid*," or simply "*a mixture*"; and the fraction represented by the ratio of the weight of dry vapor to the weight of the entire mixture is called the *Quality Factor*, or *Quality of the mixture*, which will be represented by the symbol x.

(**b**) The term quality may also be applied to a definite amount of a fluid that may be made to exist in a container partly as a vapor and partly as a liquid, and both at the same temperature, the two components being in contact with each other but not mixed. Thus, in the case of the formation of the vapor in which one pound of the liquid was heated in the cylinder in Sect. **173**, the quality of the entire pound of the material may sometimes be used to advantage. For example, if ten pounds of a fluid are in a container and one pound in the form of a dry saturated vapor fills the upper portion, while nine pounds of liquid fill the lower part, it may be said that the *quality of the material present, or of the wet vapor*, or of the "mixture" inside this container is 10 per cent, while the quality of the dry vapor, considered by itself, is 100 per cent.

(**c**) In general then, referring to Fig. 140, in state c the liquid is saturated and any further addition of heat at constant pressure will cause some vapor to form, hence it may be said that $x_c = 0$. In state d, $x_d = 100$ per cent since the *entire weight under consideration* has become a *dry saturated vapor*. Then, for any state between the saturated liquid line and the saturated vapor line the quality will be intermediate between 0 and 100 per cent.

(**d**) Sometimes the term quality is used when referring to superheated vapor. In this case the quality of the vapor is not given by a percentage greater than 100 per cent, but *the degree of superheat* may then be said to express the quality of the vapor.

(**e**) It is possible to have superheated vapor flowing at a high velocity in the upper portion of a pipe line, and at the same time have a small stream of liquid flowing along the bottom of the pipe. Such cases, however, are not those of thermal equilibrium as the liquid is being constantly evaporated and the degree of superheat is being continuously reduced. If given time enough equilibrium would be established, and the quality of the substance could then be determined. Another case, similar to the foregoing, is that in which the superheated vapor actually carries entrained drops of liquid until sufficient time has elapsed for equilibrium to be attained.

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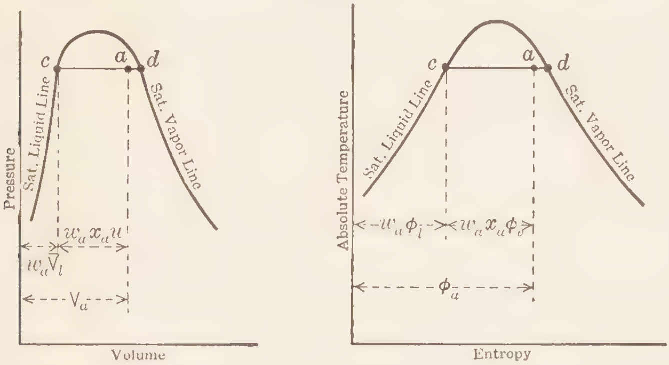
[7] Also the less satisfactory term "wet saturated vapor" is sometimes used.

178. Properties of a Mixture.—(a) Consider any state of a *wet vapor*, or mixture, say a, such that the quality x_a is less than unity.



(a) (b)

FIG. 143.

In Fig. 143 the state a is shown on the PV- and $T\phi$-diagrams, and it is now desired to find the values of t_a, V_a, Φ_a, H_a, and I_a, it being assumed that the weight of the mixture w_a, the pressure p_a, and the quality x_a are known, and that suitable vapor tables are available.

(b) *The temperature of the mixture*, t_a, may be found directly from the saturated vapor table, since $t_a = t_d = t_c = t_{sat}$.

(c) *The volume of the mixture*, V_a, is obtained as follows: The specific volume of the saturated liquid $\overline{V}_l$ at the temperature t_a may be obtained from the vapor table, and since $\overline{V}_c = \overline{V}_l$ at t_a, it follows that $V_c = w_a\overline{V}_l$, or in other words, the volume at c is that which would be occupied by the entire weight w_a if it were all in the form of a liquid at the temperature t_a. The increase in volume of one pound during complete vaporization at constant pressure has been designated by the letter u, hence $u = \overline{V}_d - \overline{V}_c$, both of these volumes being read directly from the vapor table. For a quality x_a the increase in specific volume during partial vaporization would be $x_a u$; hence

$$V_a = w_a(\overline{V}_l + x_a u)_{p_a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (206)$$

It is important to observe that Eq. (206) gives the correct volume for any specified weight of a mixture whose quality and pressure are known. For very low pressures the value of u is extremely large relative to $\overline{V}_l$, and for such cases, if the quality is fairly large, the term

$\overline{V}_l$ becomes negligibly small. On the other hand, at very high pressures, the value of $\overline{V}_l$ may become even greater than u.

(d) The *entropy of the mixture*, Φ_a, will now be determined:

The specific entropy of the saturated liquid, ϕ_l, at the pressure p_a is found from the table; hence $\Phi_c = w_a\phi_l$. The specific entropy of vaporization, ϕ_v, is either given in the table, or may be obtained by subtracting the entropy of the liquid from the entropy of the dry saturated vapor. Thus $\phi_v = \phi_d - \phi_l$. Hence

$$\Phi_a = w_a(\phi_l + x_a\phi_v)_{p_a}. \qquad (207)$$

(e) *The heat content of the mixture*, H_a, is found as follows: The specific latent heat of vaporization, r, for the pressure p_a may be obtained directly from vapor table, as can also the specific heat content of the liquid at state c. Then, since $p_c = p_a$, it follows that the heat content of the mixture must be:

For one pound,

q

$$h_a = (h_l + x_ar)_{p_a}, \qquad (208a)$$

or, for w_a pounds,

$$H_a = w_a(h_l + x_ar)_{p_a}. \qquad (208b)$$

(f) The *internal energy*, I_a, of the mixture may be obtained as soon as the volume and heat content have been found. Thus

$$i_a = h_a - AP_a\overline{V}_a, \qquad (209a)$$

or for w_a pounds,

$$I_a = w_a(h_a - AP_a\overline{V}_a) = H_a - AP_aV_a. \qquad (209b)$$

If the vapor tables give the specific internal latent heat of vaporization, ρ, it is sometimes more convenient to use the following relations for determining the internal energy of the mixture:

$$\begin{aligned} i_a &= (i_l + x_a\rho)_{p_a} \\ &= h_l - AP_a\overline{V}_l + x_a\rho, \qquad (210a) \end{aligned}$$

or

$$I_a = H_l - AP_aV_l + w_ax_a\rho. \qquad (210b)$$

If the pressure is not very high, or if the volume of the liquid is negligible, the term AP_aV_l becomes extremely small and in such cases may be neglected, thus giving $i_a = h_l + x_a\rho$. However, in general, it will be found better to use Eqs. (209) or (210).

179. Specifying the State of a Vapor.—When dealing with *ideal gases* it was found that the properties p, $\overline{V}$, t, ϕ, i, and h were so interrelated that fixing *any two* of them determines the others.

In the case of *saturated liquids*, or of *saturated vapors*, these properties are so related that the fixing of *one* of them determines the others.

[illegible]

[illegible] - [illegible]

In the case of a *mixture* of a liquid and its vapor, or a wet vapor, any *two* of these six properties, except pressure and temperature, will determine the state of the vapor. If the pressure and temperature be given it is just the same as though only one is known since either one of these determines the other. Hence, for this case it is essential to specify an additional property, or else give the quality.

For a *superheated vapor* the state is completely specified by giving any *two* of its properties.

180. Evaporation.—(a) There is sometimes difficulty in harmonizing the phenomena of vaporization at constant pressure previously described, with what is commonly known as "evaporation." There is no basic difference in the phenomena, vaporization as so far considered being only a special case of evaporation at constant pressure. It should be remembered that the so-called *temperature of vaporization* at any pressure is really the temperature of the saturated vapor or liquid at that pressure, and it is therefore preferable to use for it the term *saturation temperature.*

(b) *Experiment shows that when the surface of a solid or liquid is exposed to any space that is not already filled with the saturated vapor of that substance, vapor is generated until the space is filled with such saturated vapor,* unless the substance present is insufficient in amount.

If the condition of equilibrium is reached, the saturated vapor must exert the pressure corresponding to the temperature. Until this equilibrium is attained, any vapor present must be *superheated vapor* because the number of molecules in a given space is less than would be the case if the space were filled with saturated vapor. Superheated vapor, however, exerts a pressure less than that of the saturated vapor at the same temperature.

If follows that the pressure under which the liquid thus changes to vapor must constantly increase until a maximum is reached, when the space becomes filled with saturated vapor. After that there can be no further change in the relative quantities of liquid and vapor present unless the temperature changes.

Since heat is required to change a liquid to a vapor under the above conditions, a supply of it must be obtained from some source to cause "evaporation." If heat is not supplied from external sources, it is taken from the liquid and surrounding matter; hence the sensation of cold when alcohol, or other volatile liquid, is quickly evaporated from the skin.

(c) Usually the space into which the vapor passes contains other material beside the vapor; for example, some air is almost always present. *Dalton's law*[8] states that *each constituent of such mixtures behaves as though the others were not present.* Therefore, the phenomenon is not in any way complicated, although the evaporation is retarded somewhat, by the presence of any number of other vapors and gases. The evaporation goes on slowly until the space is filled with the saturated vapor of the material in question, and only then is equilibrium reached. The vapor will then have all the properties given numerically in its saturated vapor table opposite the existing temperature, but the total pressure in the closed vessel is made up of the individual pressures of the several gases and vapors composing the mixture.

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[8] See Sect. 75.

When some of the constituents of such a mixture are saturated vapors, the ideal gas laws cannot ordinarily be used if great accuracy is desired. When, however, the quantity of such vapors is minute as compared with that of the gases present, the error resulting from the use of the gas laws is small, and for the sake of simplicity those laws are often used and the error is neglected.

When a space is thus filled with the saturated vapor of a material, it is said to be *saturated with that vapor or with respect to that vapor.* Because of a peculiar construction of this expression an incorrect idea has become fixed in engineering language. It is somewhat common to speak of air saturated with water vapor, whereas the real meaning is that a space occupied by air is also occupied by saturated water vapor.

181. Boiling.—(a) Heat is often added to a liquid at such a rate and such a way that the temperature of one part becomes higher than that of adjacent parts; that is, local heating takes place. This is the result when the rate of the addition of heat at any point exceeds the rate of heat conduction through the material. Such heating raises the temperature locally to the saturation value corresponding to the pressure, after which further addition of heat would cause local vaporization; that is, a small amount of the liquid inclosed within the rest would be converted into vapor which would appear as a bubble.

The *pressure* at any point within a liquid at rest must be that due to the *static head* of the liquid above that point *plus the pressure due to any material resting upon the surface.* Therefore, the bubble of vapor would be formed under that pressure and, during formation, would have to displace the column, or "piston," of liquid above it against that pressure.

The bubble, being less dense than the surrounding liquid, would rise, but if the temperature of the liquid thus encountered were lower than its own, it might entirely condense before reaching the surface. This process continued long enough would bring all the liquid approximately to the same temperature, after which the bubbles traveling upward through the liquid would escape as vapor from the surface.

Liquid is said to be in a state of ebullition or to be *boiling* when it is in such a state that bubbles of vapor formed within its mass pass up and out through its surface.

(b) The *boiling point* of any substance is defined as the temperature at which there is equilibrium between the liquid and its vapor, the liquid being under a total pressure equal to the vapor pressure. Unless otherwise specified the pressure is usually understood to be that of the standard atmosphere; but for this condition it is better to use the term *normal boiling point.*

In case the substance is a *mixture* of two or more liquids, the boiling point is that temperature at which the sum of the several vapor pressures becomes equal to the total external pressure on the mixture. In general, the boiling point of a mixture cannot be inferred from the boiling points of its components. The composition of the vapor mixture is usually not the same as that of the liquid mixture in thermal equilibrium with it.

182. Unstable States of Liquids and Vapors.[9]—(a) In the previous discussion in this chapter it was assumed that the liquid and its vapor were always in thermal equilibrium, except in the cases mentioned in which it was possible to have superheated vapor in contact with its liquid, equilibrium not being established because of lack of time for its accomplishment.

(b) There are certain other unstable states, of very different types from the foregoing, and these will now be mentioned. For example, it can be shown by a very careful experimenter that pure water, from which all absorbed gases have been removed, may be heated very slowly and very quietly to a temperature much above 212 deg. fahr., although under an absolute pressure of only one atmosphere. When in this condition the liquid is termed a *superheated* one and it is very unstable, the least bit of stirring or jarring being sufficient to cause it to boil most violently, or almost explode, as a small amount of vapor is formed, thus absorbing the surplus heat and restoring equilibrium conditions between the temperature and pressure.

Similarly, if pure water under atmospheric pressure is cooled very slowly and quietly, its temperature may be lowered to about 20 deg. fahr. without the formation of ice, but the liquid is then in a very unstable state. In this condition it may be called an *undercooled* or a *supercooled* liquid, and the least disturbance of it will cause crystals of ice to appear throughout its mass, thus giving up sufficient latent heat of fusion to bring the mixture up to the normal freezing temperature.

These cases of unstable states of liquids are not, in general, encountered in ordinary engineering processes.

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[9] This Section may be omitted in short courses.

(c) If a dry saturated vapor have heat abstracted from it very slowly and quietly it may be cooled considerably below the saturation temperature corresponding to its pressure, without any condensation taking place. It is then said to be a *supercooled* or *supersaturated* vapor; and it is very unstable. If a single drop of liquid or any dust be introduced into vapor in this state, condensation will immediately begin and thus soon will restore the equilibrium. This case of vapor supercooled quiescently is not, however, so likely to occur in engineering as is the one in which cooling results by reason of a very quick expansion. A good illustration of the latter occurrence is found in the modern steam turbine, in the nozzle of which the vapor may pass from one region to another of lower pressure in an almost inconceivably short period of time—say, $\frac{1}{20000}$ second,—and under these conditions condensation probably does not take place rapidly enough to maintain equilibrium and the vapor may therefore become supersaturated. Vapors in this state will be discussed further when dealing with steam turbines. When not otherwise stated, however, it will be assumed that all of the discussions that follow are for vapors that are in equilibrium states.

183. Continuity of the Liquid and Gaseous States.—(a) It has already been stated that no real gases obey exactly the laws of ideal ones, but that it may be assumed without great error that those which are farthest removed from the conditions of liquefaction do obey these laws.

The study of materials in the liquid, vaporous, and gaseous states shows clearly that any material may be made to pass directly from one of these states to the other. This may be presented by means of Fig. 144. It should be clearly understood, however, that this figure is qualitatively but not quantitatively correct; that is, it is not drawn to scale, nor does it exactly represent the behavior of any real material. However, for all known materials, it shows the general *nature* of the processes under consideration.

(b) The diagram is for unit weight of material on pressure-volume coordinates, *and each of the lines acdz is an isotherm.* Starting with the lowest one of the series, the point a represents the pressure and volume of a unit weight of liquid at temperature T. It should be noted that the pressure p_a is much higher than the saturation pressure p_c for this temperature. If the pressure is decreased while the temperature is maintained constant, the volume of the liquid will increase very slightly until the saturated liquid line is reached at the point c. Further expansion at constant temperature will result in a progressive vaporization at constant pressure, as shown by the horizontal line cd. At d all of the liquid has become transformed into a saturated vapor. A further decrease of pressure at constant temperature will cause this vapor to become superheated and to be more and more like an ideal gas the further the expansion is continued. Since the isothermal expansion of an ideal gas is according to the law $PV = a\ const.$, the isotherm dz approaches this form of curve as the expansion proceeds to still larger volumes and lower pressures.

Starting from a_1, with the material in liquid form at a temperature T_1 higher than T, a similar process carries the material isothermally to z_1. The same statements can be made for all other starting points at different temperatures up to some such value as T_3, when the process will be somewhat similar to that shown by the curve $a_3c_3z_3$. In this case the points c_3 and d_3 have become coincident, the saturated liquid having the same volume at pressure P_3 as does its saturated vapor, and there is no distinction between them. This point represents the **critical state,** and the critical isotherm is shown by T_3.

(c) At temperatures higher than the critical, such as T_4 and T_5, the isothermal expansions begin with the substance in the states represented by a_4 and a_5, and the curves become more nearly rectangular hyperbolas ($PV = a\ const.$) as they are drawn for higher and higher temperatures. Until recently it has been considered that a substance when kept above its critical temperature could exist only as a gas, but the work of Bridgman[10] and Tammann[11] at extremely high pressures shows that it may be made to solidify. The only requirement seems to be the application of sufficient pressure. In some of his experiments Bridgman used pressures of about 600,000 lb. per square inch. He states that " an ordinary liquid under a pressure of 200,000 lb. per

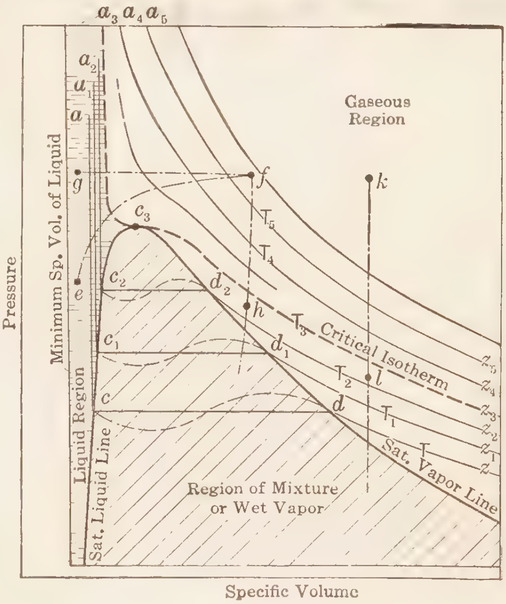

FIG. 144.

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[10] Properties of Matter under High Pressure, by P. W. Bridgman, Mech. Eng., March, 1925.

[11] The States of Aggregation, by Gustav Tammann. D. Van Nostrand.

square inch loses in volume about 33 per cent," and that "the facts seem to be that by applying enough pressure to any liquid you can force it to freeze."

(d) Above the critical pressure *the distinction between liquids and gases ceases to exist.* Condensation and vaporization phenomena in which part of the substance is a liquid and part a vapor, as previously described in this chapter, no longer occur. Instead, the substance can be made to pass *continuously and homogeneously* from the so-called liquid field into the adjoining one. This continuous transformation is one in which the entire mass of substance remains homogeneous throughout the process which may be made to occur in any manner desired, provided it is not carried across the saturation curves. For example, a liquid under pressure such as shown by the state *e* in Fig. 144 might be heated under increasing pressure and volume, as shown by the line *ef*, until it crosses the critical isotherm at a pressure higher than the critical value. No evidence would be given by any of the characteristics of the substance to show that it changed from the liquid to the gaseous phase, because above the critical pressure there is **no strict boundary** between these regions.

(e) On the other hand, by carrying out the processes represented by the isotherms below the critical, such as $acdz$, $a_1c_1d_1z_1$, etc., **real boundaries,** represented by the saturation curves, may be determined experimentally simply by observing the abrupt changes that take place at the saturation lines. These same boundary curves may also be located experimentally by following out a series of constant pressure processes as previously described in connection with Fig. 141(*a*). After the critical temperature and pressure have been found for a substance, the critical isotherm may be drawn as shown in Fig. 144, as soon as sufficient values of pressure and volume at this temperature are known. Note, however, that the critical isotherm could never be found experimentally by following such paths as are represented by the dotted lines *ef*, *fg*, and *kl* because during such processes there is no abrupt change in any property of the substance to indicate when the critical isotherm is crossed. Any process, such as a constant volume one, crossing the critical isotherm at the critical state c_3 would give evidence of that fact because it would also then be crossing the real boundary made by the saturation curves. Similarly, if the processes *fh* and *kl* are continued far enough, the pressure will finally become sufficiently low to cross the saturated vapor line, and, if the substance be in a glass tube, experimental evidence of the change of phase would then appear because the material would then cease to be homogeneous and drops of liquid could be seen separated from the vapor.

On the other hand, any constant pressure process, such as *fg*, above the critical pressure, would give no evidence when the so-called liquid field is entered, because the material would at all times be homogeneous. Similarly, a line might be drawn through *f* representing an isothermal compression above the critical temperature, and even though carried to extremely high pressure, such as were used by Bridgman, there will be no discontinuity unless, or until, a solid phase appears.

(f) The region just above the critical state is of great importance when dealing with certain substances, such as CO_2, in refrigeration, and it may sometimes become desirable to use steam in power plants at pressures greater than its critical value, for example as has been done in an experimental way by Benson.[12]

It is apparent that the usual definition of a superheated vapor as given in Sect. **173** (1) is not broad enough to apply to pressures greater than the critical. Furthermore the term, degree of superheat, ceases to have any significance above the critical pressure. Hence a **superheated vapor** may be defined *in its broadest sense as a vapor in the region intermediate between the gas and its adjoining fields.* Regardless of the name given to the substance in this region, the main point to remember is that it is a *vapor* and must be treated as such. The properties of some working substances have been well determined for this field, while for others considerable work remains to be done. The following table gives the approximate values of the critical pressures and temperatures of a number of substances of special interest to engineers:

TABLE X

CRITICAL TEMPERATURES AND PRESSURES

Substance	Critical Temperature, Deg. Fahr.	Critical Pressure Lb. per Sq. In. Abs.
Air	−220	544
Ammonia	271	1657
Carbon Dioxide	88	1070
Ether	379	543
Helium	−450	40
Hydrogen	−402	294
Nitrogen	−230	515
Oxygen	−180	735
Sulphur Dioxide	311	1160
Water	706	3200

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[12] See Power, May 22 and 29, 1923.

184. Van der Waals's Equation.[13]—(a) From the preceding section it is obvious that any gas, except possibly those at excessively high pressures, is really only a very attenuated liquid, differing in its properties from the liquid because its molecules are much farther apart, and possibly of simpler structure. If this be true, it ought to be possible to express by one equation the relation between pressure, volume and temperature of a material in either the liquid or the gaseous form. Several attempts have been made to do this, and one in particular is of great interest. It is due to Van der Waals and was developed in 1873 by modifying Boyle's law to take account of the two following assumptions:

(1) The space filled by a gas is partly occupied by the molecules of that gas, and it is only in case the molecules are infinitely small relatively to the space they traverse in their vibratory movements that the gas obeys Boyle's law. The space available for such movements is the volume within the container minus that occupied by the molecules themselves.

(2) In no real gas are the molecules far enough apart to be absolutely independent of one another; certain intermolecular forces still exist. These forces make the gas behave as though subjected to a pressure greater than the real external pressure.

The equation for one pound material is as follows:

$$\left(P + \frac{a}{\overline{V}^2}\right)(\overline{V} - b) = RT, \quad . \quad . \quad . \quad . \quad . \quad (211a)$$

in which a and b are constants, differing with the kind of gas. The constant b is often called the "co-volume" since it represents the reduction in specific volume due to the molecules. Since the constant a is to compensate for the intermolecular forces, it seems reasonable to make it proportional to the square of the density, or $1/\overline{V}^2$.

(b) This equation can be rearranged to read

$$\overline{V}^3 - \overline{V}^2\left(\frac{RT}{P} + b\right) + \overline{V}\frac{a}{P} - \frac{ab}{P} = 0, \quad . \quad . \quad . \quad (211b)$$

a cubic equation in terms of the specific volume $\overline{V}$. Then for a given temperature and pressure there may be three values of $\overline{V}$ which satisfy the equation. There must be three if proper account is taken of multiple and imaginary roots.

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[13] Sections **184** and **185** may be omitted in short courses.

If the curves obtained by substituting in the equation are drawn for constant temperatures, they resemble the lines *acdz*, etc., in Fig. 144, except that the horizontal lines *cd*, etc., are replaced by the dotted curves as shown. If the equation is really true, the process of vaporization must be more complicated than at first appears. The fact that the phenomena corresponding to certain parts of the dotted curve from *c* downward and from *d* upward can be realized experimentally gives evidence in support of this law. Under special conditions a liquid may have its pressure reduced slightly below the saturation pressure corresponding to its temperature, and similarly, a vapor may be compressed, without condensation, to a pressure somewhat above the saturation pressures corresponding to its temperature. The condition of the material thus carried into the dotted position of the curve is, however, very unstable, and the substance suddenly assumes the condition shown by the horizontal line if disturbed, as discussed in Sect. **182.**

(**c**) The critical state may now be said to be the one at which all three roots of the equation coincide, i.e., at which the equation has a triple root.

(**d**) The equation of Van der Waals, though better than that of Boyle, does not fully express the truth. If it did it would hold for material in the solid as well as in the liquid state. It really recognizes no such condition as solid. If it did, its graph, continued far enough back in the direction *zdca*, should, for all ordinary pressures, at least, show another jog similar to, but shorter than, *dc*, representing the constant-pressure, constant-temperature change from liquid to solid. This it does not do, and hence it is imperfect.

(**e**) The phenomenon of zero volume at absolute zero temperature to comply with Charles's Law of a gas cooled at constant volume can now be explained. According to the simplest kinetic theory of gases, the temperature is supposed to be a measure of the translational energy of the molecules, and the pressure is the result of the bombardment of the containing walls by the rapidly moving molecules.

Assuming, with Van der Waals, that the volume to which the ideal laws refer is not the total space occupied by the gas, but equals the latter corrected for the volume of the molecules present, the limiting case of the ideal laws is easily explained. When absolute zero of temperature is reached the molecules of a gas must be assumed to be devoid of translational motion and in such positions that the volume referred to above has become zero. Then as the molecules at rest could not bombard surrounding surfaces the pressure would also be zero.

185. The Clapeyron Equation.—(a) In preparing a Vapor Table for any substance it is necessary to obtain as accurately as possible a large number of values for each of the properties included. The numerical magnitudes of many of these properties may be obtained by careful experiment, but, for use in verifying these, for computing the values of other properties, and for other purposes, it is essential also to know what relations exist between certain of them. One important relation, first developed by Clapeyron, is that between the latent heat, r, the increase in volume during vaporization at constant pressure, u, and the rate at which the saturation pressure varies with its temperature, $(dP/dT)_{\text{sat.}}$ Clapeyron's relation may be stated thus:

$$r = AT\left(\frac{dP}{dT}\right)_{\text{sat.}} u. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (212)$$

(b) This equation may be derived in the following manner: In Sect. **119** (e) it was shown that for any substance the Carnot cycle is bounded by isentropic and isothermal lines, and is therefore represented by a rectangle on the temperature-entropy diagram. In Figs. 145(a) and (b) consider the small Carnot cycle $abcd$, in which the line ab represents the isothermal addition of a quantity of heat that is just sufficient to cause complete vaporization at the constant pressure P, and the corresponding temperature T_1. The adiabatic expansion from b to c is taken as that which will cause a small temperature drop $\delta T = T_1 - T_2$, as shown in Fig. 145(b) The isothermal and adiabatic compressions needed to complete the cycle are shown by cd and da respectively in both diagrams.

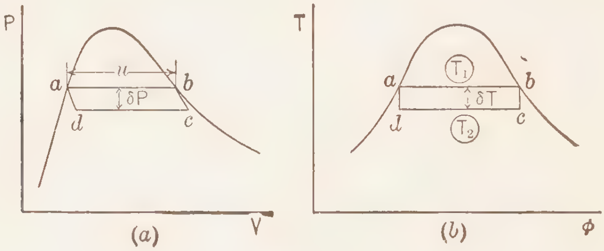

FIG. 145.

In this figure $u = V_b - V_a$, and for a very small drop in pressure δP, the area $abcd$ on the PV-diagram becomes very nearly equal to $\delta P \cdot u$ ft-lb. On the $T\phi$-diagram the area $abcd$ is exactly equal to $(\delta T)(\Delta\phi_{ab}) = (\delta T)(r/T)$ B.t.u. Each of these areas expresses the net work of this small Carnot cycle. As the drop in temperature, δT, and the corresponding drop in pressure, δP, become smaller and smaller, the closer does the value of $\delta P \cdot u$ express the net work of the cycle in ft-lb., or $A\,\delta P \cdot u$ in B.t.u. Therefore, in the limit, with δP becoming indefinitely small, it follows that $AdP \cdot u = dT\frac{r}{T}$; hence, $u = \left(\frac{dT}{dP}\right)\frac{r}{AT}$, or $r = AT\left(\frac{dP}{dT}\right)_{\text{sat.}} u$, as given in Eq. (212). This equation shows the exact relation that must exist between the latent heat, r, and the increase in volume, u, during vaporization at the constant absolute temperature T, provided the derivative $\left(\frac{dP}{dT}\right)_{\text{sat.}}$ for the saturated vapor or liquid is known.

(c) Since the pressure-temperature measurements of a saturated liquid or vapor are the two easiest properties to determine experimentally it follows that the relation between these two variables may be shown graphically, as in Fig. 146, from which $\left(\frac{\delta P}{\delta T}\right)$ may be found, or it may be expressed analytically, as by Eq. (213) in Section **187** for steam, thus enabling the derivative $\left(\frac{dP}{dT}\right)_{\text{sat.}}$ to be obtained. After the derivative has been determined Clapeyron's relation becomes of the greatest assistance in checking the values of r and u, or in determining one if the other be known.

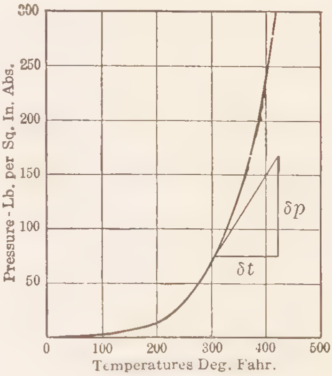

FIG. 146.

186. Comparison of Different Vapors.—(a) The vapors formed from water, ammonia, carbon dioxide, and sulphur dioxide are in frequent engineering use. Steam is, however, much the more common because it is used very extensively in many kinds of power plants, and also for heating purposes, both domestic and industrial.

(b) The pressure-volume diagrams to show the "vapor domes" of steam, ammonia, sulphur dioxide and carbon dioxide have been drawn to scale in Fig. 147(*a*) and (*b*). The corresponding temperature entropy diagrams of these substances are shown in Fig. 148. Some of the values near the critical state are rather uncertain at the present time, hence the corresponding parts of these curves are drawn with dotted lines in this region.

(c) Aside from the wide variation in the critical pressures and temperatures the most striking feature is the magnitude of the latent heat of steam as compared with the others, especially CO_2 and SO_2, all taken at some common temperature. For example, at an absolute temperature of 500 deg. fahr., the latent heats of the four vapors are shown in Fig. 148 by the respective areas of the rectangles having the common height of $T = 500$ and the various widths shown by the four corresponding entropies of vaporization at this temperature.

(d) It should be noted that in Fig. 148 the zero of entropy for ammonia was chosen as saturated liquid at a temperature of -40 deg. fahr., or $T = 420$, whereas for the other three substances the zero was taken as the saturated liquid at 32 deg. fahr., or $T = 492$. Had the values for ammonia been plotted with the same zero of entropy as the others, the shape of its saturation curves would remain just the same, but they would be shifted to the left about 0.2 unit of entropy.

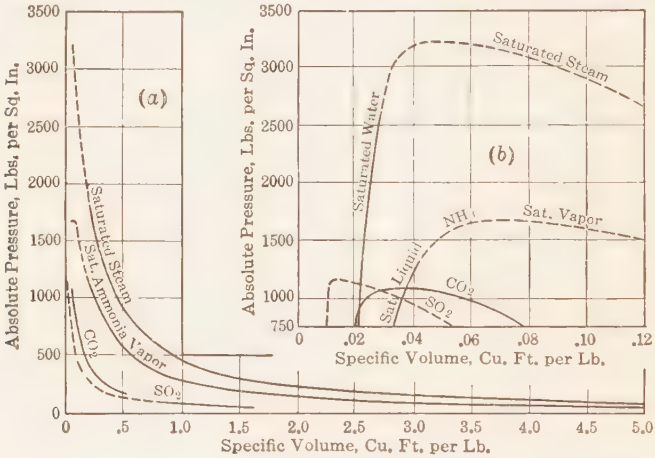

FIG. 147.

(e) In Table XI are given values from which the curves of Figs. 147 and 148 were plotted. The latent heat (r) and the entropy of vaporization (ϕ_v) may be found directly from this table by remembering that $r = h_{sat} - h_l$, and that $\phi_v = \phi_{sat} - \phi_l$. Since, by definition, $u = \overline{V}_{sat} - \overline{V}_l$, it is apparent that these short tables will enable all of the properties of vapors having a known quality less than unity to be calculated for the given pressures.

In the Appendix will be found somewhat more complete tables of the properties of steam and ammonia.

187. The Properties of Steam.[14]—(a) Since *steam* is the most important working substance used, this section will be devoted to a very brief discussion of steam formulas and the numerical values of its properties. However, the beginner should realize that it is extremely difficult to obtain accurate data by direct measurement of even such properties as pressure and volume over a wide range. Pressure and temperature may be measured more easily. In all cases it is desirable to determine the correct thermodynamic relationship existing between certain properties and then by means of all available and reliable experimental data, formulate an equation that agrees with the most trustworthy and at the same time is consistent with the thermodynamic relations. Some such formulas are very long and include many terms in order to cover a wide range of pressures and temperatures. Other formulas are much shorter and may be very reliable over a much smaller range of pressure and temperature. For an excellent discussion of the various steam formulas, see the article by Professor Heck in A. S. M. E. Transactions, 1920, p. 711.

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[14] In short courses Sect. **187** (a), (b), (c), (d) and (e) may be omitted.

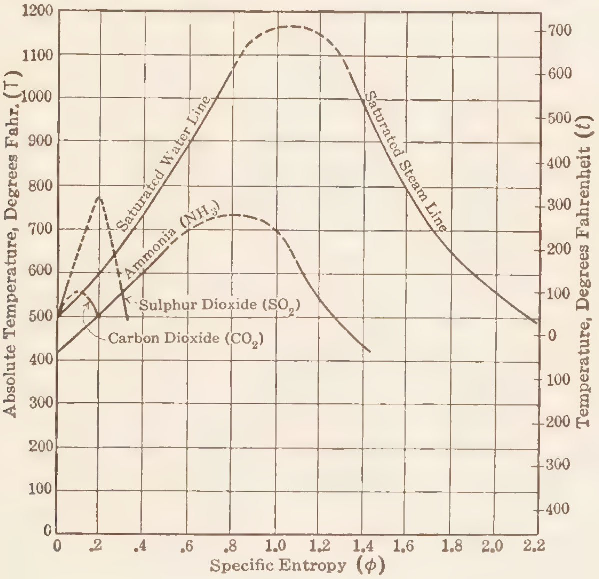

FIG. 148.

(b) The *steam formulas*, as developed by the several authors of steam tables, such as Callendar, Goodenough, Heck, Knoblauch, Marks, and Davis, and Stodola, yield values of the heat content and entropy that are not in close agreement at the higher pressures, but it will be found later that the most important *differences* in these quantities between two given states, as usually required in engineering calculations,

TABLE XI

Properties of Saturated Liquids and Vapors

	Temp. Deg. Fahr.	Absolute Pressure Lb. per Sq. In.	Volume		Heat Content		Entropy	
			Liquid	Vapor	Liquid	Vapor	Liquid	Vapor
			Cu. Ft. per Lb.	Cu. Ft. per Lb.	B.t.u. per Lb.	B.t.u. per Lb.	Per Lb.	Per Lb.
	t	p	$\overline{V}_{liq.}$ or $\overline{V}_l$	$\overline{V}_{sat.}$ or $\overline{V}_d$	$h_{liq.}$ or h_l	$h_{sat.}$ or h_d	$\phi_{liq.}$ or ϕ_l	$\phi_{sat.}$ or ϕ_d
Steam*	32	0.0886	0.01602	3294	¶0.00	1073.4	0.0000	2.1832
	100	0.946	0.01613	350.8	67.97	1103.6	0.1295	1.9800
	200	11.52	0.01663	33.60	167.94	1145.8	0.2937	1.7761
	212	14.7	0.01671	26.79	180.0	1150.4	0.3118	1.7565
	300	67	0.01744	6.46	269.6	1179.1	0.4371	1.6343
	400	247.1	0.0187	1.866	374.1	1201.3	0.5663	1.5286
	500	678.7	0.0206	0.677	484.0	1209.0	0.6860	1.441
	600	1574	0.024	0.27	604.0	1176.0	0.799	1.339
	706§	3200§	0.049	0.049				
Ammonia†	−40	10.41	0.02322	24.86	0.04	597.6	0.000	1.4242
	0	30.42	0.02419	9.116	42.92	611.8	0.0975	1.3352
	25	53.73	0.02488	5.334	70.2	619.1	0.1551	1.2879
	50	89.19	0.02564	3.294	97.93	625.2	0.2105	1.2453
	75	140.5	0.02650	2.125	126.25	629.9	0.2643	1.2065
	100	211.9	0.02747	1.419	155.21	633.0	0.3166	1.1705
	125	307.8	0.02860	0.973	185.1	634.0	0.3679	1.1358
	175	593.5	0.03160	....	(248)			
	250	1347	0.0422	....	(365)			
	271.4§	1657§	0.0686	....	(433)			
Carbon Dioxide‡	0	309	0.016	0.292	−14.0	103.0	−0.032	0.223
	32	505	0.018	0.167	1.68	101	0.000	0.202
	50	650	0.019	0.120	11.5	97.9	0.019	0.189
	75	906	0.022	0.071	30.5	88.1	0.054	0.162
	88.4§	1070§	0.035	0.035	63.0	63.0	0.112	0.112
Sulphur Dioxide‡	−25	5.1	0.011	13.89	−17.2	148.7	−0.037	0.342
	0	10.4	0.011	7.07	−9.9	152.0	−0.020	0.332
	25	19.3	0.011	3.94	−2.3	156.0	−0.005	0.322
	32	22.6	0.011	3.40	0.05	157.0	0.000	0.319
	50	33.0	0.011	2.34	5.8	159.5	0.012	0.313
	75	54.1	0.011	1.47	14.4	162.2	0.028	0.304
	100	84.6	0.011	0.97	23.1	163.9	0.044	0.295
	311§	1160§	0.011					

* Obtained from the tables of Marks and Davis and from the A. S. M. E. "Progress Report on the Development of Steam Charts and Tables from the Harvard Throttling Experiments," reported by J. H. Keenan in *Mechanical Engineering*, Feb., 1926.

† Obtained from Circular No. 142, U. S. Bureau of Standards.

‡ Obtained from the Report of the Refrigeration Research Committee of the Institution of Mechanical Engineers given in their Oct. 1914 Proceedings, p. 645.

§ Critical Value. ¶ Strictly 0.003.

will agree quite closely regardless of what steam table is used. It is due to the fact that certain differences in heat content are so important to engineers, that the several steam tables give these values to a larger number of digits than would be justifiable at the present time, if only absolute quantities were involved instead of differences.

A discussion of the research carried on in this country to obtain more accurate information concerning steam at high pressures will be found in the several progress reports made to the A. S. M. E. Steam Table Research Fund Committee.[15]

(c) The *pressure temperature* relation of saturated steam over a wide range is now known with extreme accuracy. The experimental work of Regnault in 1847 gave very accurate results up to 363 deg. fahr., the maximum temperature used by him. Since that time the work of many investigators has established the relationship for pressures up to about 2000 lb. per square inch. The remarkable agreement of the values of Holborn and Baumann with the ones of Keyes and Smith is shown in a short table given in "Mechanical Engineering" for Feb., 1926, p. 155.

The equation of Goodenough gives results that are in very close agreement with those obtained by Keyes and Smith. His equation is as follows:

$$\text{Log}\, p = A - \frac{B}{T} - C \log T - DT + ET^2 - \Delta, \quad . \quad . \quad (213)$$

where

$$A = 10.5688080 \qquad \log D = \bar{3}.6088020$$

$$\log B = 3.6881209 \qquad \log E = \bar{6}.1463000$$

$$C = 0.0155 \qquad T = t + 459.6$$

and

$$\Delta = 0.0002\left[10 - 10\left(\frac{t - 370}{100}\right)^2 + \left(\frac{t - 370}{100}\right)^4\right].$$

The numerical values corresponding to this equation may be read from Fig. 149.

It should be noted how very rapidly the pressure increases with each degree rise in temperature at the higher values. This fact is shown in Fig. 149, but the graphical representation of it, as in Fig. 146, shows very closely how the ratio $\delta p/\delta t$ varies.

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[15] Mechanical Engineering, March, 1923, p. 165; Feb., 1924, p. 81; Nov., 1924, p. 808; March, 1925, p. 174; Feb., 1925, p. 103; Feb., 1926, pp. 144–160.

(d) The *heat content* of saturated and superheated steam cannot be accurately expressed in terms of pressure and temperature except by equations that are somewhat complex. It is difficult at the present time to say whose equation is the most reliable one throughout the entire range from 32 deg. fahr. to the critical temperature. However, amongst the most reliable ones is Goodenough's which is as follows:

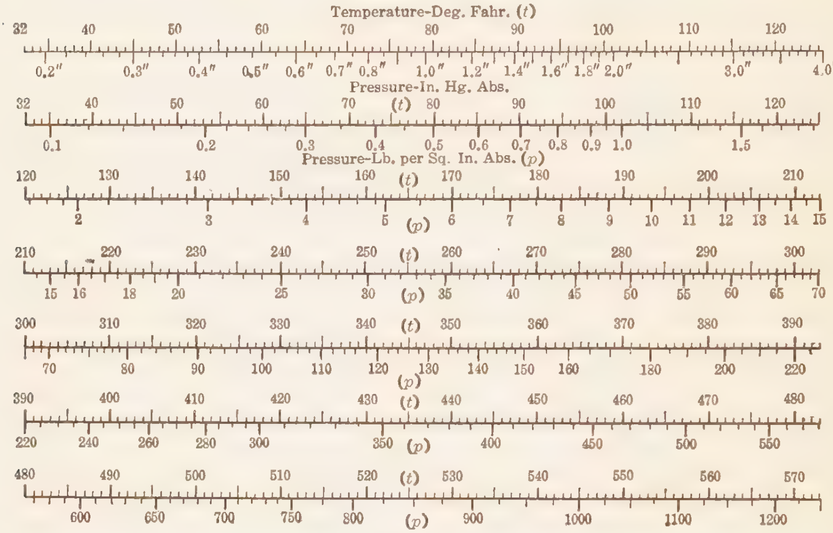

FIG. 149.

$$h = \alpha T + \tfrac{1}{2}\beta T^2 - \frac{\gamma}{T} - \frac{Ep}{T^4}(1 + 2ap^{1/2}) + 0.1852p\overline{V}_l + h_0. \quad . \quad (214)$$

in which

$\alpha = 0.320$ $\qquad a = 0.0171$

$\beta = 0.000126$ $\qquad \log E = 10.79155$

$\gamma = 23583$ $\qquad h_0 = 948.54$

In Fig. 150 may be seen the graphical representation of Goodenough's equation, with pressure and heat content as coordinates. It is apparent from the small insert (*a*) that the uniform scale of pressures, extending over a wide range, is not satisfactory, hence, a logarithmic scale has been used for the ordinates in the main figure. The maximum value of the heat content of saturated steam is seen to be at a pressure much below the critical. This figure is not intended to be used for the solution of numerical problems, but to show approximately how the heat content varies over a wide range of pressures, temperatures, and qualities.

(e) The *specific volume* of *steam*, saturated or superheated, may be obtained from the table given by Mr. J. H. Keenan in the Progress Report on the Development of Steam Charts and Tables[16] from the Harvard Throttling Experiments, in "Mechanical Engineering" for Feb., 1926, or from Goodenough's equation which is as follows:

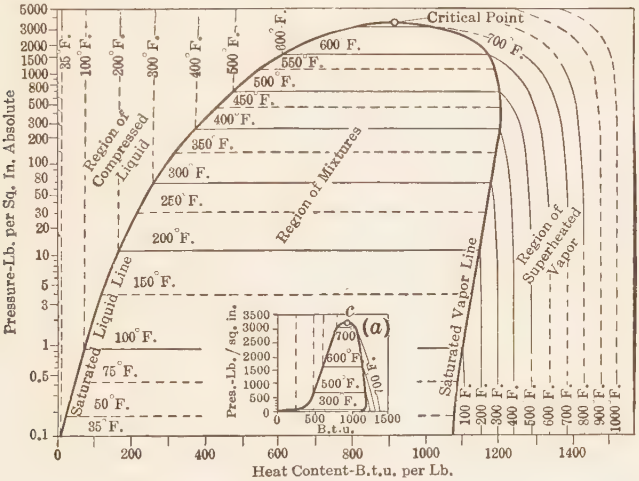

FIG. 150.—The Relation between Pressure and Heat Content of Steam.

$$\overline{V} = \overline{V}_l + \frac{BT}{p} - (1 + 3ap^{1/2})\frac{m}{T^4}, \quad . \quad . \quad . \quad . \quad (215)$$

where

$$\log B = \bar{1}.77448$$

$$\log m = 10.82500$$

$$\log 3a = \bar{2}.710000$$

(f) The *specific heats* of a *superheated vapor* are extremely variable when considered over a wide range of pressure and temperature. For steam the *mean values* of the constant pressure specific heat, $\bar{c}_p$, are given in Fig. 151, from which it is apparent that for a small degree of superheat and a high pressure the values of c_p become very large relative to their corresponding values at low pressures.

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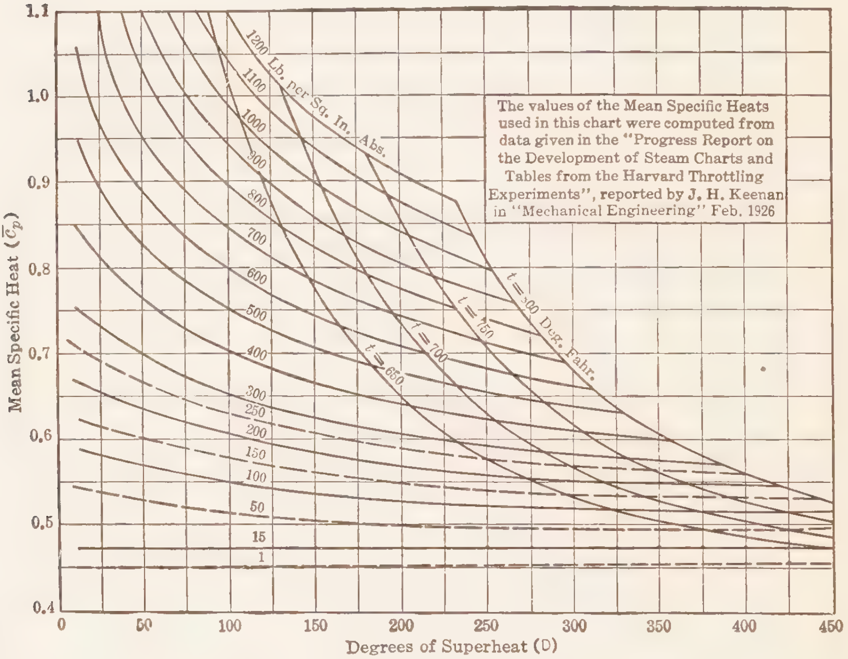
[16] See the Appendix for an abbreviated form of this table.

It should be noted that each of the mean values in Fig. 151 is given for a constant pressure process from the saturation curve to the particular degree of superheat or temperature under consideration. This is done because ~~of~~ chart of this kind is most useful in finding the *heat of superheating*, $\bar{c}_p D_a$, or the *entropy of superheating* [see Eq. (201)], for some given state a in the superheated region.

The corresponding *instantaneous* specific heats, c_p, for superheated steam are given in Fig. 152.



The values of the Mean Specific Heats used in this chart were computed from data given in the "Progress Report on the Development of Steam Charts and Tables from the Harvard Throttling Experiments", reported by J. H. Keenan in "Mechanical Engineering" Feb. 1926

FIG. 151.—Values of Mean Specific Heats of Superheated Steam.

The values of the specific heat at constant volume, c_v, are not needed for most engineering work and consequently they are not given here.

(g) The *instantaneous specific heat of water* under constant pressure is far from constant, as may be seen from Fig. 153. However, for low temperatures, that is, below 200 deg. fahr., its value may be considered as unity for all work in which great accuracy is not essential. The method of obtaining the mean value of the specific heat between two different temperatures from curves such as those given in Figs. 152 and 153 is indicated in Fig. 33, p. 82.

188. Summary of Important Relations.—(a) In the next chapter a number of thermodynamic processes that are frequently used will be studied, and it is essential that the properties of vapors shall be well understood before beginning this work. A brief summary of the most important relations just developed are therefore appended, it being assumed that Table XI, or a much more comprehensive one, is available.

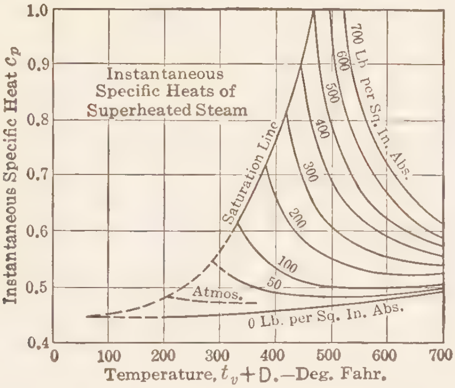


FIG. 152.—Values of the Instantaneous Specific Heat c_p, of Superheated Steam.

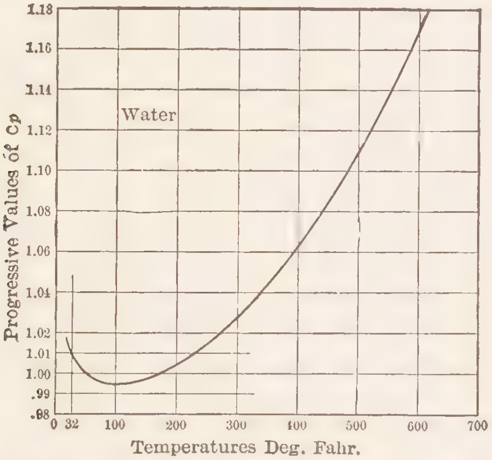


FIG. 153.—Instantaneous or Progressive Values of Specific Heat c_p, of Water.

(b) The *Definitions* give the following equations:

$$r = h_d - h_l = h_{sat} - h_l,$$

$$u = \overline{V}_d - \overline{V}_l = \overline{V}_{sat} - \overline{V}_l,$$

$$x = (\text{wt. of dry vapor})/(\text{wt. of mixture}),$$

$$\rho = r - APu,$$

$$\phi_v = r/T_v = \phi_d - \phi_l = \phi_{sat} - \phi_l.$$

(c) For a *Saturated Liquid* specified by its pressure or temperature, the values of h_l, $\overline{V}_l$, and ϕ_l are found directly from the tables. Then,

$$i_l = h_l - AP\overline{V}_l.$$

(d) For a *Non-Saturated Liquid*, or one having a pressure higher than the saturation value corresponding to its temperature, the approximate value of the heat content may be found by using Eq. (205).

(e) For a *Mixture* in state a, specified by p_a and x_a,

$$h_a = h_l + x_a r.$$

$$i_a = h_a - AP_a\overline{V}_a = i_l + x_a\rho,$$

$$\overline{V}_a = \overline{V}_l + x_a u,$$

$$\phi_a = \phi_l + x_a\phi_v$$

(f) For a *Superheated Vapor* in state e, specified by t_e and p_e, which is less than the critical pressure, the values of, h_e, $\overline{V}_e$, and ϕ_e may be found directly from the Superheat Tables. Also

$$D_e = t_e - t_v = t_{sup} - t_{sat},$$

$$h_e = h_d + \bar{c}_p D_e = h_{sat} + c_p D_e,$$

$$i_e = h_e - AP_e\overline{V}_e = h_{sup} - AP_e\overline{V}_{sup},$$

and, very closely,

$$\phi_e = \phi_d + \bar{c}_p \log_e \frac{T_e}{T_v} = \phi_{sat} + \bar{c}_p \log_e \frac{T_{sup}}{T_{sat}}.$$

(g) For a superheated vapor, having a pressure higher than the critical, the properties may be found directly from the superheat tables carried to pressures beyond that value.

CHAPTER XVI

THERMODYNAMIC PROCESSES OF VAPORS

189. Introductory.—Just as in the case of gases, the study of thermodynamic processes of vapors naturally follows immediately after the mastery of their six characteristics, P, V, T, Φ, H and I, whose relations have been discussed in the preceding chapter for all possible states of the substances. The various processes that will now be considered involve only the application of the *Energy Equations*, as given in Chap. IV, to the many processes that are needed by the engineer who is dealing with vapors. The several cases will first be solved analytically, assuming the vapor tables to be available, as that is the best method to be followed by the beginner who intends to acquire a thorough understanding of the subject. Then, at the end of the chapter, there will be given a discussion of several forms of vapor charts that will enable numerical results to be obtained much more rapidly than is possible by the first method.[1]

190. The Isobaric, or Constant Pressure, Process.—(*a*) In Sects. **173** to **179** the constant pressure process of vapors is discussed to a considable extent, hence the treatment of it in this section will be very brief. Consider w pounds of some known substance to have heat added to it in such a manner [2] that the pressure remains constant, at the assigned value P, until its volume has increased from one specified value to another, or until its temperature has increased from some given initial value to another. In order to show all of the cases of the isobaric process that may need to be studied, consider the one shown by *1 2 3 4 5*, in Fig. 154. It is apparent that *1* represents a nonsaturated liquid, *2* a saturated liquid, *3* a wet vapor or mixture, *4* a saturated vapor, and *5* a superheated vapor, and that

$$P_1 = P_2 = P_3 = P_4 = P_5 = P = \text{const.}$$

(**b**) Since the pressure is constant it follows that the *work done* by the given weight of substance in expanding at constant pressure from any one of these states to another is equal to the product of the pressure and the increase in volume. Thus, in general, if a and b represent the initial and final states respectively, then

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[1] In short courses it may be advisable to use the charts for most of the numerical work in Chap. XVI.

[2] See Sect. **173** for a suitable apparatus.

$$Wk_{ab} = P(V_b - V_a) = wP(\overline{V}_b - \overline{V}_a). \quad . \quad . \quad . \quad . \qquad (216)$$

For example, in Fig. 154,

$$Wk_{1,\,5} = P(V_5 - V_1) = wP(\overline{V}_5 - \overline{V}_1)$$

$$= \text{area } \mathit{1\ 5\ k\ l}, \text{ to scale,}$$

and

$$Wk_{2,\,3} = P(V_3 - V_2) = wP(\overline{V}_3 - \overline{V}_2)$$

$$= \text{area } \mathit{2\ 3\ k'l'}, \text{ to scale.}$$

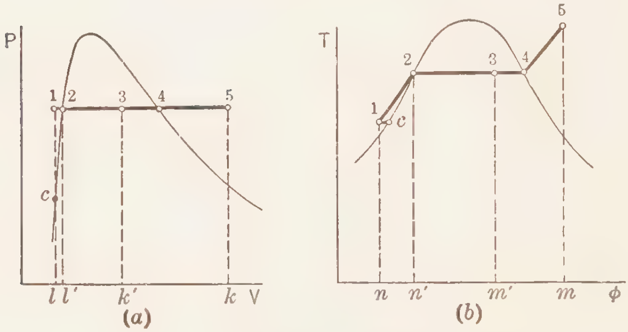


FIG. 154.—Constant Pressure Process.

The problem thus resolves itself into one of merely finding the initial and final specific volumes.

$\overline{V}_2$ and $\overline{V}_4$ may be obtained directly from the saturated liquid and saturated vapor tables, or from charts, for the given pressure or for the corresponding saturation temperature.

$\overline{V}_5$ is found from the superheated vapor table, or chart, for the given pressure and temperature, t_5, or the degree of superheat, D_5.

$\overline{V}_3 = \overline{V}_2 + x_3u = \overline{V}_l + x_3u$, it being assumed that x_3 is known.

$\overline{V}_1 = \overline{V}_c$ very closely, since an isotherm for a liquid is almost a constand-volume line, as is indicated by *1c* in Fig. 154 (a). In other words, for nearly all engineering applications the specific volume of the nonsaturated liquid, under all ordinary pressures such as are used in refrigeration or in power plants, may be taken as equal to the specific volume of the saturated liquid having the same temperature, i.e., if $t_c = t_1$, then $\overline{V}_1 = \overline{V}_c$, whose value may be found in the saturated liquid table for the given temperature t_1.

(c) Since the pressure is constant *the heat absorbed* by the substance during any part of the process is equal to the gain in the heat content between the initial and final states under consideration. Hence, in general,

$$Q_{ab} = H_b - H_a = w(h_b - h_a). \quad . \quad . \quad . \quad . \quad (217)$$

For example, in Fig. 154,

$$\begin{aligned} Q_{1,\,5} &= H_5 - H_1 = w(h_5 - h_1) \\ &= \text{area } \mathit{1\,2\,4\,5\,m\,n}, \text{ to scale,} \end{aligned}$$

or, as another illustration,

$$\begin{aligned} Q_{2,\,3} &= H_3 - H_2 = w(h_3 - h_2) \\ &= \text{area } \mathit{2\,3\,m'\,n'}, \text{ to scale.} \end{aligned}$$

Thus, to obtain the heat absorbed during an isobaric process, the initial and final values of the *heat content* are the only quantities to be found.

The values of h_2, h_4 and h_5 may be determined directly from the vapor tables, or charts, it being assumed that either t_5 or D_5 is known; $h_3 = h_2 + x_3r$, or it may be found directly from the chart; and, from Eq. (205), $h_1 = h_c + A(P_1 - P_c)\overline{V}_c$ very nearly.

191. The Isometric, or Constant Volume, Process.—(a) If a given weight of a substance be enclosed in any kind of a vessel that does not change its volume as heat is supplied to it, or abstracted from it, the substance must undergo a constant volume process. A steam boiler with all valves closed, as it is when first getting up steam, and a cask of ammonia with the sun shining on it are examples of the formation of vapor at constant volume. In both of these cases there is the possibility of the pressure rising to a dangerously high value with the application of relatively only a small amount of heat.

(b) Suppose a known weight of a mixture of a vapor and its liquid, having a given pressure p_1 and a quality x_1, is heated at constant volume until some assigned pressure such as p_2 or p_4 is obtained. It is desired to find the heat absorbed by the mixture during such a process, and to show the area representing it. Since the initial state *1* is specified by its pressure and quality, it can be located on both the PV- and $T\phi$-diagrams, as shown in Fig. 155. Then a vertical line drawn through *1* on the PV-diagram must represent the constant volume process. This line will cross the saturated vapor curve at some pressure, say p_3, whose value may be found from the saturated vapor table, or charts, to correspond with the specific volume $\overline{V}_3$, which must first be determined. This volume is readily found as follows: Since $V_3 = V_1$, and since $w_3 = w_1$, it being assumed that there is no leakage, then

$$\overline{V}_3 = \overline{V}_1 = \overline{V}_{l_1} + x_1u_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (218)$$

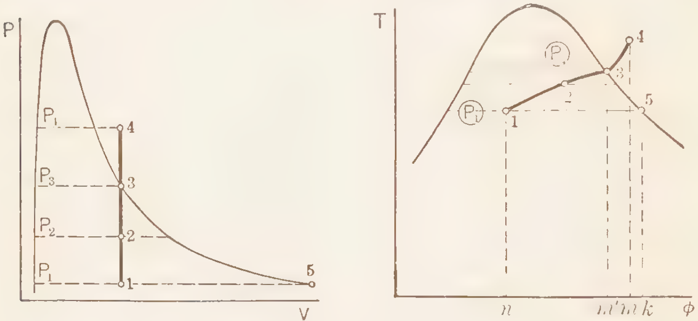
The subscripts attached to l and u in this equation simply show that these values are obtained from the table for the given pressure p_1.

Similarly, for the point *2* in the region of mixtures, since $\overline{V}_2 = \overline{V}_1$,

$$\overline{V}_{l_2} + x_2u_2 = \overline{V}_{l_1} + x_1u_1.$$

Consequently,

$$x_2 = \frac{\overline{V}_{l_1} + x_1u_1 - \overline{V}_{l_2}}{u_2}. \quad . \quad . \quad . \quad . \quad . \quad (219)$$



(*a*) (*b*)

Fig. 155.—Constant Volume Process.

By using Eq. (219) the qualities corresponding to the state of the mixture having the constant volume V_1 and any pressure less than p_3 may be obtained.

For any pressure higher than p_3, say p_4, the state must lie in the region of superheat. With p_4 given, and with $\overline{V}_4 = \overline{V}_1$, the state *4* is completely specified, and the corresponding values of t_4, D_4, h_4, ϕ_4 and i_4 may be obtained by the aid of the vapor tables or charts. Conversely, if, the pressure p_4 is not given, but is to be determined, it is essential that besides $\overline{V}_4$ the value of an additional characteristic shall be given for this state.

(c) By choosing as many pressures between p_1 and p_3 as desired, it becomes possible to compute the qualities of any number of states in the wet region, such as state *2*, that lie on the constant volume line through *1*. As soon as these qualities are determined the curve may be plotted *on the $T\phi$-diagram*, as shown by *1 2 3* in Fig. 155(*b*). After passing into the region of superheat the constant volume curve is represented by *3 4* and may be drawn after obtaining from the superheated vapor tables, or charts, the simultaneous values of temperature and entropy that correspond to the specific volume $\overline{V}_4 = \overline{V}_1$ and to the various pressures above p_3. It should be noted that the character of the constant volume line on the $T\phi$-diagram changes abruptly at the saturated vapor boundary line, just as does the constant pressure line.

(d) In general, if a and b represent the initial and final states, respectively for this process, since the volume is constant it follows that

$$Wk_{ab} = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (220)$$

and then from the Simple Energy Equation, Eq. (9), it must be true that the *heat absorbed* by the working substance during a constant volume process must be equal to the gain in internal energy. Thus, in general,

$$Q_{ab} = I_b - I_a = w(i_b - i_a). \quad . \quad . \quad . \quad . \quad . \quad (221a)$$

$$= w\ [h_b - AP_b\overline{V}_b - (h_a - AP_a\overline{V}_a)]. \quad . \quad (221b)$$

For example, in Fig. 155,

$$Q_{1,\,4} = I_4 - I_1 = w(i_4 - i_1)$$

$$= \text{area } 1\ 2\ 3\ 4\ m\ n, \text{ to scale},$$

and

$$Q_{1,\,3} = I_3 - I_1 = w(i_3 - i_1) = \text{area } 1\ 2\ 3\ m'\ n, \text{ to scale}.$$

(e) Consideration of the formation of a saturated vapor, starting with any initial state, such as *1* in Fig. 155, shows that if the *pressure* be kept constant the final state is that shown at *5* and the corresponding amount of heat absorbed is represented by the rectangle *1 5 k n*, but if the *volume* be kept constant the final state is *3* and the corresponding amount of heat absorbed is shown by the area *1 2 3 m′ n*. Note in the latter case that the temperature and pressure rise enormously with the resulting element of danger, as mentioned in (a), while in the former they are both constant. Most of the commercial applications of heat to form vapors are very close to constant pressure processes, rather than constant volume ones.

192. The Reversible Adiabatic, or Isentropic, Process.—(a) As was discussed in Sect. **81**, any substance undergoing an expansion or compression without any turbulence or transfer of heat to or from it represents a reversible adiabatic or isentropic process. For the case of ideal gases it was an easy matter to express for this process the functional relation between two characteristics, such as pressure and volume, but for vapors it is not possible to write a simple equation between these characteristics that will hold for all kinds of vapors while undergoing isentropic processes covering a wide range of pressures and temperatures. On the other hand, there is no especial need for such an equation for vapors because the relationships are given in the tables and charts. Since the entropy is constant during a reversible adiabatic, and as the vapor tables, or charts, always include the entropies, the secondary state of the working substance may be determined as soon as one additional characteristic is specified for this state.

(b) Consider w pounds of a superheated vapor inside a cylinder that is perfectly insulated thermally and that is fitted with a piston that may be moved in or out, thus permitting an isentropic compression or expansion of the vapor from some given pressure p_1 to some final pressure p_2, or volume V_2. Assume that the final pressure is known and that the initial state of the vapor is given by its pressure and temperature—or, in other words, that p_1, t_1 and p_2 are specified. It is desired to find the work that would be done during the isentropic expansion of this vapor from *1* to *2*.

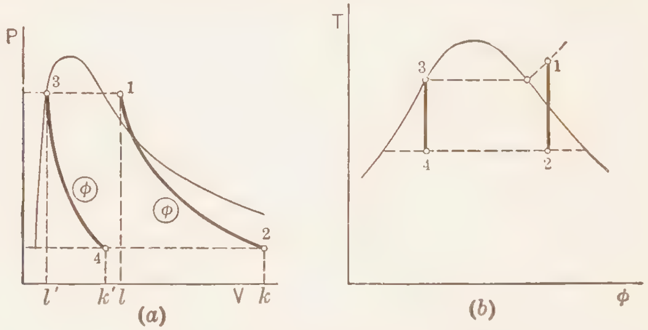


Fig. 156.—Constant Entropy Process.

In Fig. 156(*b*) the vertical line *1 2*, represents the constant entropy line drawn from the initial point *1* until the constant pressure line p_2 is intersected at *2*. The pressure-volume diagram may be drawn for the same process by determining the volumes attained at various pressures throughout the expansion. The method of finding the final volume V_2 in the region of mixtures is typical in case the final state is not in the region of superheat, hence it will be the only one illustrated. It involves the determination of x_2, which may be obtained as follows: Since, $\Phi_1 = \Phi_2$ and since $w_1 = w_2$, it being assumed that there is no leakage, then, with ϕ_{v_2} representing the specific entropy of vaporization at p_2, it follows that

$$\phi_1 = \phi_2 = \phi_{l_2} + x_2\phi_{v_2}.$$

Hence,

$$x_2 = \frac{\phi_1 - \phi_{l_2}}{\phi_{v_2}} = \frac{\phi_1 - \phi_{l_2}}{\phi_{sat_2} - \phi_{l_2}}. \quad . \quad . \quad . \quad . \quad . \qquad (222)$$

The value of ϕ_1 is determined directly from the superheated vapor table, or chart, or it may be computed from Eq. (201) and the saturated vapor table; and the other entropies may be obtained from the saturated vapor table for the pressure p_2. As soon as x_2 is found, the value of V_2 may be readily determined from Eq. (206), i.e.,

$$V_2 = w_2(\overline{V}_{l_2} + x_2 u_2).$$

In case the initial quality is less than unity, then $\phi_1 = x_1 \phi_{v_1} + \phi_{l_1}$, and with this value of ϕ_1 substituted in Eq. (222) the final quality may be found.

As will be seen later, when a vapor chart is used the quality x_2 may be read directly from it at the intersection of the constant entropy line ϕ_1 and the constant pressure line p_2. If the constant volume lines are on the same chart the specific volume $\overline{V}_2$ may also be read directly.

If, instead of knowing the final pressure p_2, the final volume V_2 (or the ratio $\frac{V_2}{V_1}$) had been specified, the solution is not so readily made by means of the vapor tables, because the pressure p_2 is needed in order to obtain the proper values of ϕ_{l_2} and ϕ_{v_2}. However, this pressure may be found by trial and is the one which will give the value of x_2 that yields the proper result for V_2 in Eq. (206). But, with vapor *charts* that give constant entropy and also constant specific volume lines, it is a simple matter to follow along the constant entropy line, ϕ_1, until the final specific volume line, $\overline{V}_2 = \frac{V_2}{w_2}$, is intersected and the state *2* is then directly and completely determined.

(c) If the process had been an *isentropic compression* from a given state *2* to some specified pressure p_1, the charts would enable the state *1* to be determined as readily as was the final state in the previous case, regardless of the regions involved. On the other hand, with the state *1* in the superheated region as shown, the complete *analytical* determination of it by means of the relation $\phi_2 = \phi_1 = \phi_{sat_1} + \bar{c}_p \log_e \left(\frac{T_1}{T_{sat_1}}\right)$ involves the use of the mean specific heat $\bar{c}_p$ for an unknown temperature range; hence it would be necessary to assume a value for this term and then check back after t_1 had been determined, thus involving a certain amount of "cut and try" procedure.

(d) If the *isentropic expansion* had started with a *saturated liquid* the process would be that shown by the line *3 4* in Fig. 156, the same initial and final pressures as before being used. For this case it is apparent that

$$\phi_4 = \phi_3 = \phi_{l_3},$$

and consequently

$$x_4 = \frac{\phi_{l_3} - \phi_{l_4}}{\phi_{v_4}} = \frac{\phi_{l_3} - \phi_{l_4}}{\phi_{sat_4} - \phi_{l_4}}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (223)$$

(e) The *work done* by any working substance during a reversible adiabatic expansion must be equal to the loss of internal energy of the substance, as was shown in Sect. **82**(c). Hence, in general, in Fig. 156,

$$Wk_{1,\,2} = I_1 - I_2 = w(i_1 - i_2) \quad . \quad . \quad . \quad . \quad . \qquad (224)$$

$$= \text{area } 1\ 2\ k\ l, \text{ to scale,}$$

and, similarly,

$$Wk_{3,\,4} = I_3 - I_4 = w(i_3 - i_4)$$

$$= \text{area } 3\ 4\ k'\ l', \text{ to scale.}$$

The value of the specific internal energy is given by Eqs. (203) and (210), thus

$$i_1 = h_1 - AP_1\overline{V}_1, \ i_2 = h_2 - AP_2\overline{V}_2., \text{ etc.}$$

(f) It should be noted from Fig. 156 that during the adiabatic expansion *1 2* the quality decreased, while for the similar process *3 4* it increased from zero to x_4. For most vapors the two saturation curves here shown are fairly typical and hence the isentropic expansion may usually be expected to produce respectively either an increase or decrease in quality, depending upon whether the initial state of the vapor is near the saturated liquid line or close to the saturated vapor curve, as shown for steam and ammonia in Figs. 157(*a*) and (*b*). However, for *ether* and certain other fluids, as the pressures are increased, the entropy of the saturated liquid increases more than enough to compensate for the decrease in the entropy of vaporization, as is shown in Fig. 157(*c*), and consequently the entropy of saturated vapor increases with the pressure. Hence for this material the quality would *always increase* during any isentropic expansion.

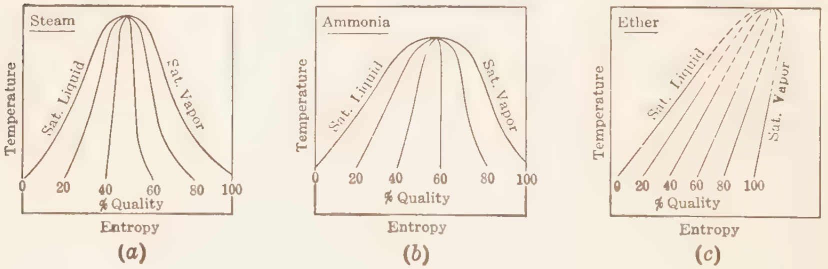


FIG. 157.

193. The Isothermal, or Constant Temperature, Process.—(a) For the *region of mixtures* the isotherm has already been covered by the discussion of the isobaric process (Sect. 190) because these processes are identical in this field. However, outside of this region they are very different. For ideal gases the relation between pressure and volume during an isothermal process is given by the equation $PV = a\ constant$, but superheated vapors do not follow this law at all closely until they have passed far out from the saturation curve. It is generally not desirable to try to find an approximate equation for a superheated vapor that is undergoing an isothermal process, because the exact thermodynamic relations are easily handled by the aid of suitable superheated vapor tables or charts.

(b) Referring to Fig. 158, consider w pounds of a vapor in state *1*, given by p_1 and x_1, to be heated isothermally until it reaches some secondary state given by the assigned lower pressure p_2. On the temperature-entropy diagram, some horizontal line such as *1 2* must represent the process, but the location of point *2* must be determined. Since p_2 is given, the constant pressure curve representing it may be drawn on this set of coordinates, and where it intersects the isotherm through *1* is the desired point, it being known that state *2* is in the region of superheat, since p_2 is less than p_1. From Fig. 158(*b*) it is apparent that the *heat added* to the vapor during the isothermal expansion is

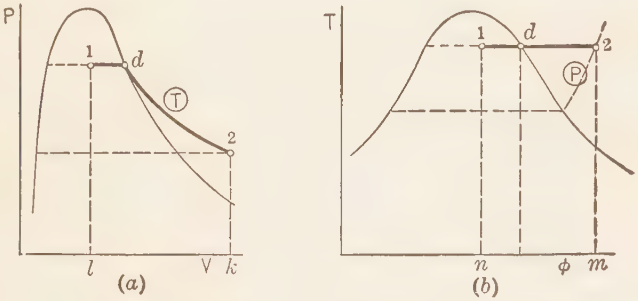


Fig. 158.—Constant Temperature Process.

$$Q_{1,\,2} = \text{the area } 1\ 2\ m\ n, \text{ to scale,}$$

$$= T_1(\Phi_2 - \Phi_1) = T_1 w(\phi_2 - \phi_1). \quad . \quad . \quad (225a)$$

The value of ϕ_1 for the given p_1 and x_1 may be found by the aid of tables or charts; and since p_2 and t_2 are known, ϕ_2 may also be similarly determined; and consequently $Q_{1,\,2}$ may be found.

By the Simple Energy Equation, Eq. (9), it then follows that the *work done* by the vapor during this process is

$$Wk_{1,\,2} = Q_{1,\,2} - \triangle I_{1,\,2} = Q_{1,\,2} + I_1 - I_2 \quad . \; . \; . \; . \; . \; . \quad (226a)$$

$$= T_1 w(\phi_2 - \phi_1) + w(i_1 - i_2) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (226b)$$

$$= w[T_1(\phi_2 - \phi_1) + h_1 - AP_1\overline{V}_1 - (h_2 - AP_2\overline{V}_2)] \quad (226c)$$

$$= \text{area } 1\ d\ 2\ k\ l \text{ to scale.}$$

The values of h_2 and $\overline{V}_2$ may be found from a suitable table or chart for the given values of p_2 and t_2.

If the final volume V_2 is specified, instead of the final pressure, the solution to determine the pressure p_2 by means of the superheat-tables involves some " cut and try " method, provided no charts are used.

194. The Polytropic Process of Vapors.[3] (a) In Sect. **80**, a polytropic process of any substance is defined as one following the law $PV^n = K$, in which n and K are constants. Hence it follows that for any substance, in any phase, Eqs. (87) and (88) may be used to find the final pressure or volume if the initial value of one of these and the ratio of pressures or volumes are known. As soon as the final values of pressure and specific volume of a vapor have been found the final temperature may be determined from the vapor table, or charts; but note that Eqs. (89) and (90) cannot be used for this purpose, unless the vapor is so highly superheated that the ideal gas laws are followed to a satisfactory degree. With both the initial and final states now completely specified by means of pressure, specific volume and temperature, it is a simple matter by means of the vapor tables or charts to determine the corresponding specific values of heat content, internal energy and entropy.

(b) The *work done* by the vapor during its polytropic expansion is given by Eq. (93); but note that Eq. (94) is not to be used for this purpose.

(c) The *heat absorbed* by the vapor during its polytropic expansion is easily determined by the Simple Energy Equation, Eq. (9), after finding I_1, I_2 and $Wk_{1,\,2}$.

(d) The *pressure-volume* and the *temperature-entropy diagrams* may be plotted as soon as sufficient values of the necessary characteristics are determined for the particular case under consideration. Obviously, the character of the polytropic curve for vapors will depend upon the value of n and also on the regions through which the vapor may pass.

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[3] This section may be omitted in short courses,

195. Throttling of Liquids and Vapors.—(a) As explained in Sect. **85**, the most important cases of throttling, of which there are many, are those that take place under the conditions of steady flow, which are given fully in Sect. **43**. Any throttling process is an irreversible one and consequently, for power purposes, it always involves a loss of available energy, but nevertheless, because of the simplicity of the apparatus required, it may be advantageous to introduce such a process with vapors for a variety of purposes. For example, a *reducing valve* is often introduced between a steam heating main and the radiators because the latter may not be strong enough to stand the pressure in the steam main, and also because the latent heat is greater at the lower pressure and therefore the condensate from the radiators will not carry away as much heat as it would at a higher pressure.

In *refrigeration*, an *expansion valve* is commonly used for the purpose of throttling the liquid ammonia, carbon dioxide or other liquid employed, from an initial state in which the fluid is at high pressure and at a temperature as low as can be attained conveniently by use of cooling water, to a secondary state in which its pressure and temperature are much lower than originally. Here the low temperature is desired because the working substance in a refrigerating plant must become sufficiently reduced in temperature to be able to absorb heat from the things that are to be kept cool.

In *steam engines* the *throttling* of the steam is one of the simplest means of reducing the power of the engine, as will be shown later.

In *research work on vapors* the *throttling* process is one of the most important methods of obtaining fundamental thermodynamic relations.[4]

For *determining the quality of steam the throttling calorimeter* is often used. The discussion of this instrument and the equations for its use will be given in Sect. **196**.

(b) Consider any working substance that is being throttled under steady flow conditions from state *1*, which is fully known, to some observed lower pressure p_2. It is desired to determine the secondary state completely, or, in other words, to find t_2, $\overline{V}_2$, h_2, i_2 and ϕ_2. For the first case to be considered, assume that the initial state of the fluid is as shown by *1* in Fig. 159 and that p_1, x_1 and p_2 are given. If it be assumed that the throttling from *1* to *2* takes place so quickly that there is no time for any appreciable transfer of heat to or from the fluid, the process must be an adiabatic, and consequently Eq. (120) or (121), page 112, is to be used. If the velocities of the fluid at sections *1* and *2* are such that $(v_2^2 - v_1^2) \div 50{,}000$ is an insignificant term, then Eq. (121)

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[4] See *The Total Heat of Saturated Steam* by Dr. Harvey N. Davis, A.S.M.E., Trans., 1908, p. 741; also Mech. Eng., Feb., 1926, p. 144.

applies,—that is, $H_1 = H_2$, or $h_1 = h_2$. If the throttling process takes place through a valve or thin orifice, the transfer of heat to or from the fluid is certain to be very small; but if it takes place through a long porous plug, or if, after throttling, it passes an appreciable distance through a pipe line, or other chamber, before reaching the section *2* at which the readings are made, then this transfer of heat cannot usually be considered zero, but should be included in the equation.

Assuming now that the conditions are such that $h_1 = h_2$, it is apparent that the state *2* becomes completely specified by knowing p_2 and h_2. Then, by the aid of the vapor tables, or charts, the other characteristics at *2* may readily be found by the methods previously given. For example, suppose that x_2 is needed. It follows that:

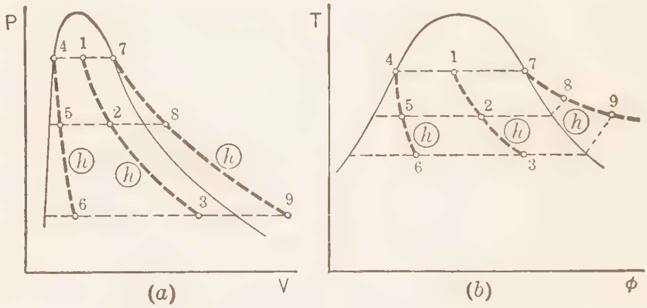

FIG. 159.—Lines of Constant Heat Content.

$$h_1 = h_2 = x_2 r_2 + h_{l_2}$$

$$\therefore x_2 = \frac{h_1 - h_{l_2}}{r_2},$$

in which all of the factors on the right of the equality sign are obtainable from the table. If the vapor *chart* be used, all that is necessary is to obtain the intersection of the line of constant pressure p_2 with the line of constant heat content h_2, and then read all of the values needed.

If the throttling process had been carried to a still lower pressure, say p_3, the state *3* would be found in the same manner as was *2*. This method of procedure could be repeated for as many pressures as desired and then a curve joining these points, such as *1 2 3*, would be called the *line of constant heat content*. It must be carefully noted, however, that in throttling from state *1* to state *3* the vapor did not pass through state *2*. It is only after the turbulence produced in the throttling process has become extremely small that any of the measurements, such as those for temperature or pressure, can be made accurately.

The discussion of graphical representation, given in Sect. **55**, should be carefully reviewed at this point.

(c) If the state of the fluid before adiabatic throttling had been a *saturated liquid*, as shown by point *4* in Fig. 159, the states *5* and *6* obviously would be found by making use of the relation $h_4 = h_5 = h_6$. Then with the temperature or pressure of the saturated liquid at *4* given, the states *5* and *6* would be completely determined as soon as p_5 and p_6 are known. This case corresponds to the passing of a saturated liquid through an *expansion valve* to the cooling coils in a refrigeration plant. If the liquid is a *non-saturated* one upon entering the expansion valve, the solution of the problem is just the same as before except that the proper value of the heat content of the non-saturated liquid must be obtained from a suitable chart. or be calculated by means of Eq. (205).

(d) If a *saturated vapor*, such as is represented by *7* in Fig. 159, be throttled adiabatically, the secondary states would be indicated by *8* and *9*, for the same set of pressures as used for the other two cases shown in this figure. It is apparent from the previous discussion, and from the figure that during these throttling processes the quality, specific volume and entropy all increase, while the temperature and pressure decrease. If the line *7 8 9* should be carried far into the superheated vapor field it will become more and more like an isotherm, because, as was shown in Sect. **85**, if the heat content of an ideal gas remain constant, its temperature must also be constant.

196. Steam Calorimeters.—(a) These instruments are used for determining the quality of wet steam, and since the throttling process, which was discussed in the preceding section, is such an important factor in steam calorimetry, the consideration of these devices is introduced at this point. The *separating* and *throttling* types of calorimeters, or a combination of these in one instrument, usually termed the *universal* type, are the ones most commonly used and will be the only ones treated here.[5]

(b) An old and simple form of the **throttling calorimeter** is shown in Fig. 160. In using it, the sample of steam, received through a "sampling tube," at a known pressure p_1, is made to expand through a small orifice into the chamber C in which the temperature t_2 and pressure p_2 of the expanded vapor are measured. This gives sufficient data for determining the quality x_1 of the original sample, provided that the steam is superheated at the end of the throttling process. Thus, if

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[5] For a discussion of the other types see Carpenter and Diederichs' "Experimental Engineering," John Wiley & Sons; also see A. S. M. E. Code on Instruments and Apparatus.

$t_{\text{sat.}}$ is the saturation temperature corresponding to p_2, the steam in the calorimeter chamber must be superheated an amount

$$D_2 = (t_2 - t_{\text{sat.}})_{p_2}.$$

Then, if the conditions are such that the throttling process may be considered one of constant heat content, it follows that

$$h_1 = h_2, \quad \text{or} \quad x_1 r_1 + h_{l_1} = (h_{\text{sat.}} + \bar{c}_p D_2)_{p_2},$$

from which the initial quality of the sample is

$$x_1 = \frac{h_2 - h_{l_1}}{r_1} = \frac{(h_{\text{sat.}} + \bar{c}_p D_2)_{p_2} - h_{l_1}}{r_1}. \quad . \quad . \quad . \quad (227)$$

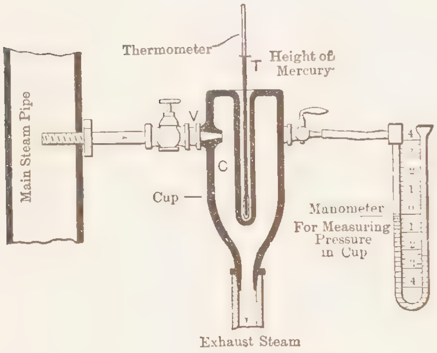

FIG. 160.—Throttling Calorimeter.

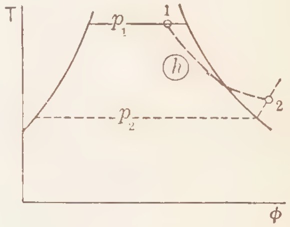

FIG. 161.

Usually the calorimeter exhausts to the atmosphere, and under this condition $\bar{c}_{p_2}$ is about 0.47, as given in Fig. 151. If suitable *Vapor Charts*, such as those that will be discussed in the next section, are available, the quality x_1 may be found directly from them as soon as p_1, p_2 and t_2 have been determined.

(c) On the $T\phi$-diagram in Fig. 161 the state of the steam in the calorimeter is represented by *2*. If through this point a line of constant heat content is drawn and extended until the pressure p_1 is reached, the initial state point *1* is then determined, and the quality x_1 follows. Note that the dotted line *1*, *2* is not a representation of the irreversible process of throttling, but is simply the locus of points that would show the state of the steam in the calorimeter if its pressure were taken at successively higher and higher values. Obviously for the throttling calorimeter to be used, there must be sufficient difference in pressure between p_1 and p_2, and a high enough value of x_1, to produce a reasonable degree of superheat D_2 in the calorimeter. If x_1 is very low, it is apparent that this form of calorimeter cannot be used.

(d) In such a calorimeter the sample of steam passing through the small orifice attains considerable velocity which is soon dissipated by reason of the turbulence produced within the instrument. The jet of steam should not impinge directly on the wall of the thermometer well, as it is important to measure the temperature of the steam in the calorimeter after the velocity energy has been reduced to a very low value, or, more strictly, to the same velocity as that in the pipe from which the sample is taken. Also, there should be no radiation or conduction losses which would affect the temperature reading.

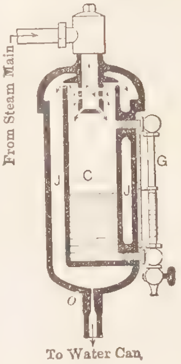

FIG. 162.

(e) A **separating calorimeter,** similar to the one shown in Fig. 162, may be used for determining the quality of steam in case the percentage of moisture is very high. With this type, a sample of steam is first led to the separating-chamber C where the moisture is thrown out and collected, the amount w being shown by the gage glass G; then the resulting dry steam passes into the jacket J and out through the small orifice O, and may be delivered to a can of water in which it is condensed and its weight W determined. This weight may also be found by calibrating the flow through the orifice. Using simultaneous values of W and w, the quality evidently is

$$x = \frac{W}{w + W}. \qquad (228)$$

(f) When the amount of moisture present in the steam is small, the throttling calorimeter gives more accurate results than does the separating one, mainly because it is less affected by radiation losses; but, as already seen, it is not applicable when the quality is low. Hence it is often desirable to have a *combination* or **universal type of calorimeter.** In such a device the steam may be tested in whichever part is best suited for the particular sample that is being analyzed. One arrangement[6] is given in Fig. 163, which shows a very compact form of this instrument that may be constructed easily. When used as a *throttling* calorimeter this design has certain advantages over the older one shown in Fig. 160. Its nozzle is in a shut-off cock, and therefore if it becomes clogged can be easily cleared by reversing the cock. The temperature readings are more accurate because the thermometer well is provided with ribs, is located in a jacketed chamber, and is not impinged upon by the incoming jet. Also the *separating* portion is an improvement over the form shown in Fig. 162 because the chamber, gage glass and blow-off cock sustain only steam that has been throttled to a low pressure, instead of being subjected to the pressure of the original sample, and because the radiation correction is less and better separation of moisture occurs. Furthermore, the nozzle can be calibrated so that the entire weight of vapor flowing through the instrument per unit of time is known, and hence no provision need be made for condensing and weighing the discharge.

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[6] See "Steam Calorimetry," by G. B. Upton, *Sibley Journal of Engineering,* March, 1921.

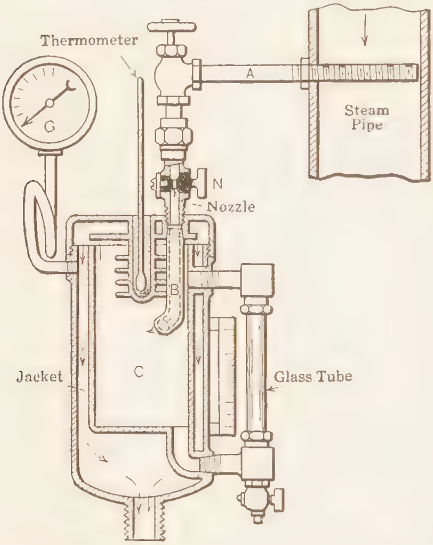

FIG. 163.—Universal Type of Calorimeter.

(**g**) When the quality of the steam to be measured is sufficiently high to permit the instrument shown in Fig. 163 to be used as a *throttling* calorimeter alone, the discussion given in (**b**) of this section applies without change; but, when there is moisture present after throttling the *separating* element must also be employed, in which case the following treatment should be used.

Let the weight of steam passing through the orifice to the separating part of the calorimeter be W_2 lb. per hour, and let w_2 represent the weight of that part of this flow that has been separated out as a dry vapor after throttling. Then the quality of the steam, after throttling from the steam-main pressure p_1 to the calorimeter pressure p_2, must be $x_2 = w_2/W_2$. Consequently with p_2 and x_2 known, the value of h_2 may be found. Since $h_1 = h_2$, and p_1 is observed, then x_1, the quality of the steam in the sampling pipe, is found directly from the charts, or by the following equation:

$$x_1 = \frac{h_2 - h_{l_1}}{r_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (229)$$

The states *1* and *2* may be represented as shown in Fig. 164.

In order to use the separating portion of this calorimeter, the actual rate of flow through the particular size of orifice used must first be obtained by a test run, preferably with a pressure p_1 and quality x_1 that agree fairly closely with those in the steam main. For any other pressure p' the flow may be taken without much error as directly proportional to p'/p_1.

(h) The chief source of difficulty in using any steam calorimeter is to **obtain a true sample**. The best means of doing this are fully discussed in various papers dealing with such experimental work.[7] The sample should be taken from a vertical pipe, preferably with downward flow, by a sampling tube having openings as indicated in Fig. 163.

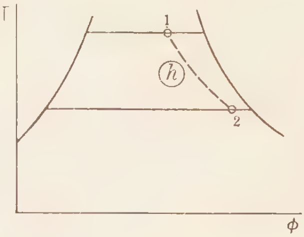

FIG. 164.

197. Vapor Charts.—(a) The pressure-volume and the temperature-entropy diagrams will always be of the utmost importance in the analysis of thermodynamic problems, but for the purpose of obtaining simultaneous values of the six important characteristics of a vapor (p, $\overline{V}$, t, h, i and ϕ) in any specified state, other sets of coordinate axes are usually found more suitable. If a sufficiently large number of points for these properties are plotted in a clear manner and curves are drawn through them the resultant chart becomes of the greatest aid in numerical work involving vapors.

(b) **The Temperature-Entropy Chart** may have lines of constant pressure, volume, heat content and quality drawn upon it, as previously shown individually, and as now represented collectively in Fig. 165. However, the constant volume lines are usually omitted, as they do not make very satisfactory intersections with the constant pressure ones, especially in the superheated region; and even in the region of mixtures these intersections are at very bad angles when the pressures are low. Furthermore the constant entropy lines and those of constant heat content do not make very satisfactory intersections in the region of mixtures at high pressures. For these reasons the $T\phi$-diagram, while of great value for analytical work, is not generally used for obtaining numerical results. Figure 165 is the diagram drawn to scale for steam.

(c) **The Mollier Charts.**—In 1904 Dr. R. Mollier published[8] a number of new charts that have been of great service in solving problems involving vapors. The one that is most often used is that in which the coordinates are the specific values of heat content and of entropy, and which covers the range of values most needed in engineering applications. A typical form of this diagram for steam is shown in Fig. 166; and Plate I in the Appendix is a corresponding chart from which values may be read with sufficient accuracy for most purposes. From these charts it may be observed that the intersections of all the lines representing constant values of pressure, temperature, quality, heat content and entropy are very definite. However, the constant volume lines do not plot at all well on this form of chart, hence Dr. Mollier also used another diagram, as shown by Fig. 167, in which pressure and heat content are the coordinates, and in which the volume lines are satisfactory. However, in this latter chart the constant entropy lines do not give good intersections with the other lines if the scales are uniform, hence the pressure scale is often made logarithmic, as was

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[7] *Ibid.*; also A.S.M.E. Code on Instruments and Apparatus.

[8] See "Zeitschrift des Vereines deutscher Ingenieure," 1904, p. 271.

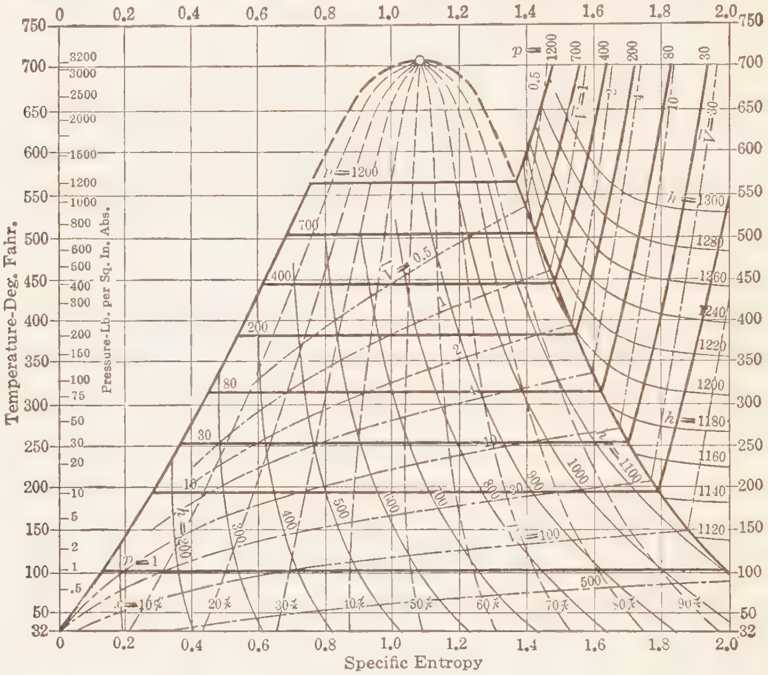

Fig. 165.—Temperature-Entropy Chart for Steam.

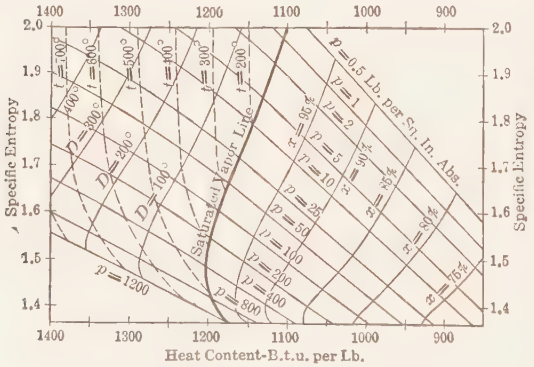

Fig. 166.—Mollier Chart for ϕ and h.

done in the Chart for Ammonia that was prepared by the U. S. Bureau of Standards,[9] parts of which are shown in Plate IV in the Appendix.

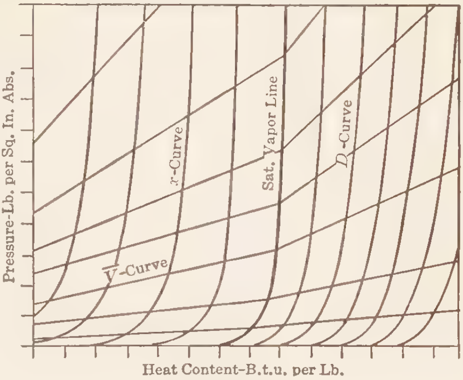

FIG. 167.—Mollier Chart for p and h.

(d) **The Ellenwood Charts** were first published in 1914. As illustrated in Fig. 168, the **main chart** is one in which heat content and specific volume are coordinates, and on which are drawn lines of constant entropy, pressure and quality. In the lower part of it a curve of heat content of the liquid, $h_{\text{liq.}}$-curve, has been added and along this line has been placed a scale of saturation temperatures. Thus, extending a constant pressure line to intersect this latter curve establishes a point whose ordinate is $h_{\text{liq.}}$ for this pressure, and whose location on this curve determines the corresponding value of t_v. A **supplementary chart** with $AP\bar{V}$ and $\bar{V}$ as coordinates, as shown in Fig. 169, is also provided.

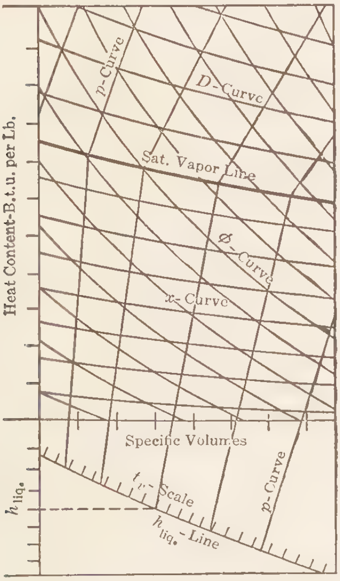

FIG. 168.—Ellenwood Chart of h and $\bar{V}$

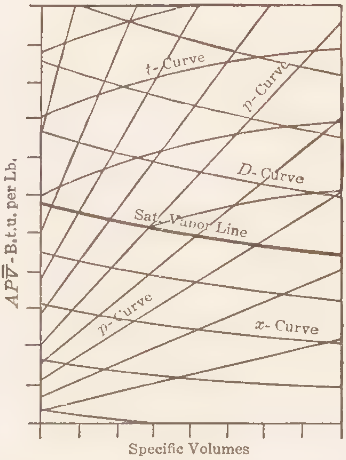

FIG. 169.—Ellenwood Chart of $AP\bar{V}$ and $\bar{V}$.

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[9] See Circular No. 142, U. S. Bureau of Standards.

Thus, from the two charts may be obtained values of p, $\overline{V}$, t, ϕ, h, $AP\overline{V}$ and i for any state of a vapor that is within the range covered, and in addition the values of the heat content of the saturated liquid may also be determined. Since specific volumes are given, these charts are particularly useful in connection with steam turbine calculations and in any other problems that involve either volumes or their ratios, whether for expansions or compressions.

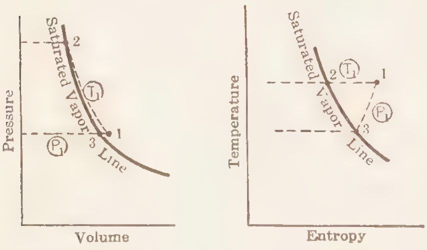
In Plates II and III in the Appendix these charts are shown in greater detail. They are, however, greatly reduced in size from the original and more accurate ones which were published in book form,[10] and in which the first chart covers twelve large pages, and the second two pages.

Unless otherwise stated, the term " Ellenwood Chart " will be used as referring to the main one which has $\overline{V}$ and h as coordinates.

(e) Vapor charts may also be constructed with other characteristics as coordinates. But, since those already given are typical, and as they are the ones most generally used, others will not be considered in this text.

198. Moisture in the Atmosphere.—(a) The earth's atmosphere consists of a mixture of gases and a small quantity of water vapor. The amount of this vapor per unit volume of the atmosphere varies widely from day to day, season to season, and place to place. Since the atmosphere is an important factor in many engineering problems, and since its vapor, commonly called its moisture or **humidity**, must usually be considered, certain technical terms relating to the latter need to be defined.

(b) Since this atmospheric vapor is generally in a superheated condition at a very low pressure, its state may be represented by some such point as *1* in both the $T\phi$- and PV-diagrams in Fig. 170. Through this point draw a constant temperature line until it intersects the saturated vapor curve at *2*; and let the respective pressures at these points be p_1 and p_2, and the specific densities be δ_1 and δ_2. Based on these properties is a very useful relationship called the **Relative Humidity**, $R.H.$ This was originally defined as the *ratio of the actual*



(*a*) (*b*)

FIG. 170.

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[10] Ellenwood's "Steam Charts," John Wiley & Sons, publishers.

weight of moisture in the atmosphere per unit volume to the weight required for saturation at the same temperature; i.e., $R.H. = \delta_1/\delta_2 = \overline{V}_2/\overline{V}_1$.

Since the absolute vapor pressure in the atmosphere is very low, always below 2 inches of mercury, the isothermal curve *1 2* almost exactly follows the law $PV = a\ const.$ If this law is assumed to apply accurately, then, $R.H. = p_1/p_2$. Thus, on this basis, the *Relative Humidity* may also be defined as the *ratio of the actual pressure of the vapor in the atmosphere to the saturation pressure at the same temperature.* As the pressures of vapors are usually more readily and more accurately determined than their densities, this second definition is becoming more generally preferred and therefore is the one recommended in this text.

(c) The relative humidity of the atmosphere can be readily determined experimentally by methods which need not be discussed at this point; and, the saturation pressure p_2 corresponding to the actual atmospheric temperature t_1, may be found from a vapor table or chart. Then, the actual vapor pressure corresponding to the given conditions follows from

$$p_1 = R.H. \times p_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (230)$$

The higher the relative humidity for the given temperature the nearer will point *1* be to point *2* and the more nearly will the actual density δ_1 of the vapor coincide with δ_2. If $R.H.$ is unity, $p_1 = p_2$ and $\delta_1 = \delta_2$.

(d) The **dew point** is that temperature at which the condensation of moisture in the atmosphere (or, in general, in any mixture) will begin, if cooling takes place at constant pressure. Since Dalton's law of partial pressures holds well for the atmosphere, it follows that when this reduction of temperature takes place, the pressure of the *vapor* also remains constant. This constant pressure process is shown in Fig. 170 by the line *1 3*. As the cooling proceeds, the superheat decreases, finally reaching zero at *3*, at which point the vapor has become saturated, the pressure being the same as that originally at *1*. Any further abstraction of heat will cause condensation, hence t_3 *is the dew point* for the case here represented. With a higher value of relative humidity than that shown, the dew point is raised, and *vice versa.*

(e) When *heating a mixture of air and water vapor at constant pressure* the amount of heat absorbed is equal to the increase in the heat content of the mixture. The gain in the heat content of the mixture may be found most readily by applying Eq. (55) or (56) to the dry air, and then adding to this result the increase in the heat content of the vapor.

For pressures less than 5 lb. per sq. in. abs. it has been shown by

Berry[11] that the value of the heat content of superheated steam, in B.t.u. per pound, is given to the nearest B.t.u. by the equation,

$$h = 1057 + 0.46t,$$

in which t is temperature of the vapor in deg. fahr. This equation will be found to be very convenient in many problems involving a mixture of air and low pressure steam.

(f) The humidity of the air is involved in problems in heating and ventilating of buildings and in air conditioning in general, in meteorology, in the design of cooling towers and ponds, in evaporating or drying processes, etc. It can be measured by means of *hygrometers* (usually of the dew-point, chemical absorption, or evaporative types). The ordinary *wet-bulb psychrometer* makes use of readings that are obtained from *dry-bulb and wet-bulb thermometers* and is the form of hygrometer that is generally used by engineers. The wet bulb is covered with moistened cloth, or wicking, over which air is made to flow. If the air is not fully saturated this results in a lower temperature reading than that given by the dry-bulb thermometer, and the dryer the air the greater will be the difference in temperature. With these two temperatures and the barometric pressure known, the relative humidity can be computed, or may be read directly from Psychrometric Tables or Charts.

The various properties of air mixed with water vapor may be obtained for the different temperatures from special tables and curves.[12]

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[11] See: The Specific Total Heat of Low-Pressure Steam, by C. Harold Berry, Power, March 17, 1925.

[12] For various charts and tables see:

Rational Psychrometric Formula, by W. H. Carrier, Trans. A.S.M.E., 1911.
Psychrometric Tables, by C. F. Marvin, U. S. Weather Bureau Bulletin 235.
Smithsonian Physical Tables; also, Meteorological Tables.
Am. Soc. of Heating and Ventilating Engineers, "Guide" (annual).
Mechanical Engineering Handbooks.

CHAPTER XVII

THE SIMPLE CYCLES OF VAPOR ENGINES AND OF VAPORS

199. Distinction between Engine and Vapor Cycles.—(a) In the case of the usual internal combustion engine all the thermodynamic processes of major importance take place within the engine proper, which thus constitutes the complete power plant; hence, only the conditions prevailing within the engine itself are usually considered in analyzing the ideal and actual cycles of this type of machine. But when a *vapor* is the working substance, there are used, in addition to the prime mover, certain other pieces of apparatus that are of importance and that may have to be considered since the fluid passes successively through them. For example, if steam is the working substance, it is necessary to have a *boiler* to generate the vapor before it can be conducted by the *piping* to the *prime mover*; and, after some of the heat has been transformed into work in the latter, a *condenser*, or its equivalent, must be used to reduce the exhaust steam to the liquid state; and, finally, suitable *pumps* must be employed for returning the resulting condensate to the boiler, thus completing a cycle for the fluid. If the steam is exhausted into the atmosphere, where it condenses without recovery, an equivalent amount of liquid from some new source must be supplied to the boiler to complete the cycle.

Thus, if a vapor is the working substance, and if the circuit of the fluid through the whole plant is to be analyzed completely, the processes occurring in *all* the apparatus involved must be considered, and the resultant cycle may be called the "*complete cycle of the vapor*," or briefly the "**Vapor Cycle.**" If, however, the flow of the fluid through the prime mover alone is under consideration, as independent of the rest of the plant, then only the **Engine Cycle** is involved. It should be noted that all of the fluid necessarily need not leave the engine at the same point to comply completely with the conception of the "flow through the engine" in the foregoing definitions.[1]

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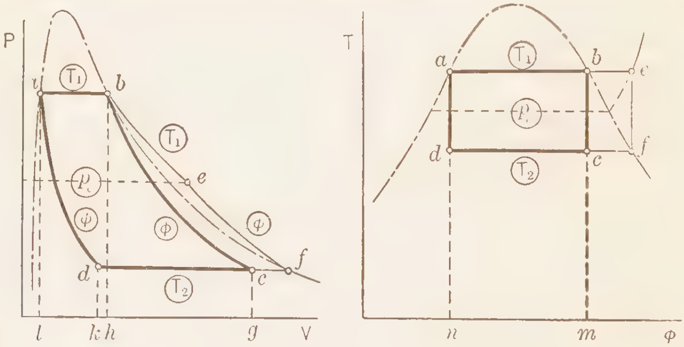
[1] In the following chapter there will be discussed certain important cycles in which some of the working substance is extracted at various points in its passage through the engine.

The distinction between the cycle of the engine and that of the vapor has heretofore been ignored in most engineering work, but with the introduction of high pressures, and of the more complex vapor cycles involved in the steam circuit, it now seems desirable to differentiate between these two cases.

(**b**) To obtain the *best commercial results* in designing or operating a power plant, or a prime mover, it is essential that the engineer responsible shall be able to analyze the thermodynamic processes involved, and in addition have a thorough knowledge of the probable costs and performances that should be expected to result with the apparatus used. The necessary analyses are best made by applying the thermodynamic processes of the previous chapters, considering them as carried out in an ideal manner, and then estimating, or determining, how closely the actual apparatus approach their ideal cycles, the more important of which will now be discussed.

200. The Carnot Vapor Cycle.—(a) The Carnot cycle for any working substance, as explained in Sect. **119**, must always be a rectangle on the temperature-entropy diagram, since it consists of two isothermals and two reversible adiabatics. The shape of this cycle when drawn on the pressure-volume diagram will of necessity depend upon the states of the vapor at the beginning and end of the isothermal processes. Thus, in Fig. 171 are shown three possible Carnot cycles, *abcd*, *abefd*, and *befc*. In the first of these the isothermal expansion is represented by *ab*, which also shows the process of vaporization at constant pressure. In the second case, the isothermal expansion line is *abe*, which has the same portion *ab* as before, but in addition the expansion continues into the superheated region to *e*. For the third cycle shown, the isothermal expansion line is *be*, which is wholly within the superheated region.



(*a*) (*b*)

FIG. 171.—The Carnot Vapor Cycle.

(**b**) The efficiency of each of these ideal vapor cycles is the same, viz., $\frac{T_1 - T_2}{T_1}$, as shown in Sect. **119** (**e**).

(**c**) The *heat supplied*, Q_1, during any Carnot cycle may be found by multiplying the isothermal increase in entropy, $(\Delta\Phi)_T$, by the absolute temperature T_1 of the fluid during that expansion. Hence, for this case,

$$Q_1 = T_1(\Delta\Phi)_T.$$

Note that $(\Delta\Phi)_T$ depends upon both the weight of fluid and the amount of isothermal expansion involved, and that in case the cycle considered is the one shown by *abcd* in Fig. 171, the value of Q_1 becomes simply the latent heat of vaporization at the constant temperature T_1.

The *heat rejected* is obviously $Q_2 = T_2(\Delta\Phi)_T$.

(**d**) The *net work per cycle* may be found for any Carnot cycle by any of the three following methods:

$$(1)\ \text{Net work} = Q_1 - Q_2 = (T_1 - T_2)(\Delta\Phi)_T. \qquad (231a)$$

$$(2)\ \text{Net work} = Q_1 \times \text{Eff.} = \frac{Q_1(T_1 - T_2)}{T_1}. \qquad (231b)$$

(3) Net work = Algebraic sum of the amounts of the work done by the fluid during each of the processes in the cycle . . . . (231*c*)

Since the only processes involved are the isentropics and isothermals, the application of the methods given in Sects. **192** and **193** will determine the heat absorbed and the work done during each process, whether the vapor is superheated or not.

(**e**) The *apparatus required* to carry out the ideal cycle *abcd* of Fig. 171 is that which was described in Sect. **119** (**b**). Consider the cylinder to contain initially a unit weight of the saturated liquid in a state represented by *a*. First apply the hot body to the cylinder head so that the liquid is vaporized at constant temperature, and therefore at constant pressure. This will cause an expansion and, when the entire weight is vaporized, the piston will have moved far enough to cause its displacement to be equal to $V_b - V_a$, and the work done during this process will be represented by the area *abhl*. Next apply the heat insulator to the cylinder head so that the adiabatic expansion *bc* may take place, thus delivering to the piston the amount of work shown by the area *bcgh*. Now remove the heat insulator and apply the cold body for abstracting heat from the vapor in such a manner that it will be condensed at constant temperature, and consequently at constant pressure, as shown by the line *cd*. The abstraction of heat must cease when the quality of the mixture in the cylinder has become equal to x_d, such that adiabatic compression from p_d to p_a

will result in causing the mixture to become a saturated liquid, as initially. Evidently the energy expended upon the piston during the isothermal and adiabatic compressions is represented by the area *cdalkg*. Consequently the net area *abcd* represents the net work delivered to the piston by the vapor during the cycle.

(f) It is very important to note that, as long as the given processes making up the ideal vapor cycle *abcd* are carried out for a given weight of fluid, the net work developed per pound of fluid completing this cycle will be the same whether all the processes take place in one cylinder of an engine, or in separate organs. Thus, if an attempt were actually made to try to follow the Carnot cycle *abcd* with modern apparatus, the complete vapor circuit would be obtained in the following manner: A suitable boiler would continuously vaporize the fluid at the constant temperature T_1 and deliver it to a pipe line leading to the prime mover. The engine would be thermally insulated so that the adiabatic expansion would take place within it until the exhaust temperature T_2 is reached, when the fluid would be delivered to a suitable condenser. In this condenser sufficient latent heat would be removed at the temperature T_2 to give the quality x_d. The mixture with this quality would then be delivered to a suitable pump, or pumps, in which the adiabatic compression would take place with the delivery into the boiler of all the fluid in state *a*. The prime mover would probably be a turbine, and the pumps would undoubtedly be centrifugal ones, so that the flow of the fluid through all of the apparatus involved would be continuous.

It is apparent that the particular weight of working substance considered in Fig. 171 for the single organ need be only a portion of the total amount of fluid that might be kept continuously circulating through the several organs. Just so long as each portion of fluid is made to undergo the processes specified by *abcd* in the figure, the net work delivered by each pound thus circulating through the ideal machines will be the same.

(g) As will be seen later, **actual vapor engines** of all types generally have higher engine efficiencies when using a fluid having a considerable amount of *superheat*, than they have when the vapor is merely in the saturated condition. But the apparatus required for extending the isothermal expansion of the Carnot cycle into the superheated region would be very complicated. Furthermore, comparison of cycles *abcfd* and *abcd* of Fig. 171 will show that with the former the mean effective pressure is lower, i.e., the size of unit will be larger for a given power output, than with the latter. These facts regarding the use of superheated steam in the Carnot cycle make it one that cannot be easily approached in practice. Whether any superheat is used or not there still remains the difficulty of effecting the adiabatic compression of the wet vapor. It is for these reasons that other vapor cycles which do not have these difficulties are the ones most often followed in power plant work.

(h) The performance of **actual power plants** is sometimes compared with that which would occur if the ideal Carnot cycle could be used with T_1 and T_2 taken equal respectively to the temperature of the boiler furnace (say, 2500 deg. fahr.) and to that of the circulating water going to the condenser (e.g., 70 deg. fahr.). Such a comparison shows how much more heat energy might be transformed into work if it were possible to use a vapor having a temperature very close to that of the furnace. Unfortunately, however, materials that are available at present for making boilers, piping, and prime movers, cannot apparently be used with safety much above 800 deg. fahr.

The *Carnot Cycle* is sometimes used as a standard of comparison for *actual engines* using vapors, and in such cases the upper and lower temperatures are taken respectively as those of the vapor at the inlet and outlet of the machine; but generally the other ideal engine cycles which are more closely followed by the actual prime movers serve as better standards for comparison than does the Carnot. These other cycles will be considered in this and the succeeding chapter.

201. Significance of the Difference in Heat Content at Constant Entropy $(\Delta H)_\phi$.—**(a)** Before proceeding further with engine or vapor cycles it is very desirable to obtain a clear conception of the full significance of the difference in the heat content of any working substance between any two states having the same entropy but different pressures.

(b) Consider a pound of any vapor, having the known volume $\overline{V}_a$, admitted without heat transfer and at the constant pressure p_a to an engine, either reciprocating or turbine, in which this fluid expands adiabatically and reversibly (and therefore isentropically) to some specified pressure, p_b, at which it is exhausted from the engine without heat transfer; also assume that the differences in the elevations and in the velocities of the fluid at entrance and exit to be negligibly small. Then, since the initial state of the vapor is known when p_a and $\overline{V}_a$ are given, the value ϕ_a may be determined by the aid of a vapor table, or chart; and, with ϕ_b equal to ϕ_a and with p_b known, the value of h_b may be readily determined.[2]

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[2] The initial state may be specified by characteristics other than p_a and $\overline{V}_a$,—thus, most commonly the state of a superheated vapor entering an engine is specified by p_a and t_a, or by p_a and D_a; but, in order to obtain a graphical representation of the work done by this fluid in passing through the engine in the manner specified

The *work done* by the pound of vapor during its constant pressure admission to the engine is represented by the area *lago* in Fig. 172 and is equal to $P_a\bar{V}_a$ ft-lb., or $AP_a\bar{V}_a$ B.t.u. During the isentropic expansion from a to b the work done by the vapor is shown by the area *abfg,* and its magnitude in B.t.u. is equal to $i_a - i_b$, because the work done by any working substance during a reversible adiabatic expansion is equal to its loss of internal energy. Since the substance having the volume V_b is exhausted from the engine at the constant pressure p_b, an amount of work is done upon it during this process equal to $AP_b\bar{V}_b$ B.t.u. as shown by the area *bkof.* Consequently the *net work* done by this fluid in passing through the engine under these conditions must be that represented by the area *labk.* Thus, in B.t.u., for any weight w,

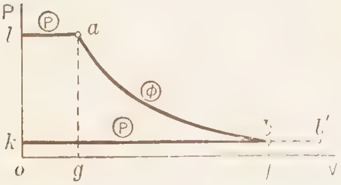


FIG. 172.

$$Wk_{labk} = AP_aV_a + I_a - I_b - AP_bV_b, \quad . \quad . \quad . \quad (232a)$$

$$= H_a - H_b = w(h_a - h_b). \quad . \quad . \quad . \quad . \quad (232b)$$

(c) In order to grasp the full significance of Eq. (232), assume that the engine is one in which the expansion takes place between the same initial state and final pressure as before, that there is no loss due to radiation, conduction or convection, but that, as in the case of the real engine, there is *turbulence and fluid friction* that cause the adiabatic expansion to be *irreversible.* Then in this case, at the time the fluid is exhausted, it would be in some state such as b' (in Fig. 172) in which the volume, internal energy and heat content will be greater than those at state b, for the reasons explained in Sect. **44** (1); and, consequently, according to Eq. (23), the work delivered to the machine by the fluid would be $H_a - H'_b$, which would be less than $H_a - H_b$.

In other words, Eq. (232) expresses the fact that, with negligible difference in the energy of the fluid at exit and entrance due to velocity and elevation, the **maximum work** that may be obtained from any working substance flowing through a machine that is thermally insulated from this fluid is equal to the heat content at the entering state minus its heat content at the exhaust pressure and at the initial entropy, or, expressed mathematically, it equals $(\Delta H)_\phi$.

(d) If the engine is a reciprocating one, the value of $(\Delta H)_\phi$ would

above, the volume $\bar{V}_a$ is needed, and its value may be readily determined to agree with p_a and t_a.

represent the energy that could be *delivered to the piston* under ideal conditions. In case a turbine is used, $(\Delta H)_\phi$ is then the energy that could be *delivered to the fluid* itself to increase its velocity while passing through an ideal nozzle,—the resulting kinetic energy being converted into useful work by the action of the jet on the turbine blades which move at a suitable velocity to absorb this energy and deliver it to the rotating elements of the turbine. It is important to note, however, that only in *ideal* machines of either type is it possible to have the processes take place as assumed, because in real engines and turbines, during admission, expansion and exhaust there will be heat transfers, fluid friction and turbulence, all of which decrease the work that may be done by the fluid.

(**e**) It is now evident that the drop in specific heat content at constant entropy, $(\Delta h)_\phi$, may be spoken of as the **available mechanical energy** *per pound of working substance* flowing through any machine that has constant pressure admission and rejection of the fluid, and in which there is no appreciable change in energy due to variation in velocity or in elevation, and no transfer of heat to or from this fluid.

Note that, in order to fulfill these conditions, steady flow is essential for the ideal turbine, but not for the ideal reciprocating engine. In the latter the flow is intermittent, but it must be without turbulence or fluid friction. For the machine having steady flow, the same conclusion regarding the meaning of $(\Delta h)_\phi$ may be seen to follow from Eq. (24), p. 47, if E'_h is zero and $\phi_1 = \phi_2$, as they must for the case of a reversible adiabatic flow.

(**f**) The significance of the term $(\Delta h)_\phi$ having been established, it is now pertinent to inquire what phrase is suitable to apply to it. Unfortunately there is no short one that is entirely satisfactory. The terms "Adiabatic Heat Drop," "Isentropic Heat Drop," and "Heat Drop" are frequently used in the literature; and until a better name for h than "Heat Content" or "Total Heat" is found, some or all of the foregoing expressions will probably continue to be used. However, they are all lacking in precision and are therefore less satisfactory than the use of the symbol $(\Delta h)_\phi$ itself, or the term "**isentropic drop in heat content.**"

(**g**) From the discussion already given, and from Fig. 172, it is apparent that for a *reversed direction of flow* of the fluid through the ideal machine, with the other conditions remaining as specified in (**b**), that the *increase in heat content* of the fluid at constant entropy then represents the **minimum work** that must be expended *upon* the working substance to force it without heat transfer from one container to another at higher pressure. Evidently, for this case the machine must be some kind of a compressor, or pump, instead of a prime mover.

(h) Referring to Fig. 172 it is seen that the area $labk = -\int_{P_a}^{P_b} VdP$. In case the fluid is one, such as an ideal gas, that has a simple relation existing between P and V during a reversible adiabatic expansion (or compression) this integral may be readily evaluated. In general, however, most of the vapors used do not have this simple relation, and it is therefore much easier to determine the values of $h_a - h_b$ for the same entropy from **Vapor Tables or Charts,** and hence this method is the one usually followed. This shows the importance of having the constant entropy lines on such diagrams.

(i) Before proceeding further, it should be carefully observed that *in Fig. 172 the fluid does not complete a cycle.* It enters the ideal machine in state a and leaves it in state b which is entirely different except as to its entropy. The lines la and bk merely serve to show, by the areas under them, the work done during the admission and rejection of the fluid at the two constant pressures, p_a and p_b, respectively. Later it will be seen that this same figure may also be taken as representing the indicator diagram that would be obtained from an ideal reciprocating engine having no clearance. In other sections there will be treated complete cycles of a known amount of fluid, and it is important to distinguish between the two kinds of pressure-volume diagrams involved. When considering such figures as 172 to represent the indicator diagram of an ideal engine without clearance, it is apparent that the *engine* has completed a cycle, although the *fluid* has not.

202. Ideal and Actual Steam Rates.—(a) The **ideal steam rate** is defined as the quantity of steam required to produce a unit amount of work in the ideal engine. When a unit weight of steam is considered to flow through any ideal engine, it develops a certain amount of work, which is called the work of the ideal engine cycle, and which will be designated as Wk_i. If this value is expressed in B.t.u. per pound, it is evident that, in order that this ideal engine may deliver an amount of energy equal to a horse power-hour (2545 B.t.u.), or a kilowatt-hour (3413 B.t.u.), a certain weight of steam would be required. This quantity is the *ideal steam rate,* and its value is

$$w_i = \frac{2545}{Wk_i} \text{ lb. per hp-hr.,} \quad . \quad . \quad . \quad . \quad . \quad (233a)$$

or

$$w_{i_k} = \frac{3413}{Wk_i} \text{ lb. per kw-hr.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (233b)$$

It is evident from these equations that the engine cycle which yields the largest amount of work per unit weight of fluid will have the smallest ideal steam rate, and *vice versa.*

(b) The **actual steam rate** is the weight of steam per unit of energy developed by the prime mover, or by the unit consisting of the prime mover and the machine which it drives, depending on which is under consideration; and its value is obtained by dividing the total amount of steam used in a given time by the energy output during the same time, both of which must be determined by actual test.

With reciprocating engines the *unit of energy* is generally taken as either the i.hp-hr. or the b.hp-hr.; with turbines it may be the b.hp-hr. or the kw-hr. delivered; and it may be either of the latter if the output of a combined unit is under consideration. Consequently the actual *steam rate* may be expressed as so many pounds of steam per i.hp-hr., per b.hp-hr., or per kw-hr.

(c) The **estimated steam rate** of a proposed or actual prime mover may be determined very closely in many cases by first calculating the ideal steam rate and then applying a factor to allow for the probable losses of the actual engine as compared with the ideal. For example, a turbine manufacturer may have to guarantee the steam rate for a proposed turbine and may have available no actual test data from a similar machine operating under the same conditions. In such case a capable engineer can make an estimate of the steam rate by the method just stated, and if he has suitable knowledge of the proper correction factors to use, his results will be found to agree very closely with those obtained subsequently by test of the actual turbine.

(d) The term "*water rate*" has been very commonly used in the past in the same sense that *steam rate* is now employed. However, the American Society of Mechanical Engineers has recently recommended that the latter term be used because it is the steam and not the water that operates the prime mover. The custom of using the older term originated because the most accurate method of measuring the amount of steam passing through the machine is to condense the exhaust steam and then weigh the resulting water.

203. The Rankine Engine Cycle.—(a) The *Rankine engine*, or *complete expansion engine*, is one that admits the working substance at constant pressure, then expands it isentropically to some lower pressure, and finally exhausts all of it at this latter pressure, all three processes taking place without any transfer of heat to or from the fluid as it passes through the engine. It is evident from the discussion in Sect. **201** that Fig. 172 may be taken as the indicator diagram of such an ideal engine, and consequently the work obtainable from each pound of fluid passing through the Rankine engine is

$$Wk_{(\text{Rankine eng.})} = h_a - h_b \quad . \quad . \quad . \quad . \quad . \quad . \quad (234)$$

(**b**) It is also apparent that the Rankine engine may be taken as the *standard* for an actual prime mover of either the turbine or reciprocating type, provided the attempt is made to have all of the fluid pass through the machine in the same manner specified in (**a**). Since the work obtainable from each pound of steam passing through this ideal engine is $h_a - h_b$ it follows that the *ideal steam rate* of the Rankine engine is

$$w_{i(\text{Rankine eng.})} = \frac{2545}{h_a - h_b} \text{ lb. per hp-hr., } \quad . \quad . \quad . \quad (235a)$$

or

$$w_{i_k(\text{Rankine eng.})} = \frac{3413}{h_a - h_b} \text{ lb. per kw-hr. } \quad . \quad . \quad . \quad (235b)$$

(**c**) In the strict sense, the line *ab* in Fig. 173, is all that may be drawn on the $T\phi$-diagram for the Rankine *engine* cycle. This is because a definite weight of vapor enters the ideal engine in the state represented by *a*, and after expanding isentropically therein it is exhausted in the state *b*, and this is all that occurs while this weight of fluid is within the engine itself. For the case here shown it is assumed that the vapor enters the engine in a superheated condition. The constant pressure lines through *a* and *b* are shown dotted so that for any other initial quality, and for the same initial and final pressures, the proper vertical line may be readily imagined.

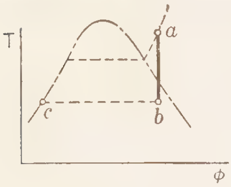


Fig. 173.

The state *c* in this figure represents the saturated liquid at the exhaust pressure and therefore it shows the condition of the fluid after coming from the condenser, but not from the engine. The significance of the *areas* of this diagram will be apparent later when dealing with the Rankine *vapor* cycle in Sect. **204** (**d**).

(**d**) *The amount of energy supplied* to a Rankine engine cannot be established until a decision is made as to what portion of the energy of the exhaust steam shall be credited to the engine. In most large steam plants the exhaust steam is delivered to a condenser, and consequently the latent heat of this vapor is wasted by being transferred to the circulating water. On the other hand, if the condenser is of the "surface" type, the condensate, or liquid resulting from condensing the steam is in such a condition that it may be used again, but its heat content is appreciably above zero, and consequently it has been agreed by engineers that the engine should be *credited* with the *heat content of the saturated liquid at exhaust pressure.*

Then, with *a* representing the state of the steam entering the engine throttle, and c the condition of the condensate corresponding to saturated liquid at exhaust pressure, the *energy supplied* to the Rankine engine by the boiler and its pumps is $h_a - h_c$ B.t.u. per pound of steam. Hence, the *efficiency of the ideal Rankine engine cycle is*

$$e_{i(\text{Rankine eng.})} = \frac{h_a - h_b}{h_a - h_c}. \qquad \ldots \ldots \quad (236)$$

This same standard is commonly used regardless of what is done with the heat content of the condensate, for then the performances of all Rankine engines operating with the same initial steam condition and the same final pressure are directly comparable. Thus, on this basis the engine which exhausts to the atmosphere, or to a heating system, may be compared with one whose condensate is returned to the boiler.

204. The Rankine Vapor Cycle.—(a) The distinction between the Rankine *vapor* cycle and the Rankine *engine* cycle is due to the fact, as explained in Sect. **199**, that in the former the working substance passes through a complete circuit, while in the latter the fluid simply passes through the Rankine engine without being brought back to its initial condition. In the vapor circuit illustrated diagrammatically in Fig. 174, the boiler (including the superheater) and the condenser are the only kinds of apparatus used to transfer heat to or from the fluid. In the boiler the volume of the working substance is increased many times, while in the condenser an enormous reduction takes place. In this circuit there are also the prime mover and the pumps, these being the only pieces of apparatus that give out or absorb mechanical energy. In the engine the volume of the working substance increases greatly and

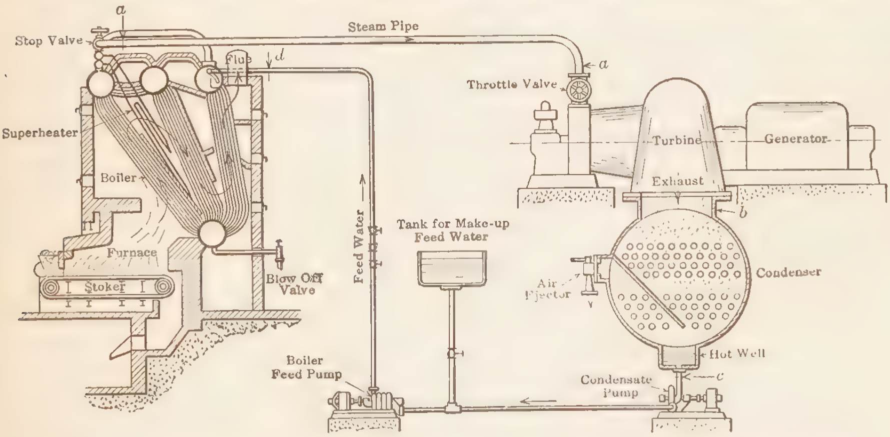


FIG. 174.—Diagrammatic Outline of Apparatus Involved in a Rankine Vapor Cycle.

in the pumps, which force the liquid from the low condenser pressure into the boiler at a much higher pressure, the volume is relatively very small. The piping is used merely as a means of transferring the fluid from one piece of apparatus to another.

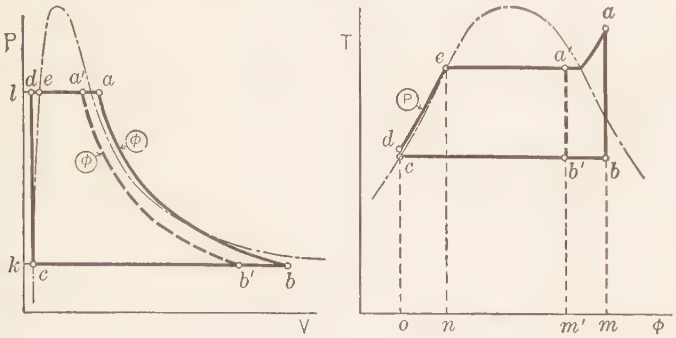
In the actual plant the *maximum pressure* in the entire circuit is at the delivery of the boiler feed pump, because to produce flow there must be pressure drops as the fluid passes through the feed line, the boiler, the superheater, and the steam pipe line. These drops are kept relatively small. In the prime mover a very *large* drop in pressure occurs with an enormous increase in volume, and this expansion is so controlled that there is developed a large amount of mechanical energy.

(**b**) In the *ideal* plant there are no losses due to fluid friction, leakage or turbulence, and no heat transfers of any kind except in the boiler and condenser. Consequently in the ideal vapor cycle the *net work* done by a given weight of fluid completing the cycle must be equal to the heat absorbed from the boiler minus that given to the condenser. This *net work* must also be equal to the work done by this fluid while passing through the ideal engine minus the work expended upon it by the ideal pumps. The energy equations involved in the Rankine vapor cycle will now be developed.

(**c**) Since any vapor cycle is made up of a series of processes that are followed by a definite weight of fluid, through a complete circuit, both the pressure-volume and temperature-entropy diagrams may be used advantageously to represent the successive state of this fluid as it passes through the various pieces of apparatus. A pound of steam, *leaving the boiler in state a*, shoves other steam ahead of it through the steam pipe, and thus constant pressure is maintained at the entrance to the prime mover. Hence, in the ideal case, the steam enters the prime mover with the absolute pressure and temperature shown by a in both diagrams of Fig. 175, it being assumed that the steam is superheated.

In the ideal prime mover this steam expands isentropically from the entering state a to a very much lower pressure, such as p_b, which in large plants is usually far below atmospheric. This expansion changes the vapor from one having considerable superheat to one with a quality appreciably less than unity, as indicated by the location of state b.

In the ideal condenser the latent heat of the exhaust steam is absorbed by the cool circulating water while the pressure remains constant at p_b, thus giving the process bc, as shown in the two diagrams. The pound of fluid is now a saturated *liquid* at the exhaust pressure, as shown by c, and is in the hot-well,—a reservoir for the liquid in the bottom of the condenser. From the hot-well, the water is pumped by the condensate pump to the boiler feed-pump which forces it into the boiler again. These two pumps in series change the state of the fluid from a saturated to a non-saturated liquid, i.e., from c to d, as indicated on the diagrams. Thus, the fluid enters the boiler feed pipe as a non-saturated liquid in state d with pressure materially increased, but with the temperature and volume changed very little. This process cd in the ideal case should be considered as a reversible adiabatic compression of the liquid, because, following the explanation in Sect. **201** (g), the minimum work that must be expended to get the liquid from the hot-well into the boiler adiabatically is represented by the area $cdlk$, or equals $h_d - h_c$. After the non-saturated liquid in state d is forced into the boiler, or other heating apparatus, it absorbs heat at constant pressure, as shown by the process dea, until it again becomes a superheated vapor in the original state a, thus completing the Rankine vapor cycle shown by *abcdea* in both diagrams.



(a) (b)

Fig. 175.—The Rankine Vapor Cycle.

(d) The *thermodynamic analysis* of this vapor cycle will next be made. Obviously, for one pound of fluid,

$$Q_1 = \text{Heat supplied by the boiler} = \text{area } deamo = h_a - h_d,$$
$$Q_2 = \text{Heat rejected to condenser} = \text{area } bcom = h_b - h_c$$

$$Q_1 - Q_2 = \text{Net work of the vapor cycle} = \text{area } abcdea = h_a - h_b - (h_d - h_c)$$

From the discussion given in Sect. **201** (g), the significance of the term $(h_d - h_c)$ is evidently the mechanical energy expended upon the fluid to pump it from the hot-well into the boiler feed line in this ideal cycle, which by hypothesis excludes fluid friction. Representing by F the energy, in B.t.u. per pound, expended upon the water by the two pumps, it then follows that

$$h_d - h_c = F, \quad \text{or} \quad h_d = h_c + F,$$

Consequently, the *heat supplied by the boiler* is

$$Q_1 = h_a - (h_c + F) = h_a - h_c - F, \quad . \quad . \quad . \quad . \quad (237)$$

and the *net work of the cycle* is

$$Q_1 - Q_2 = h_a - h_b - F. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (238)$$

Therefore the *efficiency of the ideal Rankine vapor cycle* is

$$e_{i(\text{Rankine vap.})} = \frac{Q_1 - Q_2}{Q_1} = \frac{h_a - h_b - F}{h_a - h_c - F}. \quad . \quad . \quad . \quad (239)$$

(e) In case the boiler is not equipped with a superheater, the steam entering the prime mover would have a quality less than unity, as represented by some state such as a' in Fig. 175. The isentropic expansion would then be represented by the line $a'b'$, and the corresponding ideal Rankine vapor cycle would be shown by $a'b'cdea'$. From the figure it is apparent that the net work, and the heat supplied per pound of steam, are less than for the case in which superheated steam is used, the pressures in both cases being the same.

For the cycle $a'b'cdea'$ the efficiency is

$$e_{i(\text{Rankine vap.})} = \frac{\text{area } a'b'cdea'}{\text{area } a'm'odea'} = \frac{h'_a - h'_b - F}{h'_a - h_c - F}. \quad . \quad . \quad (240)$$

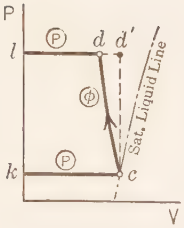


FIG. 176.

(f) An understanding of the full *significance of the factor F* in the above equations is of importance. Referring to Fig. 176, which is an exaggerated drawing to show more clearly the portion *cdlk* of the pressure volume diagram of Fig. 175, let it be supposed that the fluid is one that is slightly compressible under the high pressures that may be used in a boiler. Then the area *cdlk* represents the energy that must be delivered to the fluid by the ideal pumps. When much increase in pressure occurs, this work factor F may be large enough to be of considerable importance.

It is evident that the less compressible the fluid, the nearer does the figure *cdlk* become a rectangle such as *cd'lk*.[3] If $V_d = V_c$, the work shown by area $cdlk = A(P_d - P_c)\overline{V}_c$ B.t.u. per pound. But, since this area always represents $h_d - h_c$, it follows that for a *non-compressible* fluid,

$$F = h_d - h_c = A(P_d - P_c)\overline{V}_c, \text{ exactly,}$$

---

[3] For the case of water with $p_d - p_c = 1000$ lb. per square inch, the ratio of V_d/V_c would be about 0.993.

Thus

$$h_d = h_c + A(P_d - P_c)\bar{V}_c. \quad . \quad . \quad . \quad . \quad . \qquad (241)$$

This derivation shows that for a specified temperature and pressure, the heat content of a non-saturated liquid as given in Sect. **176** (f), is approximate only in so far as the liquid is compressible.

(g) The *efficiency* of the Rankine vapor cycle when the pressures are low may be calculated without considering the term F, which becomes negligibly small under these conditions. In this case Eq. (239) then becomes identical in form with Equation (236); but, note that $h_a - h_b$ is greater for the ideal Rankine *vapor* cycle than it is for the *engine* cycle because in the former the state a is chosen to agree with the condition of the *vapor leaving the actual boiler* while in the latter it agrees with the state of the steam *entering the engine.* Thus the losses of the steam pipe line are included when comparing the actual vapor circuit with its corresponding ideal vapor cycle whereas they are not involved when the engine cycle is considered. Of course the inequality in the entropy of the steam at the beginning of its isentropic expansion for the two cases makes the state b different in the two cycles, although they have the same exhaust pressure.

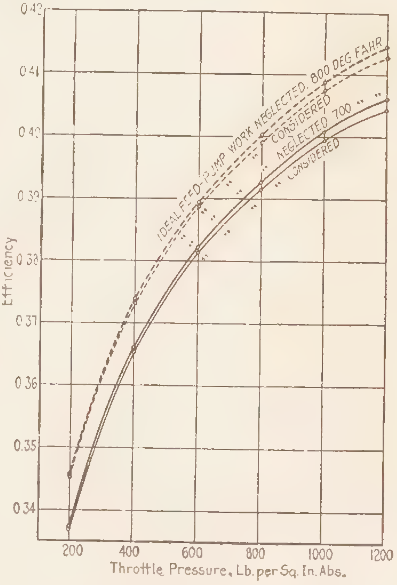


FIG. 177.—Efficiencies of Rankine Cycles ($p_b = 1''$ Hg Abs.)

For steam, the effect of neglecting this term F may be seen from the curves of Fig. 177 taken from the paper of Hirshfeld and Ellenwood entitled "High Pressure, Reheating, and Regenerating in Steam Power Plants."[4] This shows clearly that for steam pressures less than 400 lb. per square inch, the factor F need not be considered, while for the much higher ones it may often be desirable to include it.

Figure 177 shows not only the efficiency of the Rankine vapor cycle both with and without considering the pump work, but it also gives the efficiency of this cycle for a wide range of pressures (200 to 1200 lb. per square inch) and for two values of the temperature of the steam i.e., for $t_a = 700$, $t_a = 800$ deg. fahr., temperatures which mean that a high superheat is used. For any given pressure these curves show a considerably higher Rankine cycle efficiency for the higher temperature steam. The reason why the *cycle efficiency* is increased at the higher temperatures, for the same pressures, is because the average temperature of the working substance during its reception of heat is greater, thus approaching somewhat more closely to Carnot's cycle in which the working substance is at the *highest possible temperature* during *all* of the time it is absorbing heat from the hot body at a fixed temperature.

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[4] See Trans. A.S.M.E., 1923, p. 663.

In the strict limit of the application of Carnot's principle the steam should leave the boiler at the temperature of the hot body which in this case is the furnace. But in order to have a rapid transfer of heat from the furnace to the fluid and to keep within the safe temperature limits of materials, as discussed in Sect. **200** (**h**), the steam must be much below the furnace temperature.

It will be found, for reasons that will be discussed later, that the *engine* efficiencies of both the steam turbine and reciprocating engine are also materially increased by using superheated steam, which is consequently largely employed in modern power plants.

205. The Rankine Cycles as Standards of Comparison.—(a) Now that the analyses of the Rankine engine and vapor cycles have been completed, these ideal cycles may be used as the two standards with which the actual engine, and the apparatus involved in the real vapor circuit, respectively, may be compared. If the *engine alone* is under test, the corresponding Rankine engine is assumed to have the same steam pressure and temperature at the *throttle*, and the same *exhaust pressure*, as the actual engine. If the whole *vapor circuit* is being studied, the maximum pressure and temperature of the ideal cycle are chosen to agree with the vapor as it leaves the *actual boiler*, and the exhaust pressure is taken the same as the actual *condenser pressure*. The entire *vapor* circuit is not tested or analyzed as a unit very frequently, but a clear understanding of it can only come from its complete analysis. On the other hand the use of the Rankine *engine* cycle as a standard is very common in commercial testing of steam prime movers.

(**b**) To determine the most convenient forms of expressions for the **various efficiencies**[5] of any prime mover conforming to the Rankine engine cycle, let the following notation be used:

h_a = specific heat content of the *vapor* at the throttle;

h_b = specific heat content of the *vapor* at actual exhaust pressure, but with $\phi_b = \phi_a$; or, in other words, b is the state of the vapor coming from the corresponding ideal engine;

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[5] The definitions and significance of the various efficiencies as given in Chap. XII should be thoroughly understood before proceeding further.

h_c = specific heat content of the ideal condensate, or *saturated liquid* at exhaust pressure p_b;

w_i = weight of steam required by the ideal engine, in pounds per hp-hr.;

w_{i_k} = weight of steam required by the ideal engine, in pounds per kw-hr.;

w_{ind} = weight of steam used by the actual engine, in pounds per i.hp-hr.;

w_b = weight of steam used by the actual engine, in pounds per b.hp-hr.;

w_c = weight of steam used by the actual engine, in pounds per hp-hr. output of combined unit;

w_k = weight of steam used by actual engine, in pounds per kw-hr. output of combined unit.

For the *ideal* Rankine engine cycle, from Eqs. (235*a*) and (*b*) and Eq. (236),

$$\text{Ideal Cycle Efficiency} = \frac{2545}{w_i(h_a - h_c)}, \quad \ldots \ldots \quad (242a)$$

or

$$\text{Ideal Cycle Efficiency} = \frac{3413}{w_{i_k}(h_a - h_c)}. \quad \ldots \ldots \quad (242b)$$

For the *actual* prime mover following the Rankine engine cycle,

$$\underset{\text{(M. Eff.)}}{\text{Mechanical Efficiency}} = \frac{\text{b.hp.}}{\text{i.hp.}} = \frac{w_{ind}}{w_b}. \quad \ldots \ldots \quad (243)$$

$$\underset{\text{(I. T. Eff.)}}{\text{Indicated Thermal Eff.}} = \frac{2545}{w_{ind}(h_a - h_c)}. \quad \ldots \ldots \quad (244)$$

$$\underset{\text{(B. T. Eff.)}}{\text{Brake Thermal Eff.}} = \frac{2545}{w_b(h_a - h_c)}. \quad \ldots \ldots \quad (245)$$

$$\underset{\text{(C. T. Eff.)}}{\text{Combined Thermal Eff.}} = \frac{2545}{w_c(h_a - h_c)}, \quad \ldots \ldots \quad (246)$$

or

$$\underset{\text{(C. T. Eff.)}}{\text{Combined Thermal Eff.}} = \frac{3413}{w_k(h_a - h_c)}. \quad \ldots \ldots \quad (247)$$

$$\underset{\text{(I. Eng. Eff.)}}{\text{Indicated Engine Eff.}} = \frac{\text{Ind. T. Eff.}}{\text{Ideal Cycle Eff.}} = \frac{\text{Eq. (244)}}{\text{Eq. (242}a\text{)}} = \frac{w_i}{w_{ind}}. \quad (248)$$

$$\text{Brake Engine Eff. (B. Eng. Eff.)} = \frac{\text{Brake T. Eff.}}{\text{Ideal Cycle Eff.}} = \frac{\text{Eq. (245)}}{\text{Eq. (242}a\text{)}} = \frac{w_i}{w_b}. \qquad (249)$$

$$\text{Combined Engine Eff. (C. Eng. Eff.)} = \frac{\text{Comb. T. Eff.}}{\text{Ideal Cycle Eff.}} = \frac{\text{Eq. (246)}}{\text{Eq. (242}a\text{)}} = \frac{w_i}{w_c}. \qquad (250a)$$

or

$$\text{Combined Engine Eff.} = \frac{\text{Eq. (247)}}{\text{Eq. (242}b\text{)}} = \frac{w_{ik}}{w_k}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (250b)$$

Note that the last four equations reduce to the very simple ratio of steam rates because the value of $h_a - h_c$ is common to both the ideal and actual engines. This is not the case with certain other cycles to be considered later.

(c) The actual values of the *mechanical efficiencies* of modern reciprocating steam engines are generally from 85 to 95 per cent. In steam turbines this efficiency is very high, and it is not usually considered to be a significant quantity; it is practically indeterminate since an indicator cannot be used on such a machine.

The *brake engine efficiency* of both reciprocating and turbine engines will vary from about 50 per cent in the small sizes to about 80 per cent, or more, in the large ones. The losses in these two types of prime movers are of very different natures and each will be discussed in detail later. In general, however, it may be stated that the *chief causes of reduced engine efficiency in the reciprocating type* are *heat transfers* between the steam and the cylinder walls, *throttling*, and *mechanical friction;* and in the *steam turbine* they are *fluid friction, turbulence*, and *throttling.* Since the throttling loss is common to both types of prime movers, regardless of what cycle is involved, it will be considered next.

206. The Loss of Available Energy due to Throttling.—(a) Consider steam while in the state a, shown in Fig. 178, to be admitted to an engine which is assumed to exhaust against the back pressure p_b. Then, as shown in Sect. **201**, the maximum amount of work that can be obtained from each pound of steam flowing through an ideal engine under these conditions is $h_a - h_b$. Next, assume that *before* admission to the engine the steam is throttled from the same state a to some specified pressure p_d and that this throttling takes place under such conditions (see Sect. **85**) that the heat content of the steam remains constant. In the figure the

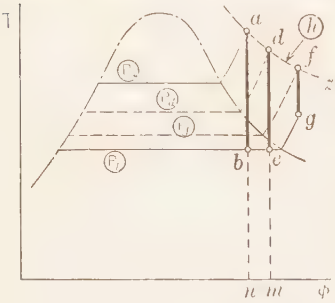


FIG. 178.

line of constant heat content for this throttling is *az*; and its intersection, *d*, with the constant pressure line p_d will therefore determine the state of the steam entering the engine. Then, starting expansion from this point and following the constant entropy line until it intersects the exhaust pressure line p_b, the state *e* at the end of expansion is determined. Thus, with throttling, the energy available for work is $h_d - h_e$, which equals $h_a - h_e$, since $h_a = h_d$. Hence the loss of available mechanical energy due to throttling, in B.t.u. per pound, is,

$$\text{Loss of A.E.} = (h_a - h_b) - (h_a - h_e) = h_e - h_b \quad . \quad . \ (251a)$$
$$= \text{area } bemn, \text{ to scale.}$$

(**b**) If the state *e* lies in the region of *wetness*, as shown in Fig. 178, it is evident that $T_e = T_b$. Consequently, for this case, the loss due to throttling, in B.t.u. per pound, may also be expressed as

$$\text{Loss of A.E.} = T_e(\phi_e - \phi_b) = T_e(\phi_d - \phi_a), \quad . \quad . \quad . \ (251b)$$

in which $\phi_d - \phi_a$ represents the gain in specific entropy of the steam due to throttling, and T_e is the absolute temperature of the saturated steam corresponding to exhaust pressure.

(**c**) In a third case, let the throttling be continued to some state *f* in Fig. 178, such that the isentropic expansion from that point to the same exhaust pressure as before results in the vapor remaining superheated, as shown by *g*. Then, for this case the loss of available mechanical energy is equal to $(h_a - h_b) - (h_f - h_g) = h_g - h_b$, which is not the same as $T_b(\phi_f - \phi_a)$, since T_g is greater than T_b.

From the preceding discussion it is evident that in general it is necessary to specify whether the *minimum temperature* or the *minimum pressure* is used as the basis of determining the available energy and therefore the loss of available energy due to throttling.

207. The Non-expansion, or Rectangular PV, Engine Cycle.—(**a**) The Rankine engine cycle just considered is sometimes called the "complete expansion cycle" because the adiabatic expansion continues from the admission pressure clear down to the exhaust pressure, as shown by the broken line *ab* in Fig. 179. From the standpoint of thermal economy this full expansion to the final volume V_b is desirable; but, from this pressure-volume diagram, it is apparent that the mean effective pressure is much less, and the engine is larger for a given output, than would be the case if the adiabatic expansion did not continue to the exhaust pressure, but terminated at some point such as *d*, with relatively small volume V_d. The steam turbine can very readily take care of the larger volume V_b, and thus utilize the heat energy as completely as possible; but the reciprocating engine is not so well suited for this purpose. For this reason the latter type of prime mover usually has "incomplete expansion," and sometimes it may even be desirable to have zero expansion, in which case the engine follows the simple "non-expansion cycle" which will now be discussed.

(**b**) Consider a reciprocating engine to have steam admitted at full throttle pressure p_a during an entire stroke. With such an engine the work done during the admission of steam would be represented by the rectangular area *zaxo* of Fig. 179, in which *zafy* may be considered as the indicator diagram of this ideal engine having no clearance. Obviously the volume V_a will depend upon the amount of steam admitted and also upon its quality. Now, assume that this engine is of such a size that it will admit just one pound of steam per stroke, that there is no leakage, and that there is no transfer of heat between the steam and the cylinder walls, then the work done during the admission *za* is $AP_a\bar{V}_a$ B.t.u. per pound. Next, let the inlet valve be closed and the exhaust valve be opened, thus permitting a large part of the steam to rush from the cylinder into the condenser, or to the atmosphere. In either case there is a **free expansion** of this steam, and the pressure of the vapor still within the cylinder drops very quickly from admission pressure p_a to the condenser pressure p_f. Assuming that this free expansion takes place while the piston is stationary at the end of the stroke, it follows that an indicator attached to this cylinder would draw a vertical line such as *af* for this process. After the pressure has been reduced to p_f, the piston is returned to its initial position, at the same time shoving the remainder of the steam from the cylinder while at the constant pressure p_f, as shown by the line *fy*.

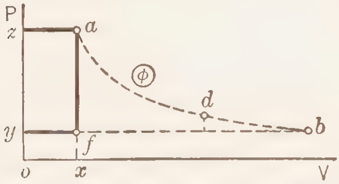


FIG. 179.—The Non-expansion Engine Cycle.

Clearly then, corresponding to the indicator diagram *zafy*, the *work done* per pound of steam is $(P_a - P_f)\bar{V}_a$ ft-lb., or $A(P_a - P_f)\bar{V}_a$ B.t.u. This represents the work that could be done by each pound of steam passing through an ideal engine having *constant pressure admission during the entire stroke, followed by free expansion until the pressure in the cylinder drops to exhaust pressure while the piston is stationary, and finally, by constant pressure rejection of the remainder of the steam.* Since the values of $AP\bar{V}$ for various states of steam are given by one of the Ellenwood Charts, Plate III, in the Appendix, those for $AP_a\bar{V}_a$ and $AP_f\bar{V}_f$ may be read directly, and consequently the work done per pound of steam is very easily found.

(**c**) The *loss of mechanical energy due to the free expansion* is represented by the area *abf* in Fig. 179, the dotted line *ab* representing the isentropic expansion of the steam that might take place, as discussed in Sect. **203**, if the engine were designed for it. However, the volume of the cylinder to permit *complete expansion* would be V_b, while for the *non-expansion* engine it need be only a small fraction of this, as shown by V_a.

(**d**) In order to obtain the *energy supplied* to the engine per pound of steam, the value of h_c, which is the heat content of the saturated liquid corresponding to exhaust pressure, is subtracted from h_a, the heat content of the steam at the throttle, for the same reasons that were discussed in Sect. **203** (**d**). Hence the *efficiency* of the *ideal* engine operating on the non-expansion cycle is

$$e_{t(\text{non-exp. eng.})} = \frac{A(P_a - P_f)\overline{V}_a}{h_a - h_c}. \quad . \quad . \quad . \quad . \quad (252)$$

(**e**) The *ideal steam rate* of the non-expansion engine would evidently be

$$w_i = \frac{2545}{A(P_a - P_f)\overline{V}_a}. \quad . \quad . \quad . \quad . \quad (253)$$

(**f**) The *thermal efficiency* of an *actual* engine operating on this cycle can be determined as soon as the actual steam rates w_{ind} and w_b are known. Thus,

$$\text{Ind. Th. Eff.} = \frac{2545}{w_{ind}(h_a - h_c)}. \quad . \quad . \quad . \quad . \quad (254)$$

$$\text{Brake Th. Eff.} = \frac{2545}{w_b(h_a - h_c)}. \quad . \quad . \quad . \quad . \quad . \quad (255)$$

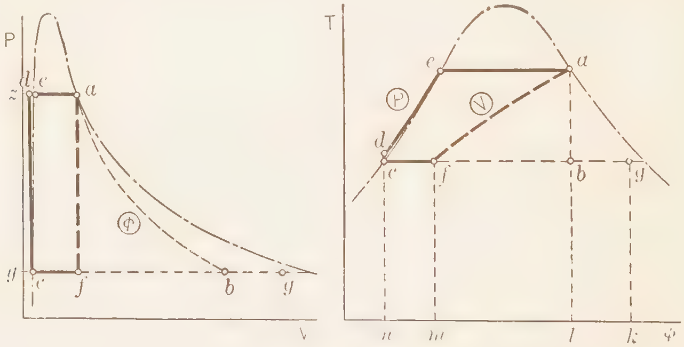
(**g**) The *engine efficiencies* of an *actual* engine that does not permit any expansion of the steam after " cut-off " may be found by dividing the ideal steam rate, w_i, as found by Eq. (253), by the actual steam rate w_{ind}, or w_b,—depending upon whether the indicated or the brake engine efficiency is to be obtained.

(**h**) If it be desired to base the engine efficiency of this type of prime mover on the Rankine engine cycle, the ideal steam rate of the latter, as given by Eq. (235), should then be used. While this procedure is often followed, it seems a questionable way of determining how closely the actual machine approaches the corresponding ideal, because the real engine in this case was never intended to use steam expansively. Such an engine efficiency will not be a true measure of the imperfections of the actual machine as compared with an ideal one functioning in a similar manner but having no friction, leakage, throttling, or loss due to heat transfers to or from the cylinder. These are the losses that the manufacturer and operator are able to reduce, and the engine efficiency should be a true measure of their success in minimizing them.

The chief argument in favor of basing the engine efficiency on the Rankine is that it may sometimes be desirable to refer all the different types of steam prime movers to the same standard. This method may simplify calculations, but it is not altogether logical, any more than it would be to compare the engine efficiencies of all internal combustion motors to one standard, say the Otto. In all cases, however, the important point is to convey *definite information* as to what the term engine efficiency means; hence, in this text it will be written "engine $\text{eff.}_{\text{non-exp.}}$," or "engine $\text{eff.}_{\text{Rankine}}$" to show upon which standard it is based.

208. The Non-expansion Vapor Cycle.—(a) In the previous section, the cycle of the *engine* using steam non-expansively was considered. Now the *working substance* will be carried through a *complete* cycle, and this means that both the PV- and $T\phi$-diagrams may be used to advantage to show the state of the fluid at various points in the power plant circuit, consisting of the boiler, non-expansion type of engine, condenser, pumps, and piping.

(b) Suppose the boiler supplies saturated steam, as shown by the state a in Fig. 180. Then, with ideal steam piping, the area *zafy* represents the work $A(P_a - P_f)\overline{V}_a$, that is done by each pound of steam passing through the engine, as discussed in Sect. **207**. The work of getting the condensate back into the boiler is represented by the area *zdey*, hence $F = A(P_d - P_e)\overline{V}_e$, as shown in Sect. **204**. Consequently



(a) (b)

FIG. 180.—The Non-expansion Vapor Cycle.

the *net work* that may be obtained from each pound of fluid completing this ideal cycle is, in B.t.u.

$$\begin{aligned} \textit{Net work}_{(\text{non-exp. vap.})} = \text{area } afcd &= A(P_a - P_f)(\bar{V}_a - \bar{V}_c) \\ &= A(P_a - P_f)\bar{V}_a - F. \quad . \quad (256a) \end{aligned}$$

(c) This same result for the net work of the cycle of the working substance may of course be obtained by taking the difference between the amount of heat supplied and the heat rejected, in B.t.u. per pound, during the entire cycle. Thus

$$\begin{aligned} Q_1 &= Q_{dea} = \text{area } dealnd = h_a - h_d, \\ Q_2 &= Q_{cfa} = \text{area } cfalnc \;= (h_f - h_c) + (i_a - i_f). \end{aligned}$$

$$\begin{aligned} Q_1 - Q_2 = \textit{net work} &= \text{area } cdeaf \text{ in both diagrams,} \\ &= h_a - h_d - h_f + h_c - i_a + i_f, \\ &= (h_a - i_a) - (h_f - i_f) - (h_d - h_c), \\ &= AP_a\bar{V}_a - AP_f\bar{V}_f - A(P_d - P_c)\bar{V}_c, \\ &= A(P_a - P_f)(\bar{V}_a - \bar{V}_c), \\ &= A(P_a - P_f)\bar{V}_a - F. \quad . \quad . \quad . \quad . \quad . \quad (256b) \end{aligned}$$

In order to obtain this result in either (*a*) or (*b*) of Eq. (256) it must be remembered that only in case $\bar{V}_c = \bar{V}_d$ does $h_d - h_c = A(P_d - P_c)\bar{V}_c = F$ exactly, as was explained in Sect. **204** (f).

(d) From the above equations it follows that for this vapor cycle *afcdea* the *efficiency* is

$$\begin{aligned} e_{i(\text{non-exp. vap.})} &= \frac{\text{area } afcdea}{\text{area } dealnd}, \\ &= \frac{A(P_a - P_f)\bar{V}_a - F}{h_a - h_d}, \quad . \quad . \quad . \quad (257a) \\ &= \frac{A(P_a - P_f)\bar{V}_a - F}{h_a - h_c - F}. \quad . \quad . \quad . \quad (257b) \end{aligned}$$

Since the factor F, representing the energy expended in pumping without any friction each pound of water into the boiler, is very small for relatively low boiler pressures, which are the only ones likely to be encountered with this cycle, its value in Eq. (257) becomes of much less importance than in the similar equations for the Rankine and other cycles in which high pressures are frequently used.

(e) The *free expansion* of the steam occurring after the exhaust valve of the engine opens involves great turbulence and consequently the states of the fluid during *this process* cannot be represented on any diagram, as discussed in Sect. **55**. The constant volume line *af* in

Fig. 180 is drawn as a broken line because it merely represents an *equivalent process* that could take place by condensing the steam inside the cylinder until its quality becomes x_f, while the piston is stationary at the end of its stroke, thus delivering no work to the piston. Here the "equivalent process" is simply one that may be substituted for the free expansion and yet the same net work will be developed per cycle. Then, since the heat supplied per cycle is independent of whether there is any free expansion or not, it follows that with the substitution of the equivalent process the same amount of heat will be rejected to the condenser.

This means that after the free expansion there may be found an **equivalent state** g of the entire pound of steam such that if its quality were reduced from x_g to x_f by rejecting heat to the condenser at the constant pressure $p_f = p_g$, the same amount of heat would be given up as though its condensation from state a to f had taken place inside the cylinder at constant volume. In other words, the state g is so located that $Q_{fa} = Q_{fg}$, or area $falm = fgkm$. Hence

$$i_a - i_f = h_g - h_f.$$

Therefore,

$$\begin{aligned} h_g &= i_a + (h_f - i_f), \\ &= (h_a - AP_a\overline{V}_a) + AP_f\overline{V}_f, \\ &= h_a - A(P_a - P_f)\overline{V}_f. \quad . \quad . \quad . \quad . \quad . \quad (258) \end{aligned}$$

It is important to observe from this equation that *this type of free expansion in which steady flow conditions are not maintained is not one of constant heat content*, since h_g is less than h_a. It should be noted also that if b represents the state of the steam after isentropic expansion from a to the same exhaust pressure, then h_g is larger than h_b. Since the free expansion takes place in the ideal cycle without any transfer of heat to or from the steam, it is an irreversible adiabatic, and the discussion given in Sect. **81** (**c**) therefore applies. Note, however, that with *vapors* the terminal temperature is usually determined by the pressure alone, since the quality is commonly less than unity.

209. The Incomplete Expansion Engine Cycle.—(a) This cycle, the ideal form of which is shown by the heavy lines in Fig. 181, is the one followed by the usual type of reciprocating engine which has "cutoff" of the steam (from the boilers) occur before the end of the stroke. Such an engine is not designed to have complete expansion because the additional energy that would be delivered to the piston by the "toe" ($bb'f$) of the diagram would be offset largely, or wholly, by the increased mechanical friction that would be caused by the additional movement of the piston and other parts of the engine, and also because there would be required a larger cylinder, and hence a more costly one, with little or no gain in power output. The ideal incomplete expansion engine cycle can now be very easily analyzed since the parts above and below the line by' represent, respectively, the Rankine and non-expansion engine cycles which have already been considered.

(**b**) Assume that an ideal engine that follows this cycle receives a pound of steam at constant pressure p_a, that the exhaust pressure is $p_{b'}$, and that, instead of having the isentropic expansion complete, as shown by the broken line ab', it ceases at b, thus obtaining a *ratio of expansion* $\bar{V}_b/\bar{V}_a$ which is considerably less than the value $(\bar{V}_{b'}/\bar{V}_a)$ for the corresponding Rankine cycle. After this expansion has taken place in the engine the exhaust valve opens at b, and the resultant drop in pressure of the vapor which remains in the cylinder, while the piston is considered stationary at the end of the stroke, is represented by the constant volume line bf, the process being one involving free expansion, such as was described in Sect. **207** (**b**). The line fy, representing the return stroke, shows the constant pressure exhaust of the remainder of the steam. This completes the cycle as far as the engine alone is concerned, because all of the steam has now passed through the machine.

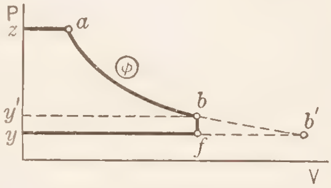


FIG. 181.—The Incomplete Expansion Cycle.

Considering the area $zabfyz$ as composed of the two parts $aby'z$ and $bfyy'$, which, from Sects. **203** and **207**, represent, respectively, $h_a - h_b$ and $A(P_b - P_f)\bar{V}_b$, the *work done*, in B.t.u. per pound of steam passing through such an ideal engine, is

$$Wk_{zabfy} = (h_a - h_b) + A(P_b - P_f)\bar{V}_b. \quad . \quad . \quad . \quad (259)$$

(**c**) The *efficiency* of this ideal engine cycle is obviously

$$e_{i\,(\text{inc. exp. eng.})} = \frac{(h_a - h_b) + A(P_b - P_f)\bar{V}_b}{h_a - h_c}, \quad . \quad . \quad (260)$$

in which h_c represents the heat content of the ideal condensate, or saturated liquid corresponding to the exhaust pressure, just as it did in the Rankine and non-expansion engine cycles having the same exhaust pressure.

(**d**) The *ideal steam rate* of this engine, in lb. per hp-hr, is evidently

$$w_{i\,(\text{inc. exp. eng.})} = \frac{2545}{(h_a - h_b) + A(P_b - P_f)\bar{V}_b}. \quad . \quad . \quad (261)$$

(e) The *engine efficiency* of an actual engine operating on this cycle is given by the ratio of the ideal steam rate to the actual one, since the energy supplied per pound of steam is the same in both cases.

The discussion given in Sect. 207 (h) regarding the inaccuracy that may result from using the Rankine cycle as a single standard of comparison for all engines regardless of the cycle actually followed does not apply with as much force in this case as in others, because the cycle under consideration does not differ greatly from the Rankine and because the loss of area due to cutting off the toe *bb′f* is relatively small in most cases. However, when engine efficiencies are used in making a *careful* study of the performance of an engine of this type operating under various conditions, the most accurate information is obtained when the corresponding ideal incomplete expansion engine is used as the standard; and, to obtain the "corresponding ideal" in this case clearly involves the use of the same throttle and exhaust pressures and the same expansion ratio, V_b/V_a, as in the actual engine.

(f) *Vapor charts* are of the greatest aid in making numerical analyses of this cycle, just as was seen to be the case with the Rankine and non-expansion cycles, which may be considered as the limiting cases of the cycle under consideration. As the pressure-volume relations of vapor during an isentropic expansion cannot, in general, be expressed very simply, the numerical solution of the incomplete expansion cycle is somewhat difficult when the initial data include only the ratio of expansion and the initial and exhaust pressures, as it usually does; on the other hand, it is relatively easy when using a chart which includes simultaneous values of volume and entropy.

210. The Incomplete Expansion Vapor Cycle.—(a) The analysis of this cycle for a pound of vapor that is carried through the complete power plant circuit involves the introduction of the pump work F, and the use of the state a in the ideal cycle as the condition of the steam as it leaves the actual boiler.

(b) The *pressure-volume* and *temperature-entropy* diagrams for one pound of working substance carried throughout the cycle are shown in Fig. 182, in which ab represents the isentropic expansion of the entire pound of steam while in the engine, and the other solid lines show the states through which the fluid passes while on its circuit through the various pieces of apparatus involved. The line bf is shown broken because the process is merely the *equivalent* of the free expansion one, as regards the net work done, as has been explained already. The process bf may be thought of as one having a constant volume abstraction of heat from the *entire* pound while its pressure drops from release to exhaust.[6] The letters y and z are placed on the pressure-volume diagram to show the significance of the area $cyzd$, or pump work F.

(c) The "*equivalent state*" g at the end of the free expansion is so located on the exhaust pressure line cf (extended) that the area $fgkm$ = area $fblm$, for the same reasons that were given in Sect. **208** (e).

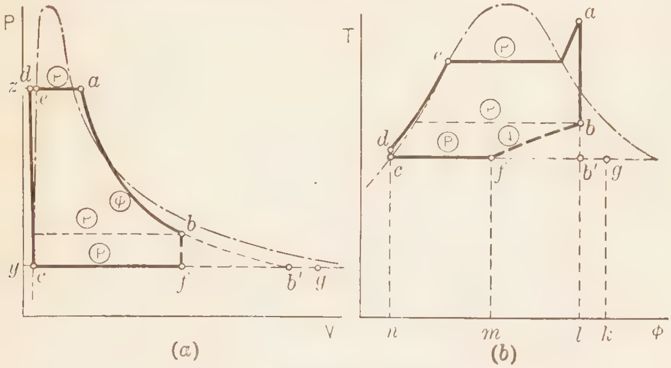


FIG. 182.—The Incomplete Expansion Vapor Cycle.

(d) From Fig. 182, and from the previous discussion of the Rankine and non-expansion vapor cycles, it follows that for this vapor cycle, the *net work*, in B.t.u. per pound, is

$$Net\ Wk_{\text{(inc. exp. vap.)}} = \text{area } abfcdea, \text{ to scale,}$$

$$= (h_a - h_b) + A(P_b - P_f)\overline{V}_b - F; \quad . \quad (262)$$

and the *heat supplied by the boiler*, in B.t.u. per pound, is

$$Q_1 = \text{area } dealnd, \text{ to scale,}$$

$$= h_a - h_d = h_a - h_c - F. \quad . \quad . \quad . \quad . \quad (263)$$

Hence, the *efficiency of this vapor cycle* is,

$$e_{t\text{ (inc. exp. vap.)}} = \frac{Net\ Wk}{Q_1} = \frac{h_a - h_b + A(P_b - P_f)\overline{V}_b - F}{h_a - h_c - F}. \quad . \quad (264)$$

As in the previous cycles, it should be noted that this expression for the efficiency of this vapor cycle becomes identical in form with its corresponding one for the engine cycle, (in this case with Eq. (260)), if the term F is small enough to be ignored, as it is when low pressures are used; but the state a for the vapor cycle is chosen to agree with the condition of the steam leaving the actual boiler instead of that entering the engine, as previously discussed.

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[6] In the pressure-volume diagram of Fig. 181 the line bf is drawn solid because this figure is merely the representation of an indicator diagram that might be obtained from the ideal engine.

211. Summary.—(a) Since this chapter deals with many important terms that are used in steam power plant calculations a brief summary of its major items now seems desirable.

(b) The modern power plant using vapor has many separate pieces of apparatus through which the fluid is kept circulating, and since sometimes the whole system may be under consideration and at other times only an individual piece may be investigated, it is important to distinguish between the complete *vapor cycle*, and what takes place in the single organ, such as an engine or a pump that may be tested alone, in which case the *engine cycle*, or the *pump cycle*, is the one involved.

(c) The application of the *Carnot efficiency*, $(T_1 - T_2)/T_1$, to the analysis of the steam power plant in the strict sense requires the use of the temperatures of the furnace and condensing water as the upper and lower limits, respectively. When used with the steam engine by itself, the limits are the temperatures of the steam entering and leaving the engine. The very great drop in temperature between the furnace and the steam is necessary, at present at least, because there are no suitable materials for the apparatus that will permit the steady flow through them of a fluid having a temperature much above 750 deg. fahr., even though medium pressures are used.

(d) *The isentropic change in the heat content of any fluid*, $(\Delta H)_\phi$, represents either the *maximum work* that can be done *by*, or the *minimum work* that must be expended *upon*, a given amount of any fluid as it passes *adiabatically* from one container, in which constant pressure is maintained, into another, at the same elevation but in which there is a different constant pressure. If this transfer of fluid is due to a *drop* in its pressure there is an isentropic *drop* in its heat content, and consequently an *ideal Rankine engine* is involved; conversely, if the fluid has its pressure *raised* by reason of its passage through an *ideal pump* there will be an isentropic *rise* in its heat content.

(e) The *Rankine Vapor Cycle* is one in which the vapor is formed at constant pressure in a boiler, then passes through the Rankine engine, after which it is condensed at constant pressure and is then pumped without heat transfer into the boiler, there being no loss by reason of friction, leakage, or turbulence in any of the apparatus. The *net work* obtainable will be less by the amount of the pump work, than that from the Rankine engine. Also the *heat supplied* by the boiler will be less, by this same amount, than the *energy* supplied to the Rankine engine, since this engine is credited with the energy represented by the heat content of the saturated liquid at exhaust pressure.

(f) The *Rankine Engine Cycle* is the standard on which is based the engine efficiencies of steam turbines which have no reheating or extraction of the steam. For reciprocating engines this same standard is also often adopted, but the non-expansion or the incomplete expansion engine cycle, as the case may be, is more suitable for showing how closely the actual engine approaches its *corresponding* ideal having the same ratio of expansion.

(g) The *engine efficiencies* of all types of prime movers, whether operating on the Rankine, the incomplete expansion, or the non-expansion cycle, may be found either from the ratio of the actual thermal efficiency to the corresponding ideal, or from the ratio of the ideal steam rate to the actual one, since the energy supplied per unit weight of steam is the same in both the actual and ideal cases in each of these three cycles. However, it should not be inferred that this ratio is also a true measure of the engine efficiencies of the more complex cycles which will be discussed in the next chapter.

(h) In case the Rankine, the non-expansion, or the incomplete expansion, ideal *vapor cycles* are used as standards with which to compare the performance of the actual vapor circuit in the real plant, the following conditions must be fulfilled: The ideal vapor cycle is to have the maximum temperature and pressure of the fluid equal to those of the actual steam leaving the boiler, and the pressure at exhaust is to be equal to that in the condenser. The net work of the actual vapor cycle is obtained by taking from the actual output of the engine the energy supplied to operate all of the pumps involved in the actual vapor circuit.

CHAPTER XVIII

THE REHEATING, REGENERATIVE AND BINARY CYCLES

212. The Principle of Reheating.—(a) Any steam prime mover, but especially the turbine, will have its engine efficiency appreciably improved by reheating the steam after it has flowed partially through it in the usual manner, and then having the vapor returned to the prime mover again, but with a high superheat, to complete its work. The increased quality of the steam at the lower pressures improves the engine efficiency to such an extent that, especially for large plants which operate under high load factors and with high pressures, the extra cost of the reheating system may be very well justified. This system was used many years ago on a few reciprocating engines, but recently the development of the large steam turbine using high pressures has made it a factor of much more importance than ever before.

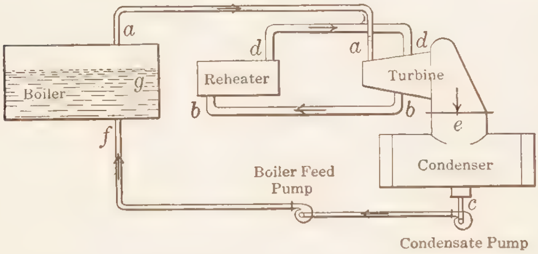

Fig. 183.—Diagrammatic Sketch of Apparatus Involved in the Reheating Vapor Cycle.

(b) An *outline of the apparatus* involved in the reheating *vapor cycle* is shown in Fig. 183. The reheater may be incorporated within the main boiler walls, or it may be a separate unit having its own furnace. Still another plan is to install a reheater which uses high pressure and high temperature steam, instead of a furnace, to supply the reheat.[1]

213. The Ideal Engine and Vapor Cycles Involving Reheating without any Losses.—(a) The distinction between an engine and vapor cycle, which has already been treated fully in the previous chapters, also applies here. For the reheating *vapor cycle*, the ideal reheating system must always be one in which there are no losses of any kind. On the other hand, for the ideal *engine cycle* it is desirable to treat two cases, in the first of which the ideal *reheating* system will have no losses, and in the second it will be assumed to have the same losses as occur in the actual reheating system. The vapor and engine cycles involving reheating without loss will be treated together in this section, and the other engine cycle, and also the reasons for its use, will be given in Sect. **214.**

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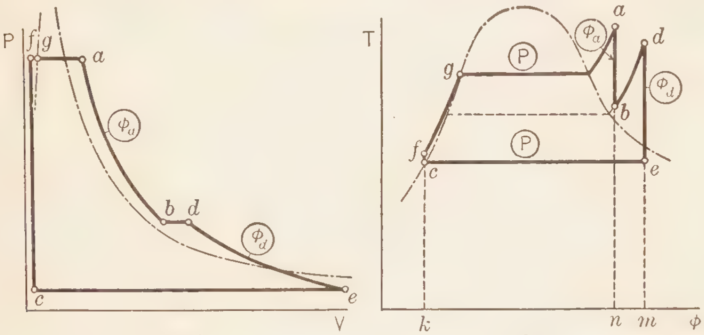
[1] See "Superheat and Reheat," by B. N. Broido, Power, May 18, 1926, p. 757.

(**b**) *The ideal reheating vapor cycle* is defined as one in which the vapor passes through its complete circuit without any loss due to heat transfer, leakage, friction or throttling, and in which the vapor, after expanding isentropically through part of the prime mover, is reheated before the further expansion takes place through the remainder of the machine. The pressure-volume and the temperature-entropy diagrams for this case are shown in Fig. 184, and on them the state of the fluid as it enters or leaves each piece of the apparatus is indicated in each case by the same letter that is used for this purpose in Fig. 183. Referring to both of these figures, the cycle is carried out as follows: The steam leaves the boiler and enters the turbine in the state *a*; it then expands isentropically in the prime mover to *b*, after which it is reheated to a temperature t_d and is then delivered to the turbine again, without loss of pressure or temperature, the state being shown by *d*. This temperature t_d is usually nearly the same as that at *a*. The expansion is then completed in the turbine, as shown by the constant entropy line *de*, and the steam is delivered to the condenser in the state *e*. It is next condensed to a saturated liquid at exhaust pressure, as shown by the state *c*, and then by means of suitable pumps the fluid is returned to the boiler in state *f*, in the same manner as discussed for the Rankine vapor cycle in Sect. **204** (**a**).



(*a*) (*b*)

FIG. 184.—The Ideal Reheating Vapor Cycle.

(**c**) The ideal reheating *engine cycle* in which there is *no loss in the reheating system* would be as given in Fig. 185. In this diagram, *la* shows the constant pressure admission of steam, *ab* the first isentropic expansion, *bd* the constant pressure reheating, *de* the second constant entropy expansion, and *ek'* the exhaust from the engine at the constant pressure.

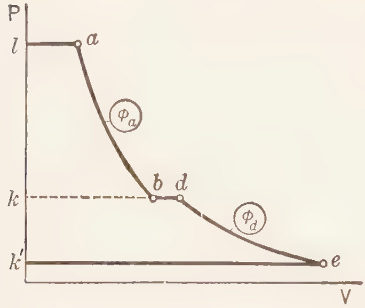


FIG. 185.—The Ideal Reheating Engine Cycle without Losses in the Reheating System.

The *work obtainable* from each pound of steam *passing through this ideal engine* is equal to the sum of the two parts represented by the areas *abkl* and *dek'k*, respectively. For each of these parts this work is equivalent to the isentropic drop in the heat content of the steam; hence, from Eq. (232), the *work* is, in B.t.u., per lb.,

$$Wk_{\text{(reh. eng.)}} = (h_a - h_b) + (h_d - h_e) \quad . \quad . \quad . \quad . \quad (265)$$

$$= \text{area } abdek'l \text{ in Fig. 185.}$$

(**d**) The *net work* obtainable from each pound of fluid completing the *vapor cycle* will evidently be equal to that obtainable from the ideal engine minus the ideal pump work F, as explained in Sect. **204** (**d**). Consequently, for the ideal reheating vapor cycle, it follows that the net work, in B.t.u. per pound, is,

$$Net\ Wk_{\text{(reh. vap.)}} = (h_a - h_b) + (h_d - h_e) - F \quad . \quad . \quad . \quad (266)$$

$$= \text{area } abdecfga \text{ in Fig. 184.}$$

(**e**) The *efficiencies* of the vapor and engine cycles may now be found by applying the same principles as for the Rankine cycles (see Sects. **203** and **204**), except that now it is also necessary to keep in mind the fact that the reheater supplies an amount of energy represented by $h_d - h_b$. Consequently, the *energy chargeable to the engine*, in B.t.u. per pound, is $(h_a - h_c) + (h_d - h_b)$; and the *heat supplied* by the *boiler* and *reheater*, in the same units, is

$$Q_1 = (h_a - h_f) + (h_d - h_b) = (h_a - h_c) + (h_d - h_b) - F \quad (267)$$

$$= \text{area } fgabdemk \text{ in Fig. 184.}$$

Therefore the *efficiency* of this ideal *engine cycle* is

$$e_{t\,\text{(reh. eng.)}} = \frac{(h_a - h_b) + (h_d - h_e)}{(h_a - h_c) + (h_d - h_b)}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (268)$$

and the *efficiency* of the *ideal vapor* cycle is

$$e_{t\,(\text{reh. vap.})} = \frac{Net\ Wk}{Q_1} = \frac{(h_a - h_b) + (h_d - h_e) - F}{(h_a - h_c) + (h_d - h_b) - F}. \quad . \quad (269)$$

As explained in Sect. **202** (d), the value of the term F (which equals $A(P_f - P_c)\overline{V}_c$) becomes of appreciable magnitude only at high pressures. But it is also true that low-pressure plants are not likely to use reheating; hence the distinction between Eqs. (268) and (269) is of more commercial importance than it is between the similar ones in the Rankine cycles, which usually involve no high pressures.

(f) Equations (266), (267) and (269) should be used when it is desired to establish a standard with which to compare the flow of the actual working substance in its *complete* circuit, because such a comparison shows the magnitude of all of the losses that are involved. Equations (265) and (268) should be used as standards with which to compare the flow of the actual working substance through the *engine and reheater only*, unless it be desired to include the reheating losses, in which case the ideal engine cycle given in the following section would be used.

214. The Ideal Reheating Engine Cycle Involving Reheating Losses.—(a) Since the manufacturer of turbines is usually not the one who furnishes the reheater and its piping, it may be desirable to compare the actual reheating turbine with an *ideal one in which the losses that are due to the drop in pressure and to heat transfer in the reheating system, are taken the same as those in the actual installation.* This is the case that will probably be met most often in the testing of large reheating turbines and it is therefore the standard recommended by the authors for such prime movers, for the present at least, pending the complete standardization of this subject by some national or international engineering society.

(b) *This ideal reheating engine cycle* is defined as fulfilling the following conditions:

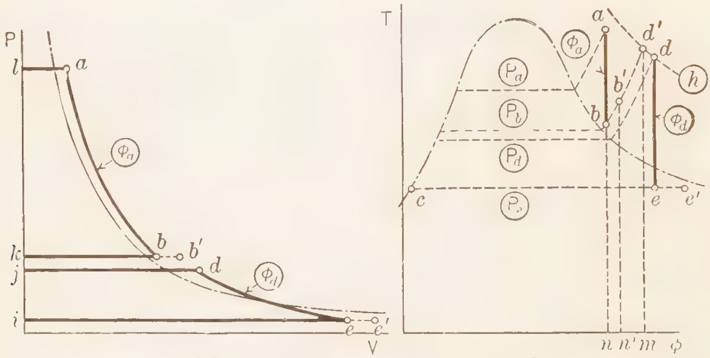
(1) The same throttle pressure and temperature are to be considered as in the actual case.

(2) The steam leaving the prime mover for reheating is taken as at the same pressure as in the actual case, but with the same entropy as at the *throttle.*

(3) After reheating, the steam is considered as returned to the prime mover at the same pressure and temperature as in the actual case.

(4) The same exhaust pressure is used as in the actual case.

With these conditions fulfilled, the pressure-volume and the temperature-entropy diagrams, as given in Fig. 186, may be used to advantage to show the states of the steam at various points as it passes through this ideal reheating engine. Thus, *ab* represents the isentropic expansion before reheating, and *de* the similar type of expansion in the turbine after reheating. The state *c* shows the saturated liquid at exhaust pressure, and the heat content h_c corresponding to it is to be credited to the engine. Since it is assumed that there is an irreversible process in the ideal reheating system, there is a break in the diagrams between the states *b* and *d*, and consequently the process between these states cannot be shown. However, by locating the point d' such that $p_{d'} = p_b$ and $h_{d'} = h_d$, then the area $bd'mn$ represents the net heat absorbed by the steam in passing through this *ideal* reheating system. Regardless of how much throttling and loss of heat by radiation and conduction may take place in the reheating system, the net amount of energy[2] absorbed by each pound of steam passing through it under steady flow conditions is $h_d - h_b$, if *b* and *d* represent the entering and leaving states, respectively. Consequently with this amount of energy added to the quantity $h_a - h_c$ that is supplied by the boiler and feed pump, it follows that the total amount of *energy chargeable to each pound of steam passing through this ideal reheating engine* is $(h_a - h_c) + (h_d - h_b)$, B.t.u.



(*a*) (*b*)

FIG. 186.—The Ideal Reheating Engine Cycle Involving Reheating Losses.

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[2] Applying Eq. (22) to the steady flow of steam through the reheating system alone, in which $v_1 = v_2$, $z_1 = z_2$ and $E_m' = 0$, it follows that the *gross energy supplied to the reheater* is $E_h' = H_2 - H_1 + R_c$. Therefore the *net energy absorbed by w pounds of the steam* in passing through this system is $E_h' - R_c = H_2 - H_1 = w(h_2 - h_1)$. Hence for the case here considered $E_h' - R_c = w(h_d - h_b) = w(h_d' - h_b) =$ area $bd'mn$.

(c) The *work done* by each pound of steam passing through this ideal engine is equal to the sum of the work done in the two portions *abkl* and *deij*, each of which represents a definite drop in the heat content of the steam at constant entropy. Hence, from Eq. (232), the *work done*, in B.t.u. per lb., is

$$Wk_{i\,(\text{reh. eng.})} = (h_a - h_b) + (h_d - h_e). \quad . \quad . \quad . \quad . \qquad (270)$$

(d) Therefore the *ideal steam rates* of this engine are

$$w_{i\,(\text{reh. eng.})} = \frac{2545}{(h_a - h_b) + (h_d - h_e)} \text{ lb. per hp-hr,} \quad . \quad . \qquad (271a)$$

or

$$w_{ik\,(\text{reh. eng.})} = \frac{3413}{(h_a - h_b) + (h_d - h_e)} \text{ lb. per kw-hr.} \quad . \quad . \qquad (271b)$$

(e) The *efficiency* of this *ideal engine cycle* is

$$e_{i\,(\text{reh. eng.})} = \frac{Wk_i}{\text{Energy Supplied}} = \frac{(h_a - h_b) + (h_d - h_e)}{(h_a - h_c) + (h_d - h_b)}; \qquad (272a)$$

or, in many cases, it may be more conveniently found by means of the ideal steam rates, thus,

$$e_{i\,(\text{reh. eng.})} = \frac{2545}{w_i(h_a - h_c + h_d - h_b)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (272b)$$

or,

$$e_{i\,(\text{reh. eng.})} = \frac{3413}{w_{ik}(h_a - h_c + h_d - h_b)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (272c)$$

(f) The *actual reheating turbine* will have the steam enter the reheating system in a state such as b' in Fig. 186, from which it is apparent that $h_{b'} > h_b$ and $\phi_{b'} > \phi_b$. This is due to the turbulence and friction of the fluid as it passes through the real machine, as discussed in Sect. **44** (1).

After having determined the state b', the net energy absorbed by each pound of the fluid passing through the actual reheating system is $h_d - h_{b'}$, which is less than in the corresponding ideal, as may be seen from the difference between areas $b'd'mn'$ and $bd'mn$. Hence the **energy supplied to the real engine** by each pound of steam passing through it is $(h_a - h_c) + (h_d - h_{b'})$. The state of the steam entering the actual reheating system, as shown by b', is found by measuring the pressure and temperature of the steam leaving the turbine. Similarly, the state d, representing the condition of the steam as it is returned to the actual turbine, as well as that of the ideal which has the same reheating losses, is determined by the actual measurement of p_d and t_d. The condition of the steam entering the condenser is shown by c for the ideal case, and by c' for the actual one. The quality at c' is fixed primarily by the state d and secondarily by the engine efficiency, which in turn is largely dependent upon the quality at d. Clearly the greater the quality at e' the larger will be the energy given to the condenser, and the designer is therefore confronted with the difficult question of deciding just how much reheating to use in any given case. He tries to have sufficiently high quality of the steam after reheating to give good engine efficiency and prevent damage of the turbine blades by moisture, but at the same time not give any more heat to the condenser than is necessary. In some cases it may even be desirable to use a second reheating when the initial pressures are very high, but this will involve no new principle, so it need not be considered here.

(g) A prime mover using reheated steam is usually a large one and generally is tested in combination with the machine to which it is connected. Regardless of what the engine may be driving, the *thermal efficiency* of the combined unit may be determined as soon as the actual steam rate and the pressures and temperatures just mentioned have been determined. Thus, let

w_b = weight of steam used by the actual unit, in lb. per b.hp-hr.;

w_c = same, except in lb. per hp-hr. output of the combined unit;

w_k = same, except in lb. per kw-hr. output of the combined unit.

Then,

$$\text{Brake Th. Eff.} = \frac{2545}{w_b(h_a - h_c + h_d - h_{b'})}; \quad \ldots \quad (273)$$

and

$$\text{Combined Th. Eff.} = \frac{2545}{w_c(h_a - h_c + h_d - h_{b'})}. \quad \ldots \quad (274)$$

or,

$$\text{Combined Th. Eff.} = \frac{3413}{w_k(h_a - h_c + h_d - h_{b'})}. \quad \ldots \quad (275)$$

(h) The *engine efficiency* of the combined unit is, by definition, as given in Sect. **144**, the ratio of the thermal efficiency of the actual engine to that of the ideal one. Hence,

$$\text{Brake Eng. Eff.} = \frac{\text{B. Th. Eff.}}{\text{Ideal Cycle Eff.}} = \frac{\text{Eq. (273)}}{\text{Eq. (272}b)}$$

$$= \frac{w_i(h_a - h_c + h_d - h_b)}{w_b(h_a - h_c + h_d - h_{b'})}. \quad \ldots \quad (276)$$

$$\text{Combined Eng. Eff.} = \frac{\text{Comb. Th. Eff.}}{\text{Ideal Cycle Eff.}} = \frac{\text{Eq. (274)}}{\text{Eq. (272}b)}$$

$$= \frac{w_i(h_a - h_c + h_d - h_b)}{w_c(h_a - h_c + h_d - h_{b'})}, \quad \ldots \quad (277)$$

or,

$$\text{Combined Eng. Eff.} = \frac{\text{Eq. (275)}}{\text{Eq. (272}c\text{)}} = \frac{w_{tk}(h_a - h_c + h_d - h_b)}{w_t(h_a - h_c + h_d - h_{b'})}. \qquad (278)$$

It is very important to note that in no case is the engine efficiency for this cycle obtainable from the ratio of the steam rates alone, because of the difference in the value of h_b and $h_{b'}$.[3]

215. The Principle of the Regenerative Cycle.—(a) From the study of the previous vapor cycles it has been found that by far the largest loss in the steam power plant is that due to the heat given up with the exhaust. It may happen, in certain instances, that the exhaust steam may be used for heating buildings or for industrial purposes. For such cases the steam system involved has, strictly speaking, an extremely high thermal efficiency; but, unfortunately, in most central stations it has thus far not been commercially feasible to utilize the exhaust steam for such purposes. However, by using a regenerative cycle, such as will now be considered, the loss in the exhaust is greatly reduced.

(b) The *regenerative cycle* is one in which the feedwater is heated to a temperature much higher than that corresponding to saturation at the exhaust pressure, and this is done by means of steam that has been *extracted*, or *bled*, from the prime mover at one or more points intermediate between the throttle and exhaust. It is apparent that the bled steam transforms some of its energy into work as it passes partially through the prime mover, and that the remainder is given to the feedwater, except for the very small portion that is lost by radiation and conduction. Since the feedwater can absorb only a limited amount of heat, the major portion of the steam used in the regenerative cycle must pass entirely through the prime mover, and consequently this part must exhaust to the condenser.

It should be noted that the regenerative cycle is different from the one in which steam is extracted from the prime mover at one or more points, for the purpose of heating something other than the feedwater. The term " extraction cycle " does not show the purpose for which the steam is bled, whereas the term " regenerative cycle " does.

(c) The *number* of *regenerative feedwater heaters* to use in an actual plant cannot be determined properly until there has been made a careful study that takes into consideration all of the factors involved in a given station. As the number of heaters installed is increased, the first cost and complication of the plant will be made much larger, but the thermal efficiency of the plant will not improve proportionately.

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[3] For an indication of the magnitude of this difference see Table 5A, Trans. A.S.M.E., 1923, p. 806, in the discussion by F. O. Ellenwood.

In general, it may be said that the greater the number of heaters in use the less is the gain in thermal economy obtainable by the addition of another one. For this reason it seldom pays to use more than four in large plants and two in small ones.

The ideal regenerative engine cycle with an infinite number of heaters is sometimes conveniently used as a standard when making a study of the probable performances of regenerative prime movers which are assumed to operate under a great variety of pressures and temperatures. It gives the best results that are obtainable with any regenerative cycle, and also has thermal efficiencies much higher than those of the cycles so far discussed, as may be seen by the comparison of such values with those of the ideal Rankine and reheating cycles, as shown in Fig. 187.[4] On the other hand, studies of this kind are not typical of the engineering work usually involved in the testing of an actual regenerative unit. Since an infinite number of heaters cannot be used in any real plant, this cycle is a less satisfactory standard for determining the engine efficiency of a regenerative unit that is being tested than is the regenerative one that has the same number of heaters as are actually used; hence only this latter case will be treated in this text.[5]

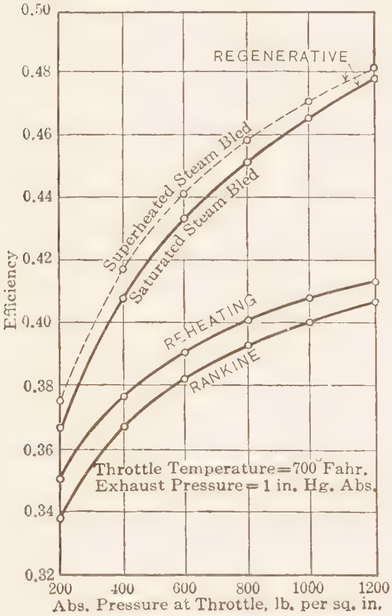

FIG. 187.—Comparative Ideal Engine Cycle Efficiencies.

216. The Ideal Regenerative Engine Cycle with a Finite Number of Heaters.—(a) There are many steam turbines operating with regenerative feedwater heaters and naturally it is desirable to be able to determine how well such a unit is performing. The term " unit " here means the turbine-generator and its regenerative heater system, because the energy chargeable to the turbine depends upon the state of the feedwater leaving the last heater, as well as upon the steam consumption, the pressure and temperature at the throttle, and the exhaust pressure. It is the purpose of this section to develop a suitable standard, or ideal cycle, by means of which the true engine efficiency of such a unit may be determined.

---

[4] In this figure the solid lines represent values taken from the paper of Hirshfeld and Ellenwood already mentioned (see Trans. A.S.M.E., 1923), in which the temperature of first bleeding in each case was taken as that for the point where the isentropic expansion line intersects the saturated vapor curve. The ideal cycle efficiency would be increased slightly by taking the first bleeding point in the superheated region, as may be seen from the broken line in the figure, which represents the efficiencies given in "The Margins of Possible Improvements in the Central Station Steam Plant," by Ernest L. Robinson, Trans. A.S.M.E., 1923, p. 644.

[5] For the method of treating the regenerative cycle with an infinite number of heaters see the articles referred to in the last footnote.

(**b**) The ideal regenerative engine cycle is defined as one fulfilling the following conditions:

(1) The same *number of ideal heaters* are involved as there are actual heaters in the real plant with which the comparison is made.

(2) The several *bleeding pressures* are chosen the same as those observed at the bleeder nozzles in the actual case.

(3) The *weight of steam bled* at each point in this ideal cycle is calculated so that it is just sufficient to heat, at constant pressure and without loss of any kind, all of the feedwater entering the heater, up to the saturation temperature corresponding to this bleeding pressure. In making this calculation the bled steam is assumed to have had isentropic expansion from the throttle condition to the bleeding pressure, hence its corresponding heat content may be readily found.

(4) The heaters in the ideal case are of the " contact " type, in which the bled steam is in direct contact with the feedwater; and each is taken as being under the same pressure as that of the steam at the bleeder nozzle with which it is connected. The fluid from each heater is considered as pumped without losses to the next heater of higher pressure,—or to the boiler, for the case of the last heater in the series.

(**c**) The diagrammatic sketch of the apparatus involved in this ideal regenerative unit is shown in Fig. 188. Four heaters are assumed, as this number is not usually exceeded in practice. In case the number is different the necessary modification of the analysis can be easily made. On this diagram are also shown the relative amounts of steam that are bled at the various points and the weights of water entering and leaving each heater in this ideal case. The method of determining these weights will be discussed in a later paragraph. The superheater will be considered as part of the boiler, as in Fig. 174.

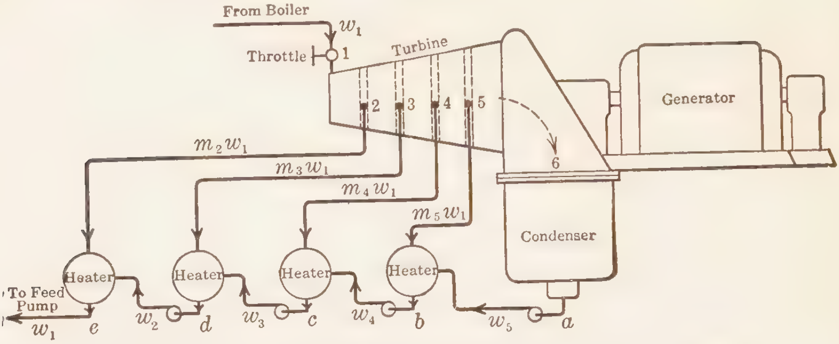

FIG. 188.—Outline of Ideal Regenerative Unit with Four Heaters.

(d) The $T\phi$-diagram in Fig. 189 shows by *1 2 3 4 5 6* the isentropic expansion line, and also by *a*, *b*, *c*, *d* and *e*, the states of the saturated liquid as it leaves the hotwell and each of the heaters respectively. The various weights of the fluid are also indicated. This figure serves merely to show the state of the fluid at various points in its passage through the ideal unit. With the various bleeding pressures chosen to agree with the actual case, as stipulated in (**b**), the several intersections of the constant pressure lines p_2, p_3, p_4, etc., with the isentropic expansion line through *1* and with the saturated liquid line may be obtained so as to determine the respective state points for the vapor and liquid. The consideration of the significance of areas on the $T\phi$-diagram will be deferred until the corresponding *vapor* cycle is discussed, as has been done with the previous cycles.

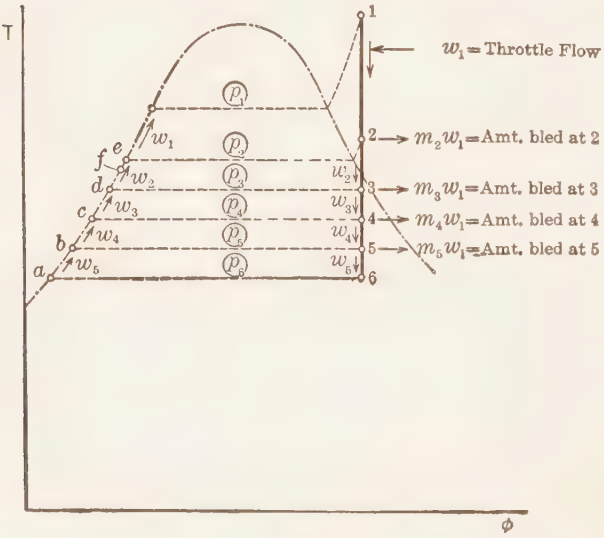
T
ϕ
1
w_1 = Throttle Flow
p_1
p_2
p_3
p_4
p_5
p_6
e
f
d
c
b
a
w_1
w_2
w_3
w_4
w_5
2 → m_2w_1 = Amt. bled at 2
3 → m_3w_1 = Amt. bled at 3
4 → m_4w_1 = Amt. bled at 4
5 → m_5w_1 = Amt. bled at 5
6

FIG. 189.

The *temperature of the feedwater* leaving the last heater in the *ideal case* is t_e, corresponding to state e, and in the actual case it is represented by t_f, corresponding to some such state as that shown by f, its value being, in general, somewhat less than t_e, due to the losses occurring in the actual regenerative heating system.

(**e**) The *amount of steam bled* at each point in the ideal cycle must be calculated before the cycle efficiency can be found. It is most convenient to express all of these amounts as fractions of the throttle flow w_1, and the symbols m_2, m_3, etc., have been used for this purpose, as indicated in Figs. 188 and 189. The determination is based on the assumption that there is no leakage of fluid and on the third condition given in (**b**). Hence,

$$w_2 = (1 - m_2)w_1$$

and

$$m_2w_1(h_2 - h_e) = w_2(h_e - h_d);$$

therefore,

$$m_2 = \frac{h_e - h_d}{h_2 - h_d}. \qquad (279a)$$

Similarly,

$$w_3 = (1 - m_2 - m_3)w_1,$$

and

$$m_3w_1(h_3 - h_d) = w_3(h_d - h_c);$$

therefore,

$$m_3 = \frac{(1 - m_2)(h_d - h_c)}{h_3 - h_c}. \qquad (279b)$$

Following the same method, it is also found that

$$m_4 = \frac{(1 - m_2 - m_3)(h_c - h_b)}{h_4 - h_b} \qquad (279c)$$

and

$$m_5 = \frac{(1 - m_2 - m_3 - m_4)(h_b - h_a)}{h_5 - h_a}. \qquad (279d)$$

(**f**) The *work done* by the steam in passing through the ideal regenerative unit,[6] in B.t.u. per pound of *throttle flow*, is

$$\begin{aligned} Wk_t = h_1 - h_2 &+ (1 - m_2)(h_2 - h_3) + (1 - m_2 - m_3)(h_3 - h_4) \\ &+ (1 - m_2 - m_3 - m_4)(h_4 - h_5) \\ &+ (1 - m_2 - m_3 - m_4 - m_5)(h_5 - h_6) \\ = h_1 - h_6 &- m_2(h_2 - h_6) - m_3(h_3 - h_6) \\ &- m_4(h_4 - h_6) - m_5(h_5 - h_6) \qquad (280) \end{aligned}$$

---

[6] Strictly speaking, the ideal unit includes the heater pumps, and, therefore, Eq. (280) should include a very small subtractive term to take care of the amount of energy expended upon them; but, since the pressure in the last heater is much less than that in the boiler, this energy is very small and may be neglected here. It will be included, however, in the vapor cycle, Sect. **217**.

(g) The *energy supplied* to the ideal regenerative unit, in B.t.u. per pound of throttle flow, is equal to h_1, the heat content of the steam at the throttle, minus h_e, the heat content of the saturated liquid corresponding to p_2, the highest bleeder pressure. Consequently the *ideal cycle efficiency* of the unit is

$$e_i = \frac{Wk_i \text{ [from Eq. (280)]}}{h_1 - h_e} \quad . \quad . \quad . \quad . \quad . \quad (281)$$

(h) The *ideal steam rate* of the regenerative unit, in lb. per kw-hr., is

$$w_i = \frac{3413}{Wk_i \text{ [from Eq. (280)]}}. \quad . \quad . \quad . \quad . \quad . \quad (282)$$

It should be observed that Eqs. (280), (281) and (282) have been put in the forms that enable them to be used very easily with any number of heaters up to four. Thus, if there are only three heaters used, m_5 becomes zero and p_5 equals the exhaust pressure. Only in case m_2, m_3, m_4 and m_5 each become zero can these three equations give the same result as the similar ones for the Rankine engine (Sect. **203**), for only then is this latter cycle carried out.

(i) For the *actual unit* the total flow of the steam past the throttle and the corresponding energy output of the generator must be determined[7] by some means. From these two factors the *actual steam rate*, w_a, in lb. per kw-hr., is found. Consequently, **the energy supplied to the actual unit,** in B.t.u. per kw-hr. output, is $w_a(h_1 - h_f)$, in which h_f is the heat content of the saturated liquid corresponding to the actual feedwater temperature, as measured upon leaving the heater having the highest pressure. Therefore, the *combined thermal efficiency* of the actual unit is

$$\text{C. Th. Eff.} = \frac{3413}{w_a(h_1 - h_f)}, \quad . \quad . \quad . \quad . \quad . \quad (283)$$

and the *combined engine efficiency* of this unit is found either from

$$\text{C. Eng. Eff.} = \frac{\text{Energy supplied ideal unit, in B.t.u. per kw-hr.}}{\text{Energy supplied actual unit, in B.t.u. per kw-hr.}}$$

$$= \frac{w_i(h_1 - h_e)}{w_a(h_1 - h_f)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (284a)$$

or from

$$\text{C. Eng. Eff.} = \frac{\text{C. Th. Eff. of actual unit [from Eq. (283)]}}{\text{Ideal Cycle Eff. [from Eq. (281)]}} \quad . \quad . \quad (284b)$$

---

[7] The water coming from the condenser may be weighed in the usual manner, but the actual flow past the throttle is not so easy to determine and the method of procedure has not yet been standardized. Note that the actual amount of steam bled to each heater is not required for determining the engine efficiency, but that the throttle flow must be found, either directly or indirectly.

It is very important to note from Eq. (284a) that the ratio of steam rates (w_i/w_a) will, in general, not give the correct value of the engine efficiency of a regenerative unit, because h_f is usually far from equaling h_e.

The *engine efficiency* of any actual regenerative unit is reduced chiefly by the imperfections of the actual prime mover and generator, but also slightly by the following additional factors that involve thermodynamic losses in the heater system of the unit:

(1) There is a certain *pressure drop* through the valve and pipe line from prime mover to the heater, and consequently the temperature in this heater is less than its corresponding ideal.

(2) *Surface heaters*, if used instead of contact ones, necessitate a certain *temperature* difference between the steam and water, since these fluids are separated by the thickness of the walls of the heater tubes through which the heat must be made to flow.

(3) The *condensate* from the surface heater is usually collected in a steam trap, or its equivalent, from which it is discharged into the next heater having a lower pressure. This arrangement avoids the use of separate pumps for each heater, but the throttling of the saturated liquid to a lower pressure involves the usual kind of loss of available mechanical energy which always occurs with a free expansion. In this case the drop in pressure of the fluid as it passes from the trap to the lower pressure of the next heater results in increasing its quality, as explained in Sect. **195** (c), and the portion that becomes a vapor separates from the liquid; or, as it is commonly spoken of, this portion "*flashes*" into steam, and mixes with that which has been bled to this heater.

An indication of the values of the engine efficiency that may be expected from actual regenerative units will be found in the problems dealing with this cycle.

217. The Regenerative Vapor Cycle with a Finite Number of Heaters.—(a) The difference between this *vapor* cycle and the engine cycle of the preceding section is due, first, to the inclusion of the feed-pump work, F, and, secondly, to the fact that the state of the vapor at the beginning of the isentropic expansion is taken the same as that at the point where the fluid enters the actual steam pipe leaving the superheater, instead of that at throttle of the real prime mover.

(**b**) The *areas on the Tϕ-diagram* of the regenerative vapor cycle cannot be compared in the simple manner that is possible for those cases in which *all* of the fluid passes through the same circuit. One form of this diagram that shows conveniently the area representing net work per pound of throttle flow in this ideal cycle is given in Fig. 190 for a unit having four heaters. This figure uses the same notation as that in Fig. 189, but in this case the diagram is so constructed that certain areas have important significances even though the weight of fluid in the turbine varies from point to point. At state *1* there are w_1 lb. of steam flowing in the prime mover per unit of time, but after the abstraction of steam at point *2* there remains in it only $(1 - m_2)w_1$ lb. of steam, consequently the ratio of the width of the diagram to that of the original is as $(1 - m_2) \div 1$. As is done in Fig. 190, it is convenient to consider the successive isentropic expansions as continuous, hence the contraction is accomplished by shifting the saturated liquid line so as to pass through d' and e'', instead of through d and e, the new axis of zero entropy not being drawn. With further extraction, the narrowing of the diagram is continued, the successive saturated liquid lines passing through d'', c'' and b'', respectively. The point e'' is so located that

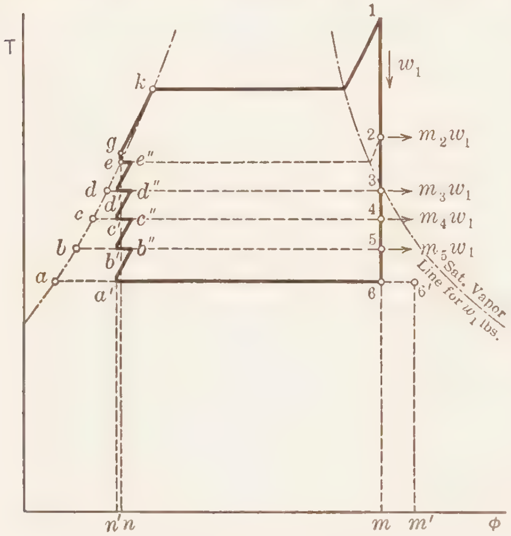

FIG. 190.—The Regenerative Vapor Cycle.

$$\frac{(\phi_2 - \phi_{e''})}{(\phi_2 - \phi_e)} = \frac{w_2}{w_1} = \frac{(1 - m_2)}{1}.$$

Likewise, for point d'',

$$\frac{(\phi_3 - \phi_{d''})}{(\phi_3 - \phi_d)} = \frac{w_3}{w_1} = \frac{(1 - m_2 - m_3)}{1};$$

and similarly for points c'' and b''; or, in general, in each case (width with bleeding $\div$ width without bleeding) $= (1 - \Sigma m)/1$, in which Σm is the sum of the fractions bled to the point under consideration. Obviously, $3\,d'/3\,d = w_2/w_1$, $4\,c'/4\,c = w_3/w_1$, etc.

(c) Using the notation given in Fig. 190, the *ideal feed pump work* for this cycle, in B.t.u. per pound of throttle flow, is

$$F = A(P_1 - P_e)\overline{V}_e = A(P_1 - P_2)\overline{V}_e. \quad . \quad . \quad . \quad (285)$$

The small amount of energy expended by the several ideal heater pumps, in B.t.u. per pound of throttle flow, is

$$\begin{aligned} F' = {} & (1 - m_2)A(P_e - P_d)\overline{V}_d + (1 - m_2 - m_3)A(P_d - P_c)\overline{V}_c \\ & + (1 - m_2 - m_3 - m_4)A(P_c - P_b)\overline{V}_b \\ & + (1 - m_2 - m_3 - m_4 - m_5)A(P_b - P_a)\overline{V}_a. \quad . \quad . \quad . \quad (286) \end{aligned}$$

(d) The *net work* of this vapor cycle, in B.t.u. per pound of throttle flow, as shown in Fig. 190 by the area bounded by the heavy lines, is

$$\begin{aligned} \textit{Net Wk}_{i\,(\text{reg. vap.})} = {} & h_1 - h_6 - m_2(h_2 - h_6) - m_3(h_3 - h_6) \\ & - m_4(h_4 - h_6) - m_5(h_5 - h_6) - F - F'. \quad (287) \end{aligned}$$

It may be noted that this equation is identical in form with Eq. (280) except for the small terms F and F'; but the states *1, 2, 3, 4, 5* and *6* are slightly different from what they are for the engine cycle because the vapor cycle starts the isentropic expansion with state *1* representing the condition of the steam as it leaves the actual superheater, instead of as it actually enters the engine.

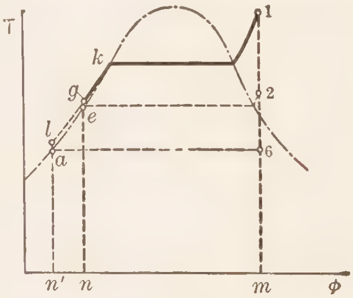

FIG. 191.

(e) The *heat supplied* by the boiler in this cycle is found in the following manner: Considering the heavy line in Fig. 191, which is redrawn from Fig. 190, the feedwater leaves the heater at e as a saturated liquid with a temperature t_e, then passes through the ideal feed pump and enters the boiler as a non-saturated liquid, as represented by g. The constant pressure formation of steam in the boiler (and superheater) then occurs, and is shown by the line *gk1*, the point *1* representing the state of the vapor leaving the superheater. Thus, the *heat supplied by the boiler,*[8] in B.t.u. per pound of throttle flow, is

$$\begin{aligned} Q_1 = h_1 - h_g = h_1 - h_e - F \quad . \quad . \quad . \quad . \quad . \quad (288). \\ = \text{area } gk1mn, \text{ to scale.} \end{aligned}$$

In case there had been no regenerative heaters used, the boiler would have supplied the additional amount of heat shown by the area *n'lgn* in Fig. 191, in which the line lgk represents the constant pressure heating of the liquid at boiler pressure.

---

[8] See Sect. **204**(d).

(f) Knowing the net work and the value of Q_1, which is the heat supplied by the boiler, the *efficiency* of this regenerative vapor cycle becomes

$$e_{i\,(\text{reg. vap. cycle})} = \frac{Net\ Wk.\ [\text{from Eq. (287)}]}{Q_1\ [\text{from Eq. (288)}]}. \quad . \quad . \quad (289)$$

(g) The heat given to the condenser in the *ideal* plant, in B.t.u. per pound of throttle flow, is represented by the area $6a'n'm$ in Fig. 190. If the state of the steam entering the *actual* condenser is shown by $6'$, then the area $6'a'n'm'$ represents the amount of heat given to the condenser in the *real plant*, in B.t.u. per pound of throttle flow. The entropy at the state $6'$ is laid off from the same zero of entropy that was used for locating a'.

Also, comparing this figure with Fig. 175, it is apparent that with the regenerative cycle the heat supplied by the boiler, the net work of the cycle, and the heat given to the condenser, are all less than they are for the Rankine for the same throttle flow; but these quantities are not all reduced in the same proportion, and hence the cycle efficiencies of the two cases are not the same, being higher for the regenerative cycle. Or, in other words, with the regenerative cycle more steam must be used to do a given amount of work than with the Rankine, the exhaust pressure and throttle conditions being the same. This increase is due to the smaller work done per pound of throttle flow, but, nevertheless, it is accompanied by a considerable improvement in cycle efficiency, and hence in a lower consumption of heat per unit of work. Obviously, comparisons of the Rankine and regenerative plants on the basis of steam consumptions, or net work per pound, are not satisfactory. Comparisons of these two vapor cycles can only be made on the basis of heat consumption per unit of energy output or thermal efficiency.

218. The Ideal Reheating-Regenerative Engine Cycle Involving Reheating Losses and a Finite Number of Heaters.—(a) If a prime mover uses a combination of the reheating and regenerative principles, which were explained in the two preceding sections, it may very appropriately be referred to as operating on the " reheating-regenerative " cycle. The reheating in such a unit is likely to take place at the same pressure as that at which the first bleeding occurs; and that is the case that will now be analyzed.[9] For the test of a unit consisting of the turbine-generator and the bleeder heaters, as shown in Fig. 192, the ideal should have the same number of bleeder heaters and the same losses in the reheating system as in the actual case. For the test of the whole system through which the working substance flows, the corresponding ideal would be the *Reheating-Regenerative Vapor Cycle,* and it would be analyzed without allowing for any loss due to radiation or drop in pressure in the reheating system. The *engine* cycle will be considered first.

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[9] There may occasionally arise conditions such that the reheating occurs at a pressure lower than that at the first bleeding point; also more than a single reheating may be used. However, the analysis of those cases will not be given in this text, because no new principle is involved..

(b) Since the main purpose in developing such an ideal is to establish a standard with which the performance of the actual unit may be compared, this *ideal engine cycle* is therefore defined as fulfilling the following conditions:

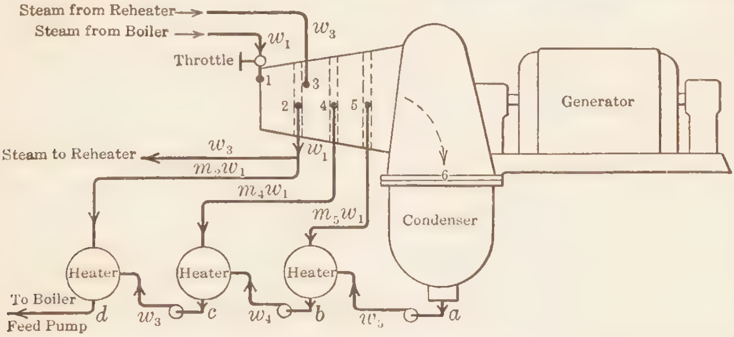

FIG. 192.—Ideal Reheating-Regenerative Unit with Three Feedwater Heaters.

(1) The steam enters the throttle in the same condition as in the actual case. This is shown by state *1* in Fig. 193.

(2) Isentropic expansion of all the steam entering the throttle takes place until the same pressure is reached as that at which the steam leaves the turbine for reheating in the actual case. This expansion is represented by the line *1 2* in Fig. 193.

(3) The same number of regenerative-heaters is used as in the actual case; and the same bleeding pressures are assumed as those actually observed at the bleeder nozzles in the real plant.

(4) Sufficient steam is bled at the state *2* to heat the feedwater to t_d, the saturation temperature corresponding to p_2.

(5) The pressure and temperature of the steam after reheating and return to the prime mover are to be the same as in the actual case. The corresponding state is shown by *3* in Fig. 193, with the pressure lower than p_2.

(6) The pressures p_4 and p_5 are taken the same as those at the corresponding bleeding nozzles in the actual case.

(7) The amounts of steam bled at *4* and *5* are just sufficient to heat at constant pressure, and without loss of any kind, the respective amounts of feedwater entering each corresponding heater, to the saturation temperatures for the pressures p_4 and p_5, respectively.

(8) Isentropic expansion is assumed to take place from *3* until the actual exhaust pressure p_6 is reached.

(9) Contact heaters, with individual pumps, are assumed, as in Sect. **216**.

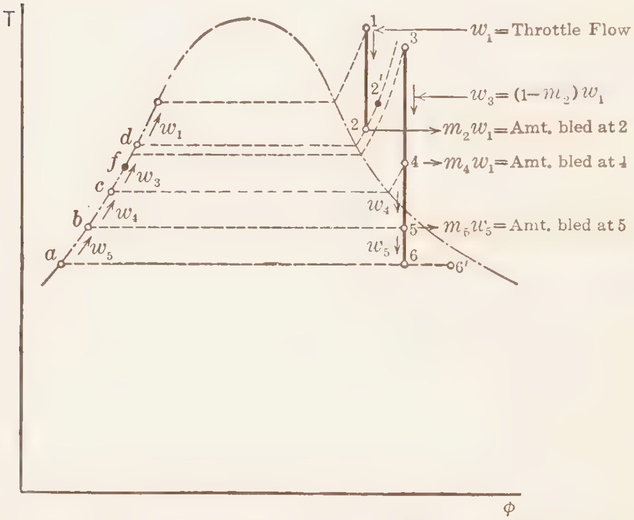

FIG. 193.—Diagram Showing the States of the Steam and Feedwater at Various Points in the Reheating-Regenerative Unit.

(c) In Figs. 192 and 193 w_1, w_3, w_4 and w_5 represent the weights of steam passing through the various parts of the ideal machine, and m_2, m_4 and m_5 are the fractional parts of the throttle flow, w_1, that are bled at the respective points. The state of the steam leaving the ideal prime mover at exhaust pressure is represented by point *6*, in the latter figure.

(d) The *amount of steam bled* at each point in the ideal case is found in the following manner. Since

$$m_2 w_1 (h_2 - h_d) = (1 - m_2) w_1 (h_d - h_c),$$

then

$$m_2 = \frac{h_d - h_c}{h_2 - h_c}. \qquad (290a)$$

Similarly, from

$$m_4 w_1 (h_4 - h_c) = (1 - m_2 - m_4) w_1 (h_c - h_b)$$

$$m_4 = \frac{(1 - m_2)(h_c - h_b)}{h_4 - h_b}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (290b)$$

and from

$$m_5 w_1 (h_5 - h_b) = (1 - m_2 - m_4 - m_5) w_1 (h_b - h_a)$$

$$m_5 = \frac{(1 - m_2 - m_4)(h_b - h_a)}{h_5 - h_a}. \quad . \quad . \quad . \quad . \quad . \qquad (290c)$$

(e) The *work done* by the steam in passing through this ideal unit, in B.t.u. per pound of throttle flow,[10] is

$$\begin{aligned} Wk_{i(\text{reh. reg.})} &= h_1 - h_2 + (1 - m_2)(h_3 - h_4) + (1 - m_2 - m_4) \times \\ &\qquad (h_4 - h_5) + (1 - m_2 - m_4 - m_5)(h_5 - h_6) \\ &= h_1 - h_2 + (1 - m_2)(h_3 - h_6) \\ &\qquad - m_4(h_4 - h_6) - m_5(h_5 - h_6), \quad . \qquad (291) \end{aligned}$$

and consequently the *ideal steam rate*, in lb. per kw-hr., is

$$w_{i\,(\text{reh. reg.})} = \frac{3413}{Wk_i\ [\text{from Eq. (291)}]}. \quad . \quad . \quad . \quad . \qquad (292)$$

(f) The *energy supplied* to the ideal unit, in B.t.u. per pound of throttle flow, is $(h_1 - h_d) + (1 - m_2)(h_3 - h_2)$ and consequently the *efficiency* of this ideal engine cycle is

$$e_{i\,(\text{reh. reg.})} = \frac{Wk_i\ [\text{from Eq. (291)}]}{(h_1 - h_d) + (1 - m_2)(h_3 - h_2)}. \qquad (293)$$

(g) For the *actual* reheating-regenerative unit the feedwater will be delivered from the last heater with a temperature t_f which will usually be lower than t_d for the corresponding ideal. The corresponding state is shown by f in Fig. 193. The steam leaving the real turbine for reheating will be at some state such as that represented by $2'$, which must always be at a higher quality than 2 for the corresponding ideal case. Furthermore, the amount of steam bled at $2'$ in the actual case will in general be different from that bled at 2 in the ideal. Consequently the weight of steam being reheated in the actual case will not be the same as in the ideal. Let the amount bled at state $2'$ in the actual case be represented in terms of the throttle flow, by $m_{2'} w_1$. Then the *energy supplied* to the actual unit, in B.t.u. per pound of throttle flow, is $(h_1 - h_f) + (1 - m_{2'})(h_3 - h_{2'})$.

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[10] This equation neglects the small amount of energy expended upon the ideal heater pumps of this unit.

If the *actual steam rate*, as determined by the throttle flow in pounds per kw-hr. of the generator output, be represented by w_a, then the *combined thermal efficiency* of this actual unit is

$$\text{C. Th. Eff.}_{\text{(reh. reg.)}} = \frac{3413}{w_a[(h_1 - h_f) + (1 - m_{2'})(h_3 - h_{2'})]}. \tag{294}$$

The *combined engine efficiency* of the actual turbine-generator and regenerative feed water heaters (but not the reheater) is, therefore,

$$\text{C. Eng. Eff.}_{\text{(reh. reg.)}} = \frac{\text{C. Th. Eff. [from Eq. (294)]}}{\text{Ideal Cycle Eff. [from Eq. (293)]}}. \tag{295a}$$

or

$$\text{C. Eng. Eff.}_{\text{(reh. reg.)}} = \frac{\text{Energy supplied per kw-hr., ideal case}}{\text{Energy supplied per kw-hr., actual case}}$$

$$= \frac{w_i[(h_1 - h_d) + (1 - m_2)(h_3 - h_2)]}{w_a[(h_1 - h_f) + (1 - m_{2'})(h_3 - h_{2'})]}. \tag{295b}$$

It is apparent from this last equation that the ratio of steam rates cannot give the engine efficiency for this case. This is because h_d is different from h_f, m_2 is not the same as $m_{2'}$, and h_2 is always less than $h_{2'}$. Hence, in making a **thermal economy test of an actual reheating-regenerative unit** it is necessary to know the weight of steam being reheated, as well as the amount of throttle flow. It is also necessary to observe the condition of the steam when it is leaving the turbine and when received by it after reheating; but it is not necessary to know the state of the steam leaving the reheater, because the reheating system is not considered as being a part of the unit under test.

(**h**) The combination of the reheating and regenerative principles in the modern steam plant is a very attractive one. The ideal cycle efficiency is high and the reheating improves the engine efficiency of the turbine, hence the thermal efficiency of the unit is high. When there is also a good boiler-room efficiency, as is the case in many modern plants, the *plant thermal efficiency*, which includes the performance of the combustion equipment, will also be high.

(**i**) In case it is desired to find *the engine efficiency of the unit* **including the reheater losses,** the corresponding ideal engine cycle is modified by moving the point *3*, in Fig. 193, up to the constant pressure line passing through p_2 and to a temperature equal to that observed for the steam as it leaves the actual reheater, instead of that at which it is returned to the turbine. With the corresponding changes in the values of h_3, h_4, h_5 and h_6 incorporated in Eqs. (290), (291), (292) and (293), these equations may then be used for this case without further modification.

219. The Reheating-Regenerative Vapor Cycle.—(a) This cycle involves the reheating-regenerative unit described in the preceding section and in addition includes the passage of the fluid through the feed pump, boiler and reheater, each having no losses. The ideal cycle may be represented by the heavy lines in Fig. 194, in which *1* shows the state of the steam corresponding to the pressure and temperature of the vapor as it enters the pipe line at the boiler in the actual plant. The condition of the steam at the beginning of reheating and at the first bleeding point is represented by *2*, while *3* shows the state of the steam having the same pressures as p_2 and a temperature equal to that of the steam leaving the actual reheater, because in this ideal vapor cycle the reheating system is assumed to cause no loss of pressure or temperature. The states *4* and *5* are found by drawing the constant entropy line from *3* until it intersects the two constant pressure lines that are in agreement with the actual bleeding pressures used; and p_6 represents the actual condenser pressure. The state d shows the saturated liquid at the pressure p_2 and represents the condition of the feedwater leaving the heater having the highest pressure in the ideal case. The feedwater as it leaves this heater in the *actual* plant will usually be at a lower temperature than t_d.

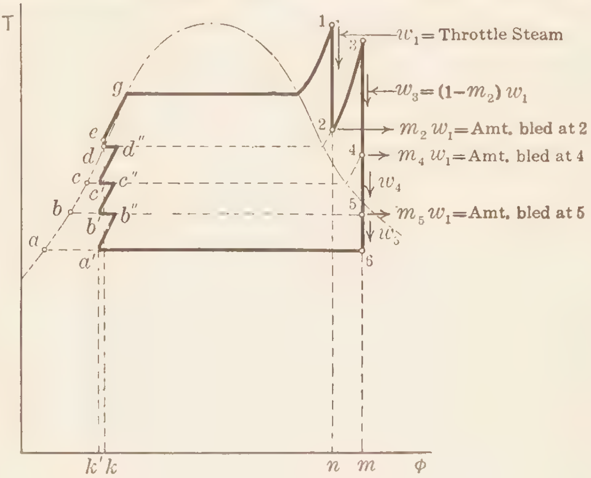

Fig. 194.

(b) The *amounts of steam bled* at *2*, *4* and *5* may be found by using Eq. (290*a*, *b* and *c*), but the results will be slightly different from those for the case considered in Sect. **218** because the values of h_2, h_3, h_4 and h_5 are now not exactly the same as when the losses in the reheating system and in the main steam line were not considered in the ideal cycle. Having found the values of m_2, m_4 and m_5, the state a', b', b'', c', c'' and d'' may be found in a manner similar to that explained in Sect. **217** (b).

(c) The *heat supplied* by the boiler, superheater and the reheater, in B.t.u. per pound of throttle flow, in the ideal case, is

$$Q_1 = (h_1 - h_e) + (1 - m_2)(h_3 - h_2). \quad . \quad . \quad . \quad (296a)$$

$$= (h_1 - h_d - F) + (1 - m_2)(h_3 - h_2). \quad . \quad (296b)$$

$$= \text{area } d\ e\ g\ 1\ 2\ 3\ m\ k \text{ in Fig. 194.}$$

The value of the *feed pump work*, in B.t.u. per pound of throttle flow, is

$$F = A(P_e - P_d)\overline{V}_d.$$

(d) The *heat rejected* to the condenser in the ideal case, in B.t.u. per pound of throttle flow, is

$$Q_2 = (1 - m_2 - m_4 - m_5)(h_6 - h_a) \quad . \quad . \quad (297)$$

$$= \text{area } a'6mk', \text{ to scale.}$$

(e) The *net work* of this ideal vapor cycle, in B.t.u. per pound of throttle flow, is

$$Net\ Wk_{(\text{reh.reg.vap.})} = h_1 - h_2 + (1 - m_2)\ (h_3 - h_6)$$

$$- m_4(h_4 - h_6) - m_5(h_5 - h_6) - F - F' \quad (298).$$

$$= \text{area } 1\ 2\ 3\ 6\ a'\ b''\ b'\ c''\ c'\ d''\ d\ e\ g\ 1 \text{ in Fig. 194.}$$

In this equation the small amount of energy required to operate the ideal heater pumps, in B.t.u. per pound of throttle flow, is

$$F' = (1 - m_2)A(P_d - P_c)\overline{V}_c + (1 - m_2 - m_4)A(P_c - P_b)\overline{V}_b$$

$$+ (1 - m_2 - m_4 - m_5)A(P_b - P_a)\overline{V}_a. \quad . \quad (299)$$

Hence the efficiency of this ideal vapor cycle is

$$e_{i\,(\text{reh. reg. vap.})} = \frac{Net\ Wk.\ [\text{from Eq. (298)}]}{Q_1\ [\text{from Eq. (296)}]}. \quad . \quad . \quad . \quad (300)$$

$$= \frac{\text{area } 1\ 2\ 3\ 6\ a'\ b''\ b'\ c''\ c'\ d''\ d\ e\ g\ 1}{\text{area } d\ e\ g\ 1\ 2\ 3\ m\ k}.$$

220. The Binary Vapor Cycle.—(a) From a study of the vapor cycles thus far considered it is apparent that the cycle efficiency is improved by increasing the temperature drop occurring in the engine, and by having the addition of heat to, and its abstraction from, the working substance occur while the latter is at the highest and lowest temperatures, respectively. As has already been explained, the upper temperature is limited by the properties of the structural materials available; and it may be further restricted by the vapor pressure that it may entail. The final temperature cannot be reduced below that of the coldest cooling medium,—usually the condensing water. If steam is the working substance, and if vaporization is to occur at the maximum temperature, no superheat will be used. If the temperature is raised much above 600 deg. fahr., the saturation pressure of this material becomes very high. Furthermore, if the exhaust temperature is greatly reduced, the saturation pressure of steam becomes very low and thus may lead to various difficulties. Hence, it may be desirable, under certain circumstances, to use in the same system two or more working substances, each functioning through the temperature range in which it is best suited. In this case the condensation of the first fluid, after passing through its engine, causes the second substance to form vapor which will have a very different pressure from the first and which is used in a second engine. The process could be continued further if commercially feasible and if suitable substances were available. The use of two fluids in this manner gives rise to what is called the *Binary Vapor Cycle.*

(b) **Steam and sulphur dioxide** were tried in a binary cycle by Professor Josse of Germany, about a quarter of a century ago. The exhaust steam from a reciprocating engine was delivered at an absolute pressure of about 5 in. of Hg to a surface condenser which was also the boiler for the sulphur dioxide. The SO_2-vapor thus formed was at an absolute pressure of nearly 200 lb. per sq. in. and was used in a second engine, the exhaust pressure of which was maintained at about 50 lb. per sq. in. abs. by use of a condenser having circulating water at about 50 deg. fahr. Tests of this apparatus are reported in Professor Peabody's "Thermodynamics of the Steam Engine," p. 282. One serious objection to SO_2 is that sulphurous acid is formed by it when combined with water.

Ether has also been tried experimentally as one of the binary fluids but it is very inflammable and there are other objections to its use.

(c) **Mercury and steam** as a binary combination have been investigated on an extensive scale by the General Electric Company. The chief advantage of mercury is that its saturation pressures are very much lower than those for steam of the same temperature. Thus for example, at temperatures of 800 and 900 deg. fahr. the saturation pressures of mercury are only about 45 and 95 lb. per sq. in. abs., respectively. Hence, if a satisfactory mercury boiler can be made, this binary vapor cycle will possess many advantages.

(d) The *mercury-steam* cycle may use a large range of temperatures and yet have only moderate pressures. One combination of possible pressures and temperatures of both fluids is given in Fig. 195, the apparatus being shown diagrammatically in Fig. 196. In this case mercury is vaporized in a mercury boiler at an absolute pressure of 85 lb. per sq. in. and a temperature of 884 deg. fahr., which is about as high as is feasible to use with any material now available. After passing through the mercury turbine, this fluid is then condensed at an absolute pressure of 1.5 in. Hg and a temperature of 438 deg. fahr. By using about 9 lb. of mercury under these conditions, sufficient latent heat is given up to vaporize 1 lb. of water at an absolute pressure of 300 lb. per sq. in.; thus the mercury condenser becomes the steam boiler. A certain drop in temperature must occur between these two fluids in order to effect a rapid rate of heat transfer between them. In Fig. 195 this temperature drop is taken as 20.5 deg. fahr. The steam is next superheated 100 deg. by means of the furnace gases that have already passed through the mercury boiler, as indicated in Fig. 196. This superheated steam may then be used in any of the cycles previously described. For the case shown in Fig. 195, the Rankine cycle, with an exhaust pressure of 1 in. Hg abs., has been represented.

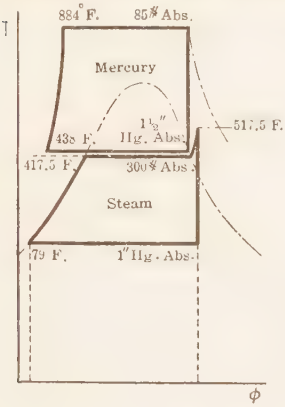

Fig. 195.

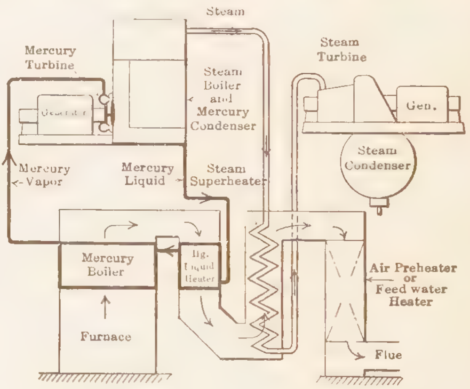

Fig. 196.

(e) The *efficiency* of this ideal mercury-steam cycle will obviously be high when such extremes of temperature obtain. The only questionable points concerning the use of this cycle in actual plants are those relating to the first cost, reliability, and maintenance. A satisfactory boiler for mercury is a most difficult piece of apparatus to design. This material does not "wet" the surfaces, and as a consequence the tubes may become overheated if the design is faulty. Mr. Emmet of the General Electric Co. has investigated many different types of boilers and a small one has been in operation in Hartford, Connecticut.

There have been many articles published in recent years on the subject of mercury-steam plants and many interesting details, including tables of the properties of mercury, are to be found in them.[11]

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[11] See The Possibilities of Mercury as a Working Substance for Binary Fluid Turbines, by Wm. J. Kearton, The Institution of Mechanical Engineers (British)

(f) **Diphenyl oxide and steam** have been suggested[12] by Dow as being the best combination of fluids for a binary vapor cycle. The temperature-entropy diagrams of this substance, together with those for mercury and water, are shown in Fig. 197 for different weights of these materials; and the values of the saturation pressure of these three fluids at various temperatures are given in Fig. 198. From these curves, it is apparent that, with a temperature of 900 deg. fahr., the saturation pressure of diphenyl oxide is slightly under 400 lb. per sq. in. This fluid could be used in the high temperature region and steam in the lower one, in the same manner that mercury and steam are employed.

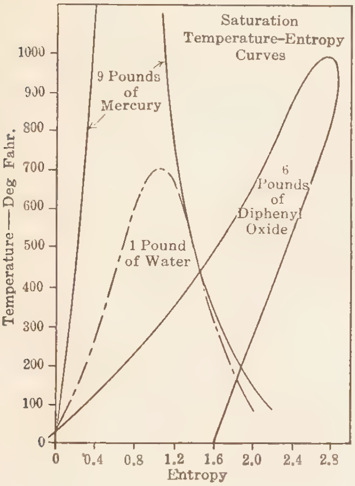

FIG. 197.

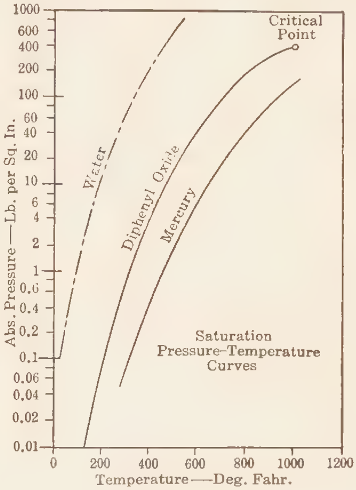

FIG. 198.

(g) The *ideal working substance* to use in a vapor cycle would be one fulfilling the following conditions:

1. The saturation pressures corresponding to the temperatures used at the throttle should not be so high that they cannot be *commercially* used.
2. The saturation pressure corresponding to the temperature of the

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Nov., 1923; The Emmet Mercury-Vapor Process, by W. L. R. Emmet and L. A. Sheldon, Trans. A.S.M.E., 1924, p. 253; also Mechanical Engineering, May, 1924, p. 235; The Binary Cycles Using Mercury and Steam, by Arthur M. Greene, Jr., Mechanical Engineering, March, 1924, p. 142; Mercury Vapor Process by W. L. R. Emmet, Franklin Institute Journal, Feb., 1925; Saturation Pressures of Mercury to 2000 kg. per sq. in., by Fritz Bernhardt, Physikalische Zeit., March 15, 1925.

[12] Diphenyl Oxide Bi-Fluid Power Plants, by H. H. Dow, Mechanical Engineering, Aug., 1926, p. 815.

cooling water for the condenser should not be so low that it involves extreme difficulty in maintaining the necessary vacuum.

3. At the throttle temperature there should be a high ratio of the latent heat of the vapor to the specific heat of the liquid, in order that the Rankine cycle may approach more closely the Carnot.
4. The ratio of the density of the vapor to that of the liquid should be small in order to keep the ratio of the work done by the engine to that expended by the feed pump as large as possible.
5. It should be available in large quantities at a low cost.
6. The liquid should have the property of wetting metallic surfaces, thus making possible the rapid transfer of heat through boiler tubes without burning them.
7. It should not be injurious to human beings or materials with which it may come in contact.
8. It should not decompose.

It is hardly necessary to add that no fluid known fulfills all of these requirements perfectly. Steam complies with the second condition, and also with the last four, better than does either mercury or diphenyl oxide; but both of the latter are far superior to it regarding the first item. Mercury is expensive, and its vapor is very poisonous when inhaled; therefore special precautions must be taken to prevent its leakage. Diphenyl oxide is superior to both steam and mercury regarding the slope of the curve representing the saturated vapor, because it can be permitted to have a large expansion ratio in a turbine without having the detrimental effects of a low quality. (Moisture is especially injurious to the blades of a turbine, as will be seen in a later chapter.) This material is inexpensive relative to mercury and can probably be obtained in large quantities. It is a new working substance that gives promise of becoming of much importance in power plants. Several materials other than those mentioned have been used in binary cycles, and still others may be found to be suitable.[13]

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[13] For a discussion of the relative merits of a number of fluids that may be used, see *Vapors for Heat Engines* by William D. Ennis, published by D. Van Nostrand Co.

CHAPTER XIX

THE RECIPROCATING STEAM ENGINE

221. Introductory.—Now that the ideal cycles of vapors and vapor engines have been analyzed it is necessary to consider the actual engines, both reciprocating and turbine. The names of the various parts of the mechanism and the method of operation of the engine will be given, the nature and magnitude of the losses that determine the engine efficiency will be discussed, and the classification and types will be illustrated. In this chapter only the reciprocating engine will be treated, while later chapters will be devoted entirely to the turbine. In case the expansion takes place in one cylinder only, the engine is called a **simple** one as contrasted with the **compound** in which it occurs in two or more cylinders in series. Because of their great importance, valve gears and governors will each be treated in special chapters in Part II.

222. Steam-Engine Parts.—(a) One of the simplest forms of a double-acting steam engine is shown diagrammatically in Fig. 199. The principal parts of the engine are generally grouped as follows:

1. **Stationary parts,**—which include the cylinder, cylinder heads (bonnets), steam-chest cover, stuffing boxes and packing, engine frame, outer bearing, and subbase, if used.

2. **Rotating parts,**—consisting of the shaft, crank (disk), fly-wheel, and eccentric.

3. **Reciprocating parts,**—the piston, piston rings, piston rod, crosshead, and connecting rod.

4. **Valve gear,**—valve, valve stem (rod), valve-rod guide (or rocker arm), eccentric rod, eccentric strap, and eccentric sheave (or " eccentric ").

5. The **governor.**

(**b**) The steadiness of the rotative speed of the engine during the revolution, that is, during the completion of one cycle on each side of the piston, is controlled by the flywheel. The number of revolutions, or number of cycles, per minute—which is usually called the engine " speed "—in " automatic " engines is regulated by the self-acting governor, which in Fig. 199 is of the " throttling," fly-ball type.

The starting and stopping of the engine is accomplished by means of the hand-operated throttle valve which, in special cases, may also be used to regulate the operation of the engine.

(c) Engines usually have the following **fittings**: drain cocks for cylinder and steam chest; cocks for attaching indicators; lubricating devices for bearings, guides, and cylinders; and shields to collect oil thrown by the crank, the connecting rod, and the eccentric.

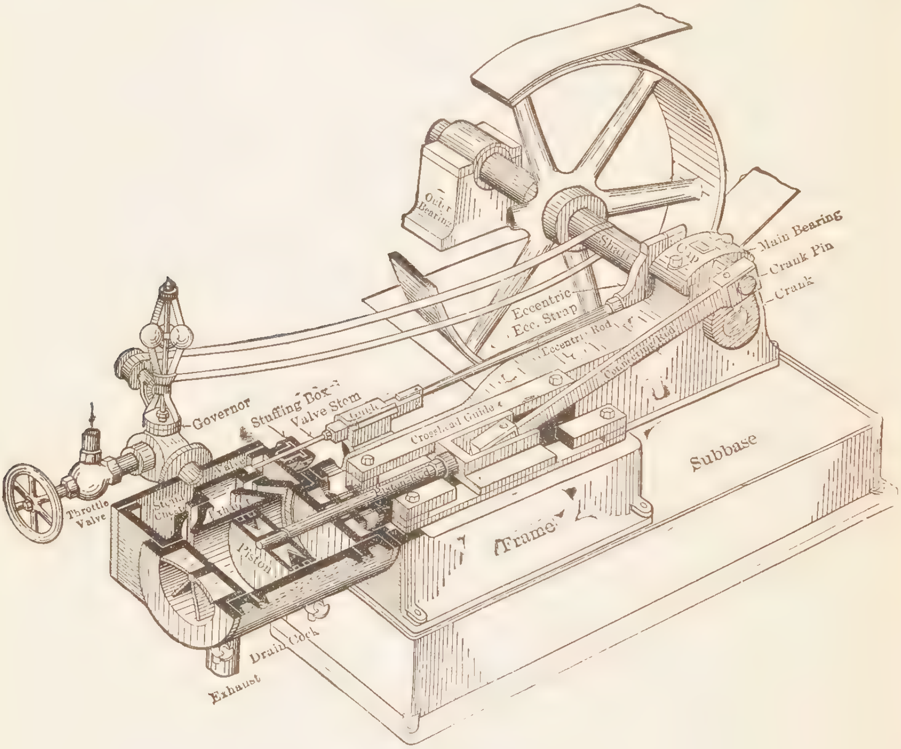

Fig. 199.

(d) Engines are mounted on masonry or concrete foundations sufficiently massive to prevent noticeable vibration being induced in the surroundings. They are fastened to the foundation by " anchor," or " foundation," bolts.

(e) The **material used** for making the subbase, frame and flywheel is cast iron. Special cast iron is generally employed for the cylinder and slide valve. The piston rod, connecting rod and shaft are made of steel. The main bearing and crank pin " boxes " are lined with babbitt or other antifriction metal. The piston rings, which prevent leakage of steam past the piston are usually of cast iron, and the packing which is used in the stuffing boxes to prevent leakage around the piston rod and valve stem is generally either fibrous ("soft packing") or metallic.

223. Method of Operation.—(a) In some reciprocating engines both the delivery of steam to and its discharge from the cylinder are controlled by the respective edges of the same valve, and in others each of these operations is accomplished by a separate valve. For the sake of clearness, only the latter arrangement will be considered at this point. In this case there is an *inlet* or *admission* valve that is so operated that it permits steam to enter the cylinder during a certain portion of the working stroke of the piston, which is therefore called the *admission period.* This period terminates when the inlet valve closes, or the supply of steam from the throttle is "*cut off.*" Clearly then the later this cut-off occurs the greater will be the amount of steam admitted per stroke. After cut-off the definite weight of steam now in the engine expands, during the so-called *expansion period,* with a rapid drop in pressure, until the piston reaches a point near the end of its stroke when "*release*" occurs because the exhaust valve then allows the steam to escape from the engine. The *exhaust period* continues until the piston has completed a certain portion of the return stroke, when the flow of steam from the cylinder is stopped by the closure of the exhaust ports. "*Compression*" then begins because a small amount of steam is still in the engine and is compressed during the remainder of the return stroke of the piston. At some point very near the end of this return stroke the admission valve is again opened, and consequently the admission period of the next cycle begins.

(b) The final *compression pressure,* p_a, is determined in any engine by the following factors:

(1) The *clearance volume,* which is fixed by the manufacturer of the engine;

(2) The *exhaust pressure,* p_k, which depends on the operating conditions, such as whether the engine is run condensing or non-condensing;

(3) The *per cent of the stroke* at which *compression begins,* or in other words, the ratio l_k/l in Fig. 200, which is fixed by the valve setting of the engine;

(4) The *amount of leakage* of steam that may take place past the piston and valves during the compression period;

(5) The *amount of heat transferred* to or from the steam by the metal walls with which it is in contact during the compression period.

Of these five factors the first three are usually by far the most important, and they are all directly under the control of the designer and operator of the engine.

(c) The *best compression pressure* to use in any particular engine is somewhat difficult to determine. In all cases, however, an appreciable amount is desirable so that the reciprocating masses may be properly "cushioned," thus making the engine run smoother. There is also a thermodynamic advantage in having, when the admission valve opens, the clearance space filled with steam at a temperature much higher than that at exhaust. Just what ratio of admission pressure to compression pressure should be used in any particular engine is a matter requiring considerable study by the designer.

(b) A typical indicator diagram of a steam engine of the usual type is shown in Fig. 200. Here the admission begins at *a* and continues to cut-off at *c*, the drop in pressure being due to throttling or "wire-drawing." The expansion period is represented by *cr*. Release occurs at *r* and the exhaust period is shown by *rek*. Exhaust closure occurs at *k*, and *ka* represents the compression period. The net area *abcreka* obviously represents the net work delivered to one side of the piston during one revolution of the engine. Of course, if the engine is a double-acting one, as is generally the case, there would be a similar diagram, but reversed, to represent the action of the steam on the other side of the piston. Having obtained such a diagram from an engine, by means of an indicator such as was described in Sect. **92**, the indicated horsepower of the engine may be found by using Eq. (130).

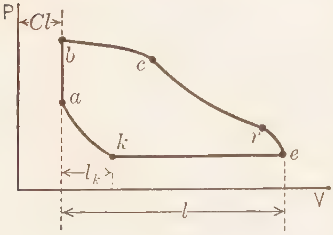

FIG. 200.

224. Clearance and Compression.—In the ideal steam engines previously discussed it was considered that no steam space existed between the cylinder head and the piston at the beginning of the stroke, and hence that the initial volume of steam in the cylinder was zero. In real engines, however, the piston must not touch the cylinder head, and the shortest distance between these parts is generally from $\frac{1}{8}$ inch to $\frac{3}{4}$ inch or more, and is called the "*mechanical clearance.*" The cubical contents of this space, including the passages to the valves and all other spaces that must be filled with steam before the commencement of the stroke, is termed the "**clearance volume.**" It equals the volume V_b shown by *Cl* in Fig. 200, and is commonly stated as a percentage of the piston displacement. It varies from 2 to 15 per cent, and depends on the type and design of the engine.

225. Mechanical Losses.—(a) In simple well-built steam engines that are in good condition the losses that are due to friction of the several parts moving relative to each other are very small when compared with the energy supplied by the working substance. The factors that contribute to the friction load are the piston, shaft bearings, crank pin, crosshead pin and guides, valves and their operating mechanism, the packing in the stuffing boxes, and the flywheel " windage."

(b) If the engine is running at constant speed the mechanical losses will be almost the same regardless of the load, and, therefore, the **mechanical efficiency** (or *b.hp./i.hp.*[1]) will be zero at no load, and will increase as the b.hp. becomes greater. Its values for good reciprocating steam engines under normal loading are usually well above 90 per cent, and consequently the main losses of this type of prime mover are not mechanical but thermodynamic, as may be seen from the next section. However, as the total amount of steam used for a given output is directly dependent on the mechanical efficiency, the friction losses should be reduced as much as possible.

226. Behavior of Steam in Passing through an Actual Engine.—(a) In the ideal engine, steam was considered to flow through it without any loss due to *throttling*, *leakage*, or *transfer of heat* to and from the engine parts; but all of these factors very materially affect the behavior of steam in real engines.

(b) As the steam passes through the throttle valve and through the more or less restricted opening of the admission valve, its pressure and temperature are reduced by **throttling.** This reduces the available energy, as shown in Sect. **206**; but, in general, this throttling does not cause a large loss because the drop in pressure is usually small in the better types of engines and because the *quality* of the steam is increased somewhat. The better quality and reduced temperature of the throttled steam both tend to diminish slightly the cylinder condensation in the manner that will be explained in a later paragraph of this section. Hence the engine efficiency is not reduced by throttling in quite the same proportion as is the available energy.

(c) Though an engine has been operating during a sufficient length of time to become thoroughly warm, the metallic surfaces, with which the admission and exhaust steam is brought in contact, will pass through a **cyclic change of temperature** during each revolution of the engine. The reasons for this, and what the result will be, will now be shown.

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[1] See definition given in Sect. **143**.

When the steam enters the clearance space in the cylinder it comes in contact with surfaces that were *cooled during the period of exhaust in the preceding cycle.* These surfaces are good conductors of heat and, since they are at a lower temperature than the incoming steam, there will be a rapid transfer of heat to them. If the entering vapor is not superheated to an appreciable degree there will be a considerable portion of it *condensed* on these surfaces, and this phenomenon is spoken of as **initial condensation.** It causes a very serious loss, since from 10 to 40 per cent of the entering steam is condensed in this way.

As the piston moves away from the cylinder head, additional cold walls are uncovered, and further condensation takes place (unless the steam is superheated). If the steam received by the engine is initially dry saturated, the *cylinder condensation* may be sufficient to reduce the quality of the steam at cut-off to 60 per cent, or even less in extreme cases. Thus, unless special means [2] are taken to prevent this condensation, it is the main factor in reducing the engine efficiency. Much of the moisture formed is deposited on the surfaces of the cylinder walls, including the heads and passages.

(d) After cut-off has occurred, the drop in steam pressure and temperature will soon become sufficient to prevent further transfer of heat to the cylinder walls; but the total amount of heat that has thus been absorbed by the cylinder up to this time is relatively very large. During the latter part of the expansion the average temperature of the steam drops below that of the cylinder surfaces, and transfer of heat from the walls to the condensed vapor, which has been deposited on these surfaces, then produces a partial reevaporation that is usually sufficient to cause the quality of the steam to increase. The only factor that tends to prevent this increase in quality is the transformation of heat into work by the expansion, which is accompanied by a certain amount of condensation, similar to that shown to be the case with the ideal engine.

(e) *Release* usually occurs before the exhaust reaches the back pressure, and there is therefore a certain loss due to **incomplete expansion.** The magnitude of this loss can be estimated from the actual indicator diagram by continuing the expansion line until it intersects the exhaust pressure line, so as to determine the area of the missing "toe."

During release the steam is dried to some extent by the heat that is given up by the cylinder wall to the steam, which is now at a low temperature, and also by the heat released from the steam itself while undergoing the free expansion.

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[2] Such as will be discussed in Sect. **230.**

During *exhaust* the confining walls are cooled somewhat by contact with the outflowing steam, and very appreciably by the **evaporation** of the film of moisture from the walls. The greater the amount of moisture evaporated the cooler will the walls become, and consequently the greater will be the *amount of steam condensed during admission in the next stroke.* The exhaust, or **back pressure,** is somewhat higher than in the corresponding ideal engine because of the resistance to steam flow offered by the valve opening and passages.

(f) During **compression** the quality of the steam is indeterminate, unless its value is known at the beginning of that process. It may be assumed, however, that the steam is dry at this point, and the results will be accurate enough for most practical purposes, because of the small weight of steam involved. During the first part of compression the steam may become slightly superheated owing to the reception of heat from the hotter walls of the cylinder. If the compression is high the temperature of the vapor may rise above that of the cylinder walls, in which case a transfer of heat takes place from the steam to the walls.

(g) The **leakage** of steam past the valves and around the piston will modify the actual diagram and cause the engine to use more steam than it should.

The loss of heat from the cylinder walls by **radiation, conduction,** and **convection** lowers the mean temperature of the walls and adds slightly to the condensation, but in the best engines the cylinders are so well lagged with non-conducting material that this loss is usually small, generally far less than 2 per cent of the energy supplied. This percentage may be more than doubled if high temperature steam is used in an engine not thoroughly lagged.

(h) From the foregoing it is evident that the main factor in reducing the indicated engine efficiency is cylinder condensation—the losses due to throttling, leakage, and incomplete expansion being relatively small. The distribution of these losses may be seen from the **energy stream** given in Fig. 201, which is for a steam engine giving a rather good performance. This figure also shows the relatively small portion of the energy supplied that is lost by mechanical friction and that eventually leaves the engine by radiation and conduction. It must be clearly recognized that the steam that is condensed upon entrance to the cylinder, and then later reevaporated during the exhaust period, is stealing its way through the cylinder without doing any useful work, and consequently the loss due to cylinder condensation appears as a large item in the exhaust steam. The losses due to throttling, incomplete expansion and leakage (except that past the stuffing boxes) also increase the energy going out with the exhaust steam. The value of E_s, the energy supplied to the engine, has previously been defined as the difference between the heat content of the steam at the throttle and the heat content of the saturated liquid at exhaust pressure. From Fig. 201 it should be carefully noted what the approximate values of the different losses are when based not only on the available energy E_a, but also on E_s.

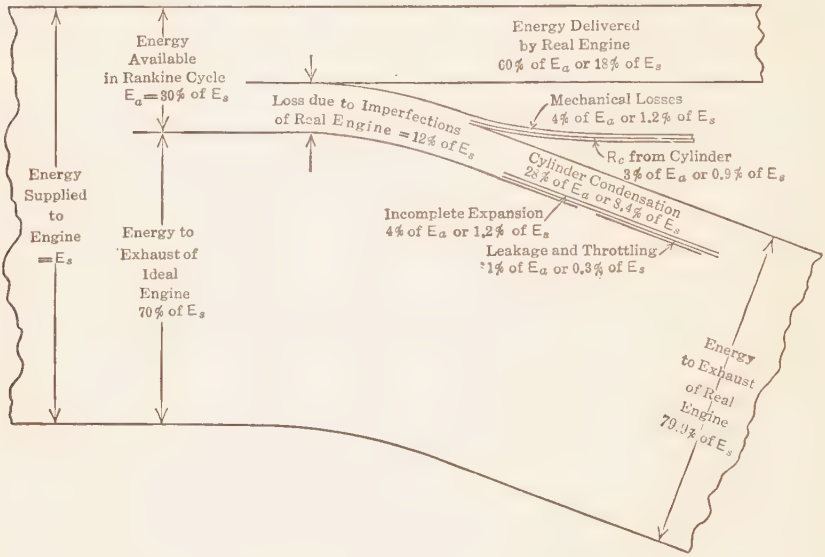
Energy Delivered by Real Engine 60% of E_a or 18% of E_s
Energy Available in Rankine Cycle E_a=30% of E_s
Loss due to Imperfections of Real Engine =12% of E_s
Mechanical Losses 4% of E_a or 1.2% of E_s
R_c from Cylinder 3% of E_a or 0.9% of E_s
Energy Supplied to Engine =E_s
Cylinder Condensation 28% of E_a or 8.4% of E_s
Incomplete Expansion 4% of E_a or 1.2% of E_s
Energy to Exhaust of Ideal Engine 70% of E_s
Leakage and Throttling 1% of E_a or 0.3% of E_s
Energy to Exhaust of Real Engine 79.9% of E_s

FIG. 201.—Energy Stream for a High Grade Steam Engine.

227. Determining the Quality of Steam in the Cylinder.—(a) Since cylinder condensation may cause such a large loss in an engine cylinder, the methods of reducing it become important. In order to determine the magnitude of these losses it is desirable to be able to calculate the quality of the steam in the cylinder. This can only be done for the expansion period. First, however, it is necessary to obtain a good indicator diagram and, by test, to find the weight of steam admitted each cycle, or the **cylinder feed**, w_f, as it is called. It is also essential to calculate the weight of steam entrapped during compression, or the **cushion steam**, w_k. Then from cut-off to release the weight of steam in the cylinder will be $w_f + w_k$, provided there has been no leakage. The quality of the steam in the cylinder during expansion may then be determined by the following method.

(b) Assume that the indicator diagram obtained from the engine under test is that given in Fig. 202, and that the clearance is known and may therefore be drawn on this diagram as shown. Then, by the aid of steam tables, the saturated vapor curve for the weight $w_f + w_k$ is drawn on this diagram. This curve ss' will usually represent a considerably larger volume of steam than does the actual expansion line cr, due to the cylinder condensation. The quality at cut-off is therefore

$$x_c = \frac{V_c}{(w_f + w_k)\overline{V}_d} = \frac{cf}{df}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (301a)$$

and at release the quality is

$$x_r = \frac{V_r}{(w_f + w_k)\overline{V}_{s'}} = \frac{rz}{s'z}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (301b)$$

By scaling the diagram the quality of the steam at any other point during the expansion may be obtained in a similar manner, and then, by establishing a suitable quality scale, as indicated in Fig. 202, the **quality curve** may be drawn, as shown. At some point after cut-off, such as g, the steam has its minimum quality because beyond that point the absorption of heat from the cylinder walls is sufficient to overcome all other effects and increase the quality of the steam.

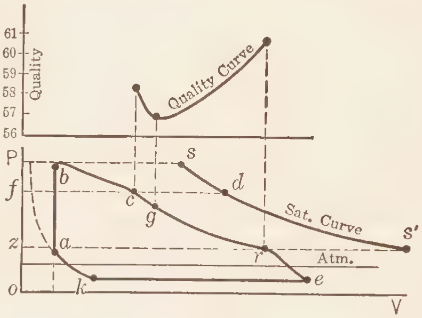

FIG. 202.

(c) The true quality of the *cushion steam* throughout compression is difficult to determine, but it is customary to assume it dry when the process begins. Since almost immediately after release the steam pressure drops to the back pressure, there is but little steam in the cylinder soon after the beginning of the back stroke. That part subsequently trapped in the clearance and compressed is subject to the higher temperature of the cylinder walls throughout nearly the whole of the return stroke, hence it must be practically dry when compression begins. The weight of cushion steam is so small relative to the cylinder feed that even though its quality is not exactly unity the results will not be affected materially.

With this assumption, the weight of the cushion steam at k, Fig. 202, can be computed from

$$w_k = V_k \div \overline{V}_k, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (302)$$

in which V_k can be scaled from diagram and $\overline{V}_k$ is the specific volume of the saturated steam at the pressure p_k existing.

The case shown in Fig. 202 is not extreme but rather typical of a simple engine of the ordinary type that is supplied with throttle steam having 100 per cent quality, or even a small amount of superheat. The quality at cut-off will probably then be somewhere in the neighborhood of 60 per cent.

228. Hirn's Analysis.—(a) With good indicator diagrams available and certain test data that are easily obtained, it is possible to analyze the diagram so as to ascertain the amount of heat that is transferred to or from the steam during each period of the cycle. Since this analysis was first given by Hirn (in 1876) it is commonly referred to by his name. The method used involves merely an application of the law of conservation of energy to the steam engine during any or all of the periods composing its complete cycle.

(b) The data needed for this analysis are:

(1) The weight, w_f, of the cylinder feed per cycle;

(2) The quality, x_k, and weight, w_k, of the cushion steam per cycle;

(3) The state of the steam at the throttle, so that its heat content, h_t, may be determined;

(4) An average indicator diagram, such as that represented in Fig. 203, from which the quality x_c at cut-off and also x_r at release may be obtained by the method previously explained. This diagram also furnishes the means of determining the amount of energy delivered *to* the piston by the steam during each period of the cycle. These amounts are shown by the areas that are cross-hatched and labeled by A_a, A_c and A_r, respectively. The areas A_e and A_k show the energy delivered *by* the piston to the steam during the exhaust and compression periods.

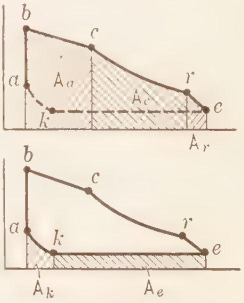

FIG. 203.

(5) The dimensions of the engine and the scale of the indicator spring, by means of which the number of B.t.u., B, represented by a unit area of the diagram may be calculated. The scale of the spring also permits finding the absolute pressures at all points desired.

(6) The weight, w_2, of circulating water used to condense w_f pounds of steam; the circulating water temperatures t_i and t_o at inlet and outlet of the surface condenser; and also the temperature of the condensate leaving the hot well, so that the heat content of this liquid may be found.

TABLE XII

HIRN'S ANALYSIS

General Equation	Internal energy of the steam in the cylinder at the beginning of the period	+ Energy added by the admission of fresh steam	= Internal energy of the steam in the cylinder at the end of the period	+ Work done on the piston by the steam in the cylinder during the period	+ Energy going to the condenser with the exhaust steam	+ Energy transferred to the cylinder walls during the period (To balance the equation)
Admission Period	$w_k i_a$	$w_f(i_t+AP_t\overline{V}_t)$ $=w_f h_{\text{throttle}}$	$(w_f+w_k)i_c$	$B(A_a)$	0	Q_{ac}*
Expansion Period	$(w_f+w_k)i_c$	0	$(w_f+w_k)i_r$	$B(A_c)$	0	Q_{cr}†
Exhaust Period	$(w_f+w_k)i_r$	0	$w_k i_k$	$B(A_r-A_c)$	$w_z(t_o-t_i)$ $+w_f h_{\text{condensate}}$	Q_{rek}‡
Compression Period	$w_k i_k$	0	$w_k i_a$	$-B(A_k)$	0	Q_{ka}†

* Always positive. † A small quantity that may be either + or −. ‡ Always negative.

(c) Now assuming that there is no leakage of steam and that the energy represented by the changes in velocity and in elevation of the steam in passing through the engine may be neglected, Table XII can be readily made up for the determination of the amount of heat transferred to the cylinder walls during each period in order to balance the energy equation. In this table all numerical values must be expressed in the same energy unit, and the B.t.u. is the one used. It will be seen that this analysis is merely an application of the Simple Energy Equation, Eq. (9), modified to take care of the admission and exhaust of a definite weight of steam during certain periods of the cycle.

(d) It should be clearly understood that the test data for this analysis ought not to be obtained until after the engine has been in operation a sufficient length of time to insure its being thoroughly warmed up. In other words, the completion of the cycle should find all the engine parts at the same temperatures that they had at the beginning of the cycle. The metal of the engine does not therefore act as a reservoir of energy, and consequently the algebraic sum of the values given in the last column of the table will represent the true loss of energy from the cylinder by radiation and conduction during a complete cycle. This loss must also be equal to

$$w_f h_{\text{throttle}} - (\text{net area of diagram}) \times B - w_z(t_o - t_i) + w_f h_{\text{condensate.}}$$

It is apparent that Hirn's analysis permits the engineer to study the effect that different designs may have on the transfer of heat to and from the cylinder during each part of the cycle, and it thus enables him to decide on the best means of reducing such interchanges.

229. Clayton's Analysis.—(a) In 1912 Mr. J. Paul Clayton[3] set forth the advantages to be gained by plotting indicator diagrams on logarithmic cross-section paper. Some of these are given on page 118, but it is now desirable to show some special features that apply directly to the steam engine.

(b) In Fig. 204 are represented the head and crank end logarithmic diagrams of a steam engine, as given in Clayton's paper. This figure shows how well the expansion and compression curves follow the polytropic law $PV^n = K$. Consequently, by obtaining the slope of the expansion or compression curve the value of n is obtained. From the figure it is apparent that for the expansion lines of both ends of the cylinder the value of n may be found from the ratio OX : OY.

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[3] See, A New Analysis of the Cylinder Performance of Reciprocating Engines, Journal Am. Soc. Mech. Eng., April, 1912, p. 539; also Bulletins No. 58 and 65 Univ. of Ill. Engineering Experiment Station.

(c) By using a large number of tests and plotting the diagrams on logarithmic paper, Clayton drew the following conclusions regarding non-jacketed steam engines exhausting at pressures near atmospheric:

(1) The expansion line for any steam engine is a polytropic curve, and when plotted on logarithmic cross-section paper it is practically a straight line; hence any variation therefrom indicates leakage.

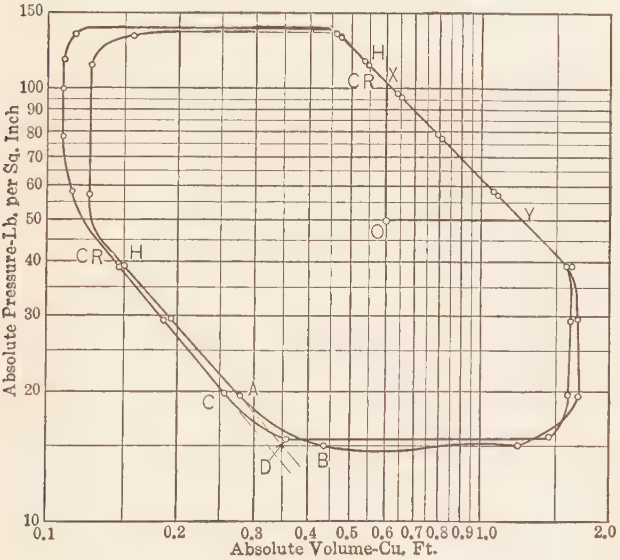

FIG. 204.

(2) The value of n for expansion may be as low as 0.7 when there is excessive condensation, and as much as 1.35 for highly superheated steam. The quality at cut-off varies from 0.4 to 0.9.

(3) There is a definite relation between the exponent n and the quality x_c at cut-off. This relation changes with the steam pressure and with the speed, but does not vary with the point of cut-off. For stationary engines running at nearly constant speed the only variation considered is the change in pressure. For a range of pressures between 75 and 150 the change of n and x_c is slight and conforms closely to the following equation:

$$x_c = 1.245n - 0.576. \quad \ldots \ldots \quad (303)$$

(d) The *steam consumption* of the non-condensing type of engine may be calculated by the aid of Eq. (303) after n has been determined from representative indicator diagrams. The method involves the same general principles that were given in Sect. **227.** There it was desired to determine the quality x_c at cut-off,—the cylinder feed per cycle, w_f, being known; whereas, now it is desired to find w_f corresponding to the value of x_c determined from the foregoing equation.

230. Methods of Decreasing Cylinder Condensation.—(a) It has already been shown that the cylinder condensation causes the largest loss in the steam engine, with the exception of that inherent in the ideal cycle. **Condensation** is evidently dependent on, but not necessarily proportional to, (1) the ratio between the condensing surface (S) to which the steam is exposed, and the volume of steam used per cycle; (2) the temperature difference (ΔT) between the entering steam and the surfaces; and (3) the time (t) of exposure, which is inversely proportional to the number (n) of cycles (i.e., to $1/n$). For computing the probable steam consumption many formulas have been proposed involving functions of S, ΔT, t, or $1/n$, and numerical coefficients determined from experimental data. Such formulas are sufficiently accurate for ordinary purposes, when there is no leakage past piston and valves.[4]

Unfortunately, while it is possible to determine experimentally whether or not **leakage** does occur, the *amount* per cycle cannot be closely evaluated; thus it is impossible in all cases to separate the loss due to leakage from that due to condensation. Hence cylinder condensation and leakage must usually be considered together.

Formulas for cylinder condensation should be derived from studies of data from engines that are known to have little or no leakage. Many of the data available are from engines which were not tested for tightness of valves and pistons and hence are unsuitable for the purpose.

(b) The **size of cylinder** has an important influence on the cylinder condensation. It can be shown by computation that large cylinders have a smaller ratio of surface to volume inclosed than have small ones of the same proportions. It is therefore to be expected that large engines will have a smaller percentage of cylinder condensation and consequently will give better economy than small ones; and that is actually the case. Very small engines may use twice as much steam per i.hp-hr. as very large ones of the same proportions and same conditions of operation.

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[4] For formulas and data see Heck's "Steam Engine," and Thurston's "Manual of the Steam Engine."

(c) The amount of **surface in the clearance space** (including that of the steam passages between valves and cylinder) has a predominating influence on the amount of cylinder condensation that takes place; for, just after admission, the piston is moving so slowly that the time of exposure of the steam to these surfaces is comparatively long, hence the amount of condensation that occurs is large. Probably the greater part of the cylinder condensation takes place in the clearance space. The cylinder passages and clearance space should therefore be designed to present the minimum amount of surface consistent with the other considerations involved.

(d) The cylinder condensation is also dependent on the **relative length of stroke** of the engine. If long and short cylinders are of the same diameter and have their passages and clearance space identically the same, and cut-off steam at the same per cent of stroke, obviously the ratio of clearance surface to the total surface exposed per stroke of the engine is smaller in the long-stroke engine than in the one with shorter stroke. Neglecting the time element, the long-stroke engine should give better economy than the short-stroke one; and in general that is the case, though this is in part due to the fact that the long-stroke engines are usually also larger, have cylinders that are better designed, and have better valve gear than those with shorter stroke.

The **time element** may have an important influence, however; for example, due to the fact that most of the condensation occurs in the clearance space and because of the shorter time of exposure, some of the short-stroke "high-speed" Corliss engines give as good or even better economy than some long-stroke low-speed Corliss engines.

(e) In many engines the **exhaust steam flows over the outer surface of the cylinder wall,** on its way to the exhaust pipe. Because of the high velocity of flow, this steam carries away heat more rapidly than would stagnant air in contact with the same surface. This lowers the mean temperature of the cylinder walls and increases the cylinder condensation. In the better designed engines the exhaust steam is not brought in contact with the cylinder walls after it leaves the exhaust valve.

(f) The **point of cut-off** affects cylinder condensation very materially. As most of the cylinder condensation occurs in the clearance space, the later the cut-off (or the greater the volume of steam admitted per cycle), the less will be the *percentage* of steam condensed, although the *amount* may be greater. (1) The percentage of steam not condensed is shown in Fig. 205 (*a*) by the ordinates, the abscissas being percentages of stroke at cut-off.[5] (2) The available energy per pound of steam

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[5] This is for large four-valve engines having little leakage. See "Engine Tests," by G. H. Barrus.

decreases as the cut-off is advanced in the stroke (because of the reduction of expansion), hence the steam consumption per unit of work in the ideal engine is greater the later the cut-off occurs, as shown by the ordinates of the curve in Fig. 205 (*b*). (3) Evidently, dividing the ideal steam rate per hp-hr., Fig. 205 (*b*), by the fraction of steam not condensed, Fig. 205 (*a*), will give the true consumption at the various cut-offs. The values of the actual steam-rate, obtained in this way, are shown by the curve in Fig. 205 (*c*).

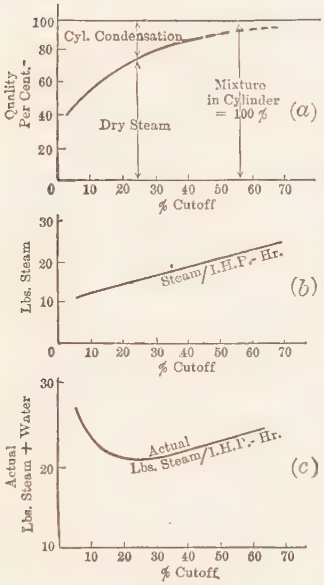

Fig. 205.

Similar steam-rate curves can be drawn by using data obtained by direct engine test, in which the steam per i.hp-hr. is measured with the engine operating under different loads (i.e., different cut-offs). Usually the steam-rates are plotted with respect to power output instead of cut-offs.

Inspection of the steam-rate curve makes it evident that, to give the best economy, *the engine should be operated with a cut-off corresponding to the lowest point on this curve.*

The most economical cut-off for non-condensing simple slide-valve engines is about $\frac{1}{4}$ stroke, and for simple Corliss engines it is between $\frac{1}{5}$ and $\frac{1}{4}$ stroke. In practice these are the cut-offs which predominate.

Usually the steam-rate curve is more nearly horizontal to the right of the lowest point than it is to the left (as in Fig. 205), hence it *is better to " overload " a steam engine than to " underload " it.*

(g) **Compounding of cylinders** is a very effective way to reduce cylinder condensation. By using earlier cut-off the amount of steam used per hp-hr. is reduced in the ideal engine because of the greater expansion of the steam. But it was seen in (f) for the case of the simple engine that the percentage of steam condensed becomes excessive with very early cut-offs and thus defeats the advantage which is gained in the ideal engine. Therefore, to use economically larger expansions than are possible with the ordinary simple engine, the cylinder condensation must be reduced in some way. It was shown in (b) that cylinder condensation can be reduced by decreasing the surface (especially that of the clearance space) to which the high temperature steam is exposed and by reducing the temperature range in the cylinder. Both of these methods can be combined by compounding the engine.

Thus, suppose a small amount of steam is admitted to a small cylinder (say with $\frac{1}{4}$ the piston area of the simple engine) and that it is expanded only enough to bring the temperature T_R (Fig. 206) of the exhaust steam part way to that of the simple-engine exhaust (say T_R is halfway between T_1 and T_2 on the temperature scale). Let the indicator diagram labeled H.P. in Fig. 206 represent this cycle. Then, owing to the smaller cylinder surface (especially that in the clearance), there is very much less initial and cylinder condensation in this case than if the same weight of steam had been expanded the same amount in the cylinder of the large simple engine.

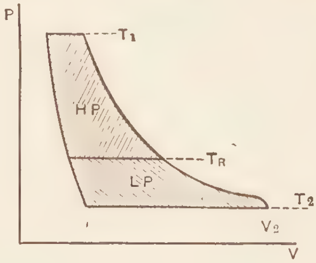

Fig. 206.

Now let the steam exhausted from the small cylinder enter one of the *same size as that of the simple engine,* and let it be further expanded in this cylinder until the back pressure of the simple engine is reached. The indicator diagram for this case is shown by L.P. in Fig. 206. During this expansion the temperature range (T_R to T_2) is low, hence cylinder condensation is also small here.

It is evident that an engine operating in this manner will use much less steam per hp-hr. than will a simple engine; roughly, it uses only about $\frac{2}{3}$ to $\frac{3}{4}$ as much. The best economy with the simple engine is obtained when the steam is expanded in the cylinder to four or five times its initial volume. In an arrangement such as has just been described, the expansion giving the best results is from 7 to 16, or more, depending upon the conditions of operation.

When an engine with two cylinders is arranged to operate in the manner just discussed, it is called a **Compound Engine.** The small cylinder is named the *high-pressure* (*H.P.*) *cylinder* and the large one is the *low-pressure* (*L.P.*) *cylinder.*

Other engines are arranged to expand the steam in three steps, or stages, using in succession three cylinders that progress in size. These are called **Triple-Expansion Engines,** and the cylinders are termed respectively the *high-pressure, intermediate-pressure* (*I.P.*), and the *low-pressure.* Triple-expansion engines use considerably less steam per i.hp-hr. than do the compound engines.

In the **Quadruple-Expansion Engine,** four cylinders are used in succession. They are termed the *H.P. cylinder*, the *first intermediate*

(I.P$_1$.), the *second intermediate* (I.P$_2$.), and the *L.P. cylinder.* **Quintuple engines** have been made, but their number is very small.

Strictly speaking, the term "Compound Engine" includes all **multiple-expansion steam engines,** but it has become customary to apply it only to those with two cylinders.

A comparison of the performance of simple, compound, and triple engines operating under the same conditions is shown[6] in Fig. 207. The triple-expansion Corliss engine in the laboratories of Cornell University was tested with the high-pressure cylinder operating alone as a simple engine, then with the high and intermediate cylinders acting as a compound engine, and finally with all three cylinders as a triple-expansion engine. The results are shown in this figure. Larger engines and those with jacketing, superheating, etc., would give better results, but this figure shows the relative value of using the different expansions.

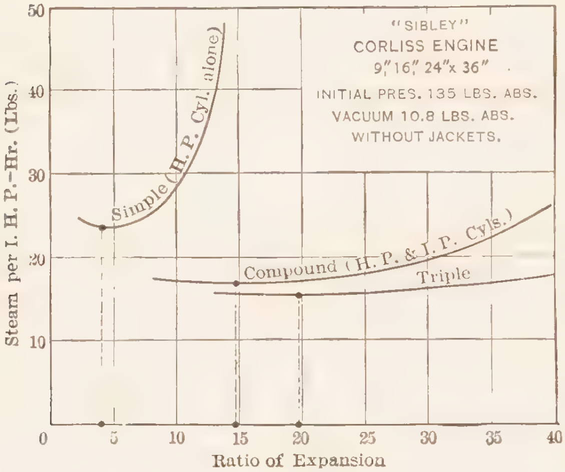

FIG. 207.

(**h**) Using **superheated steam** is one of the most satisfactory methods of reducing cylinder condensation. The cooling of the cylinder walls during exhaust is largely due to their surrender of the heat used in evaporating the moisture on their surfaces. As the latent heat of vaporization corresponding to the exhaust pressure is very large, roughly 1000 B.t.u. per pound, the evaporation of a small amount of water results in a very considerable reduction of the mean temperature of the cylinder walls, and consequently in an increase in the cylinder condensation when the steam is admitted.

When superheated steam is used there is less moisture in the exhaust steam, and partly because of this, partly because of the slow rate of heat transfer between superheated steam and metal, and partly because the incoming superheated steam can give up heat without condensing, the cylinder condensation is reduced, and the economy of the engine is improved. Thus by sacrificing superheat to heat the cylinder walls, less heat is required at the boiler for evaporating the water and for superheating the steam used. It is even possible to superheat sufficiently high to prevent all initial condensation.

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[6] See report of test, Carpenter, Trans. A.S.M.E., Vol. XVI. Also Thurston, A.S.M.E., Vol. XVIII.

In experiments by Ripper[7] on a small steam engine, it was found that $7\frac{1}{2}$ deg. fahr. of superheat would prevent one per cent of cylinder condensation. The specific heat of superheated steam under the test conditions is about 0.53, hence the amount of heat used in preventing 1 per cent of condensation was $(7\frac{1}{2} \times 0.53) = 4.0$ B.t.u. per pound of steam. For larger engines and other conditions from 15 to 25 deg. fahr. and from 8 to 12 B.t.u. per pound are used per cent of saving of condensation.[8]

In the ideal engine the use of superheated steam was shown to increase the cycle efficiency only a small amount, but by using sufficient superheat to overcome cylinder condensation the engine efficiency is very materially increased. Naturally, the saving to be expected by superheating is dependent upon the amount of cylinder condensation that would occur in the same engine if no superheat were used. Evidently the greater this condensation, the larger is the saving possible. Ordinarily the steam consumption is reduced about 6 per cent with 50 deg. fahr. and about 9 per cent with 100 deg. fahr. superheat. A reduction of 15 per cent is frequent and as much as 40 per cent has been attained.

It is found that with high temperatures of superheat there is difficulty from warping of cylinder and valves and from failure of lubricants unless they are of the highest grade. A total temperature of 600 deg. fahr. is about as high as can be used to advantage in ordinary steam engines. Cylinders and valves for higher temperatures should be specially designed for the service. Above 750 deg. fahr. there is difficulty in finding materials that will endure the temperature for long periods of time.

(i) **Steam jackets** are sometimes used to reduce cylinder condensation, and in such cases the cylinders are surrounded by " live " steam (usually at high and constant temperature). Their walls are maintained at higher mean temperature and have less temperature fluctuation than in the ordinary cylinder, consequently there is less cylinder condensation. The heat received by the cylinder wall from the " jacket steam " is the latent heat freed by the condensation of a portion of this steam, assuming that the jacket steam[9] is not superheated. If the jacket steam is at the same temperature as that entering the cylinder, the mean temperature of the walls will be but little below that of the entering steam, hence the condensation will be small.

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[7] Superheated Steam Engine Trials, Proc. Inst. C. E. (London), Vol. CXXVIII.

[8] For data and references see Mechanical Engineering Handbooks, and Gebhardt's "Steam Power Plant Engineering."

At first it may appear that the weight of cylinder condensation thus avoided cannot be more than the steam simultaneously condensed in the jacket, in cases where the condition of the steam entering both the cylinder and the jacket is the same. Because of this, and because the jacket has radiating surface which is larger, and which is maintained at a higher mean temperature, than in the case of the ordinary cylinder, it would seem that no advantage is possible from the use of a steam jacket.

That the steam jacket is beneficial is largely due to the fact that, with its use, the amount of moisture evaporated from the inner walls of the cylinder during exhaust is greatly reduced, thus less heat is abstracted from these walls by the exhaust steam and less steam is used in the cylinder. It has been seen that one pound of moisture evaporated from the cylinder walls carries away roughly 1000 B.t.u. from which there is no return. In the case of the jacket, however, the condensate formed in the jacket can be returned directly to the boiler, and, as it is at boiler pressure and temperature, it will carry back from 250 to 300 B.t.u. per pound. Thus the net result with the steam jacket may be a gain in economy.

In considering the performance of a jacketed engine the heat supplied to the jacket steam must be considered.

As most of the cylinder condensation occurs in the clearance space, this is the most important part of the cylinder to jacket. It might be desirable to jacket the piston, that is, circulate steam through it, but practical difficulties usually prevent this.[10] As there is probably no advantage in having the exhaust steam superheated, the temperature of the jacket steam should usually not be much higher than that of the steam entering the cylinder. This applies especially in the case of the low-pressure cylinders in multiple-expansion engines.

(j) **Reheating receivers** are often used in multiple-expansion engines to reduce the cylinder condensation. These have "live steam" in tubes or coils of pipe, around which the steam passes on its way from one cylinder to the next. As the steam in the coils is at relatively high temperature, it superheats (or reheats) the receiver steam, provided the moisture has been properly separated from this latter.

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[9] In some engines using superheated steam the cylinder feed passes through the jacket. A typical engine of this kind will be shown in Sect. **252**.

[10] The Prosser engine has the piston, piston rod and cylinder very completely jacketed. For description and test see "Mechanical Engineering," April, 1925.

The presence of moisture in the working substance defeats the purpose of the reheating receiver. This moisture should be removed before the steam reaches the reheater, for it can be evaporated to better advantage in the boiler.

The action of the reheating coils is similar to that of the steam jacket; and the heat surrendered by the condensation of steam in the coils of pipe is to be charged against the engine.

(k) Cylinders are always "lagged" with some **non-conducting material** such as asbestos, mineral wool, magnesia, etc., to reduce the radiation. Some small compound "Lokomobile"[11] engines, which have phenomenal economy, are so arranged that the cylinders are surrounded by the furnace gases as they pass to the stack.

(l) It is of course evident that the higher the **rotative speed** (or the greater the frequency of cycles), the less will be the cylinder condensation, because the entering steam is exposed a shorter time to the cylinder walls. For example the high-speed Corliss engines use less steam than the low-speed engines of that type, under the same conditions. There are practical considerations, however, which place limits on the speeds of rotation that can be used.

There is a special type of engine, the "Uniflow," in which the cylinder condensation is extremely small. Its importance is such that it will be considered separately in Sect. **232**.

231. Operating Engines Condensing and Non-Condensing. (a) In previous chapters it has been shown that by reducing the exhaust pressure, the amount of work that is obtainable from a unit weight of steam in passing through an *ideal engine* is made greater.

(b) In the *actual simple engine* of the usual type the reduction of the exhaust pressure below atmospheric may not always be commercially desirable. This is true because the *engine efficiency* may be decreased through cylinder condensation by a sufficient amount to offset the improvement in *cycle efficiency* resulting from the lower exhaust pressure and temperature. Furthermore, the extra cost of the condenser and its auxiliaries may be sufficient to overcome the saving in steam that might be effected by operating condensing.

(c) The *indicated power* of the *simple engine* on the other hand is very materially increased by using a low exhaust pressure. This is shown by the relative areas of the indicator diagrams represented in Fig. 208. The installation of a condenser, with a simple engine that

[11] Herr E. Josse in Zeitschrift des Vereins deutscher Ingenieure, Sept. 12, 1908. Also see report of test of a Wolf engine (London), Engineering, Oct. 8, 1909.

has been operating non-condensing, may give sufficient additional power to carry the extra load that may have developed after the original installation. For such cases, however, the purchase of a low-pressure exhaust turbine and condenser might be the best solution.

(d) With *compound engines*, and with the *uniflow* engine which will be considered in the next section, the condenser is nearly always used since cylinder condensation is relatively small with these types and their thermal economy is therefore much improved by reducing their back pressure. There is also a decided advantage in having hot clean water fed to the boiler, and this may be accomplished by the use of the condensate which has been freed from oil. In general, it may be said that the reciprocating engine is not able to utilize the available energy of low pressure steam as well as the turbine, and therefore the exhaust pressure is not usually reduced to such very low values as are now common with the latter.

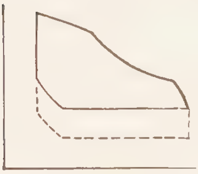

FIG. 208.

232. Uniflow Engine. (a) This type of engine was invented [12] in 1885 by T. J. Todd of England but remained undeveloped until 1908 when Professor Stumpf, of Charlottenburg University, succeeded in making it a highly perfected prime mover.[13] Since that time others have modified it to suit special conditions so that it is now widely used for a variety of purposes.

(b) The *unusual features* of the engine may be noticed by referring to Fig. 209. The piston is of the box type having a length equal to about 90 per cent of the stroke; and when near the end of its movement it uncovers the exhaust ports located around the middle of the cylinder, thus taking the place of the exhaust valve. By having the exhaust all pass out through these central ports, the admission ends of the cylinder and the clearance surfaces are not cooled by the outflowing steam, as they are in the usual "counter-flow" type of engine. Furthermore, by reducing the time during which exhaust occurs, the amount of cylinder condensation is still further decreased. The exhaust period may be kept very short without causing any appreciable increase in the back pressure because the area of the exhaust ports is relatively large. Another means of preventing (or reducing) cylinder condensation on these engines is the use of the steam jackets on the heads and part of the walls of these cylinders. Also the clearance is kept extremely small, the compression ratio is very large, and highly superheated steam is commonly used.

[12] See "The Uniflow Engine," by F. B. Perry, Proceedings of Inst. Mechanical Engineers (British), July, 1920, p. 731.

[13] See Professor Stumpf's book, "The Uniflow Steam Engine."

All these factors combine to minimize the cylinder condensation or to eliminate it altogether, even though the engine be run condensing with a high ratio of expansion.

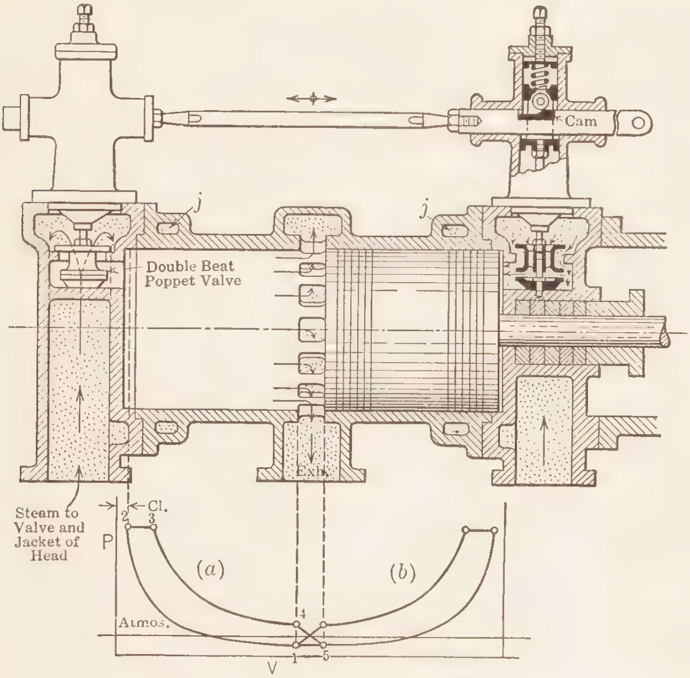

Fig. 209.—The Uniflow Engine.

(c) The admission valves shown in Fig. 209 are of the double-beat poppet type, but other kinds are also often adopted. The poppet type allows highly superheated steam to be used and the double beat feature permits a smaller lift to be employed for a given area of opening. The piston functions as the exhaust valve, as already noted.

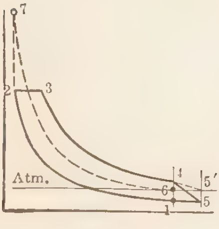

Fig. 210.

(d) The *indicator diagrams* from each end of the cylinder of a condensing engine will be similar to those shown by (*a*) and (*b*) in Fig. 209. The compression must necessarily begin very early because it is controlled by the closing of the exhaust ports by the piston.

If the engine is run non-condensing with the large compression ratio that is used with the condensing type, the compression pressure is likely to go much higher than the admission pressure, as shown by the dotted line *6 7* in Fig. 210.

This would be very undesirable, and consequently the uniflow type of engine is built in a variety of ways to take care of this feature in case it becomes necessary to run the engine non-condensing. Thus, extra clearance is often introduced by means of a special valve that opens automatically when the compression pressure exceeds the admission pressure. Sometimes some of the compression steam is by-passed to the other side of the piston, and in certain engines small auxiliary exhaust valves are employed.

(e) The *sizes* of uniflow engines that are in use vary widely. There are large numbers of them under 500 horsepower, and many above 1000. Recently a 30,000 horsepower uniflow was built.[14]

(f) The chief *advantages* of the uniflow engine are:

(1) The *thermal efficiency* is high because the cylinder condensation is small, and because the mechanical efficiency is large even though a high ratio of expansion is used.

(2) The *steam consumption* at three-quarter load is only slightly more than at full load; and at 25 per cent overload, and also at half load, it is only about 5 per cent greater.

(3) The engine itself, its floor space and foundation are much *less costly* than for the compound engine of the same economy.

(4) It is *simpler* to operate and less expensive to maintain than is the compound of the same economy.

233. The Still Engine. (a) In 1919 Mr. W. J. Still of England first gave to the public the results of his extensive research on the development of the engine that bears his name. This prime mover consists of an internal combustion motor and a steam engine combined in such a way as to use a single cylinder and piston in common and produce a marked improvement in thermal efficiency. The part above the piston is the internal combustion engine while that below is a single-acting steam engine which receives vapor produced by heat from the cylinder walls and from the gases exhausted from the upper part.

(b) This combination gives the following beneficial results:

(1) The *temperature* of the exterior wall of the cylinder is kept *uniform* because the jacket cooling medium enters (as water) and leaves (as steam) at the same temperature, its value being fixed by the pressure maintained in the jacket.

(2) The *exhaust gases* give up a large amount of heat to the water used in forming the steam, the exhaust temperature being reduced to 200 deg. fahr. or less.

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[14] See "Power Plant Engineering," July 1, 1926; also "Power," 1925, p. 122.

(3) The *steam side* of the engine operates without loss from cylinder condensation because the piston and walls are at a higher temperature than the steam.

(4) The *piston* is efficiently *cooled* by the steam.

(5) The *mechanical efficiency*, *flexibility*, and *overload capacity* of the combination are improved over those of the internal combustion engine operating alone.

(6) The Still engine is *reversible*, and this feature makes it of special importance in marine work.

(7) The *brake thermal efficiency* is very high, tests showing values from 35 to 40 per cent.

(c) The engine is more complicated, and will probably cost more than either a steam or internal combustion engine of the same capacity, but its greater thermal efficiency may cause it to become widely used in cases where fuel cost is of major importance. There have been published many articles describing the engine and giving test results, and these should be consulted for more detailed information.[15]

234. Classification and Types of Steam Engines. (a) Owing to the great variety of designs and to the diversity of uses to which steam engines are put, it is impossible to give any one classification that would be satisfactory in all cases. The usual **commercial types of stationary engines** are often classified in three groups,—" high-speed," " medium-speed," and " low-speed " engines. By " speed " is meant the rotative speed, when used in this connection.

High-Speed Engines are those which have high rotative speeds accompanied by strokes which are very short when compared to the diameter of the cylinder, the piston speed being generally in the neighborhood of 600 feet per minute.[16] The stroke is usually about equal to the diameter of the cylinder. These engines almost always have a single " balanced " valve and a " shaft " governor. They are often called " short-stroke engines," and are designed to occupy the smallest space, have the least weight, and to be "direct connected" to the smallest dynamo, for a given power, of any of the stationary commercial types. This class includes only engines of comparatively small power, the cylinders usually not being made larger than 20 inches in diameter. Fig. 211 shows a center-crank engine of this type.

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[15] Some of these are: The Still Engine, by Capt. F. D. Acland, Jour. Royal Soc. of Arts (London), May, 1919; abstracted in "The Engineer," 1919, and in the S. A. E. Journal, June, 1920. The Scott-Still Marine Oil Engine, "The Engineer," 1923, pp. 556-592; and also "Engineering," Nov., 1923, pp. 639–640. Marine Oil-engine Trials (Scott-Still Engine), Engineering, April 17, 1925, pp. 485–487.

[16] For definition of " piston speed " see Sect. **169**(e).

Low-Speed Engines have long strokes (from 2 to 4 times the diameter of the cylinder) and usually operate at less than 120 r.p.m., the speed being limited by the valve gear, the action of which becomes unreliable at higher speeds. This class includes engines having the "Corliss" and other types of "trip cut-off gear." The governor is usually of the "fly-ball" type. An engine of this kind is illustrated in Fig. 212.

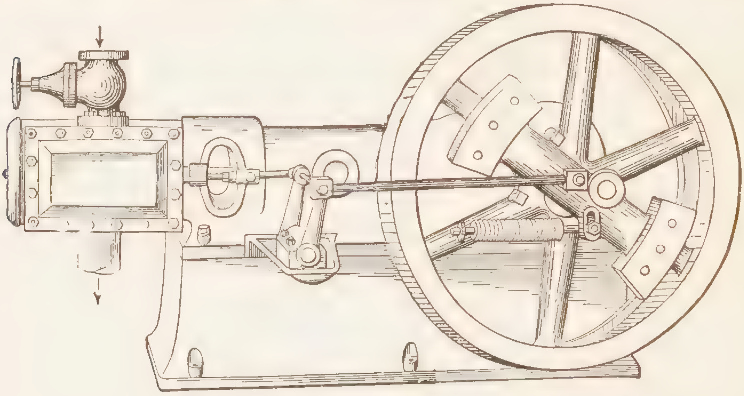

Fig. 211.—Center-Crank Engine with Inertia Type of Governor. (The engine is mounted on a cast-iron subbase.)

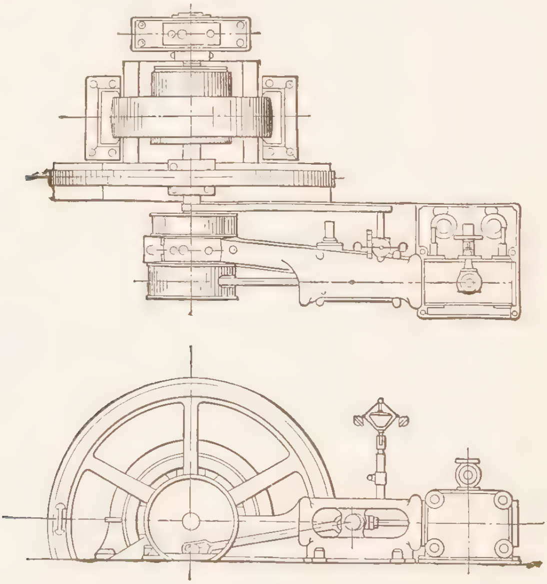

Fig. 212.—Low-speed Engine with Corliss Valve Gear. Direct Connected to an Electric Generator.

Medium-Speed Engines have rotative speeds and strokes intermediate between the foregoing. Positively driven multiple valves are generally used. The cut-off is positive and is often effected by a separate valve. The governor is nearly always of the "shaft type." The piston speed is around 600 feet per minute, being higher on the larger engines. The engine shown in Fig. 213 is of this type.

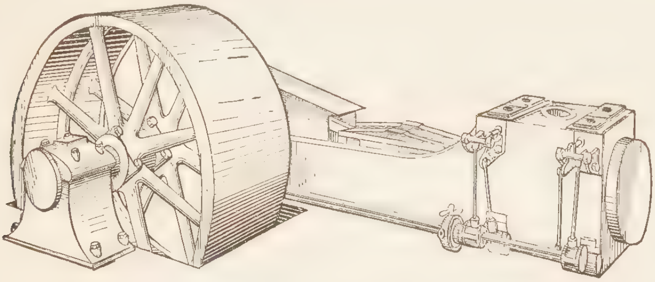

Fig. 213.—Medium-speed Engine—Shaft Governor—Positive Cut-off.

The medium- and low-speed engines are usually of larger power than the high-speed.

There is no sharp dividing line between these different types, and it is sometimes difficult to decide in which class an engine belongs.

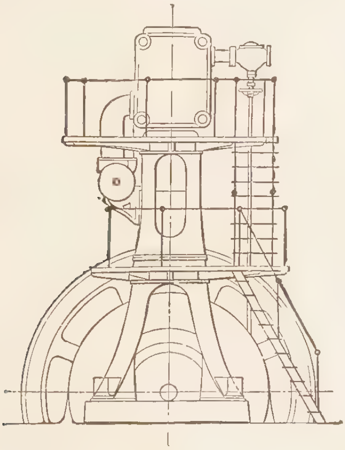

Fig. 214.—Vertical Corliss Engine.

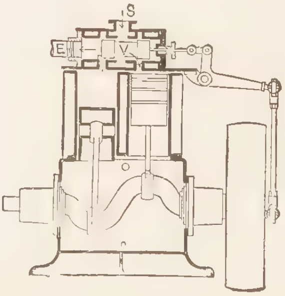

Fig. 215.—Vertical Twin-cylinder, Single-acting Engine.

(b) **Vertical Engines** (Figs. 214 and 215) occupy less floor space, have smaller foundations, less cylinder wear, and slightly greater mechanical efficiency, than do horizontal engines. When large, they are more difficult to erect, and caring for them involves more effort, as certain parts are reached only by ladders.

In some special instances engines have been constructed with axes inclined with the horizontal.

(c) **Single-acting Engines** (Fig. 215) give half as much power as do double-acting ones with the same diameter and stroke of piston, consequently a larger engine is required for a given output. They use pistons of the bucket, or trunk, type, and have no piston rod, therefore they are shorter than the double-acting engines.

(d) **Reciprocating Engines** are so called because they have pistons that reciprocate within the cylinder. They are the most common, although engines with rotary pistons would apparently be more desirable. Many unsuccessful attempts have been made to devise an engine of the latter form. The difficulty lies in the production of a machine that is economical in the use of steam after the parts have become worn. Prior to 1902 there were issued over 2000 patents on **Rotary Engines,** and none have yet been able to compete with the reciprocating engine as regards steam economy. When small size and low first cost are more important than the amount of steam required, they may be used to advantage. Fig. 216 shows one of the simplest engines of this type.

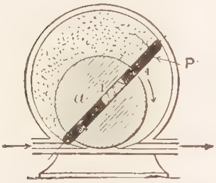

Fig. 216.—Rotary Engine.

In some instances **Oscillating Engines,** such as that shown in Fig. 217, have been used. The connecting rod and crosshead are dispensed with and the shaft is thereby brought closer to the cylinder. Steam is admitted through one trunnion and exhausted through the other. This causes a side thrust, for which proper provision must be made. When the cylinder reaches its extreme position, its inertia causes great pressure to exist between the stuffing box and the piston rod, and these parts must be designed to resist this force.

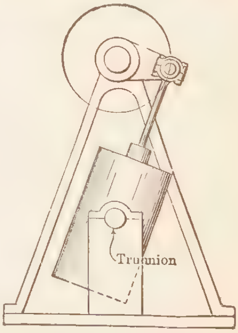

Fig. 217.—Oscillating Engine.

(e) Fig. 218 shows two horizontal **"side-crank engines"** that have the disadvantage of using a separate outer bearing which must be aligned with the main bearing. They possess the advantage that there are only two bearings to be kept in alignment even when the engine is direct-connected to an electric generator, as in Fig. 212.

A horizontal side-crank engine is said to be **right-hand** in arrangement when an observer, standing at the end of the cylinder (at S in Fig. 218) and facing the crank, finds the valve gear and governor parts are to his right; otherwise the engine is "left-hand."

A horizontal engine is said to be **running over** if the crank pin is receding from the cylinder when the crank is above the horizontal center line of the engine; otherwise it "runs under" (see Fig. 218). If a double-acting engine "runs over" the crosshead will exert downward pressure on the guides during both strokes; hence engines are usually operated in this manner.

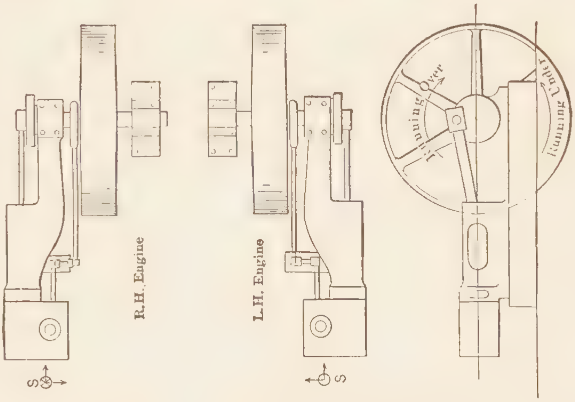

FIG. 218.—Side-crank Engine—Right Hand and Left Hand—Running Over and Running Under.

(f) Fig. 219 shows a **Center-crank Engine.**—In this type the crank is located between the two main bearings, BB, and the belt- and fly-wheels are overhung. If small, these engines may be shipped assembled, ready to be mounted on their foundations. If direct-connected to an electric generator, the latter is substituted for the belt wheel and an out-board bearing is added. There are then three bearings to be kept in line, which is a disadvantage, as very accurate adjustment of them is required.

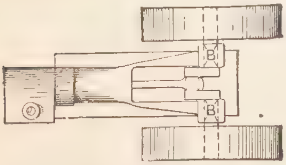

FIG. 219.—Center-crank Engine.

In many engines the crank case and crosshead-guide chamber are **inclosed** so as to be dustproof and prevent the throwing and waste of oil (see Figs. 211 and 215).

Some inclosed engines are arranged to be **self-oiling** as regards the crosshead, connecting rod, and main bearing. In these cases oil is maintained at such a level in the bottom of the crank case that the crank disk dips into it, and while rotating throws the oil into the crosshead and into collecting pockets from which it is fed to the bearings. The oil then automatically drains back to the crank case and is used repeatedly without being purified.

Other inclosed engines are provided with forced or gravity **oil-**

feeding systems, in which the lubricant is filtered each time before reusing.

(g) **Compound engines** have their cylinders arranged in many different ways.

If the two pistons are on the same piston rod, as in Fig. 220, the engine is called a **Tandem Compound.** Such an engine occupies no greater width than a simple engine of the same power and type, but has greater length. A vertical engine of this type ("steeple compound") occupies the same floor space as the equivalent simple engine.

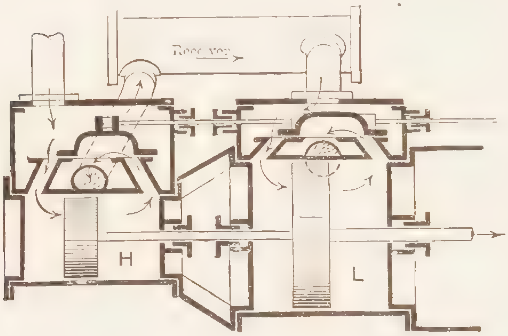

Fig. 220.—Tandem-compound Cylinders.

Either the high- or low-pressure cylinder may be placed next to the frame.

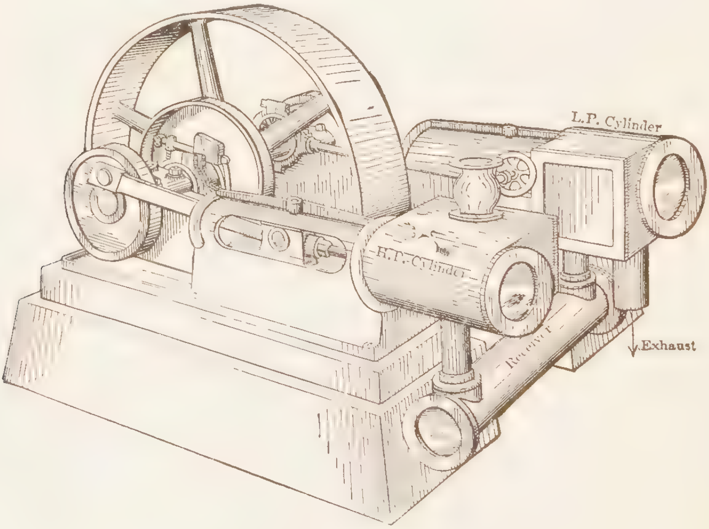

Fig. 221.—Cross-compound Engine.

If the cylinders are side by side, as in Fig. 221, the engine is called a **Cross-compound.** This engine occupies greater width than the tandem-compound, but its length is about the same as that of the simple engine. As it has two frames, and as other parts are duplicated, it is more expensive than the tandem; but because the cranks may be set at right angles it is possible to obtain greater uniformity of turning effort than with simple or tandem engines, and therefore a smaller flywheel can be used.

In some cases, with this arrangement of cylinders the cranks are placed diametrically opposite (180 degrees apart), but the turning effort is then about as variable as with the single-crank engine.

The arrangement of engine known as the **Angle-Compound,** shown in Fig. 222, occupies the same floor space as a simple engine, has the uniformity of crank effort obtained with cranks at 90 degrees (for in this case the connecting rods are at 90 degrees and are attached to the same crank pin), and is easily counterbalanced.

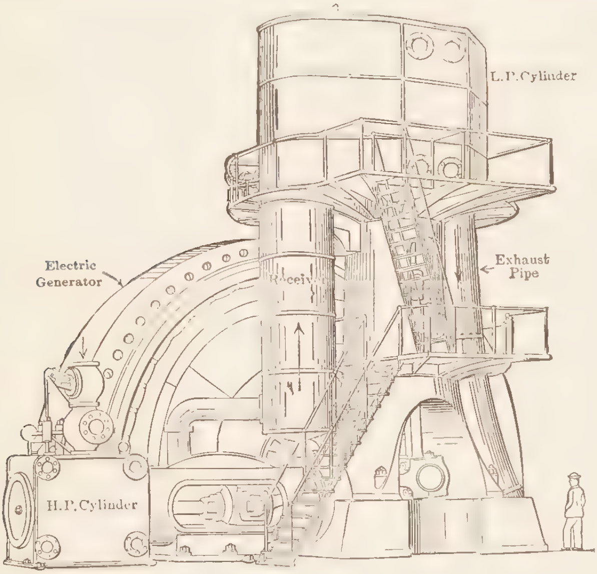

FIG. 222.—Angle-compound Engine.

In **triple-** and **quadruple-expansion** engines the cylinders are arranged in various ways and, looking at the end of the shaft, there may be various sequences with which the cranks pass a given point. The arrangement of cylinders and the sequence and angle between cranks have a predominating influence on the counterbalancing of such engines, as will be seen later when the subject of counterbalancing is discussed.

(h) Engines are used for a great variety of purposes, and are often referred to by their use; thus there are marine engines (Fig. 223),

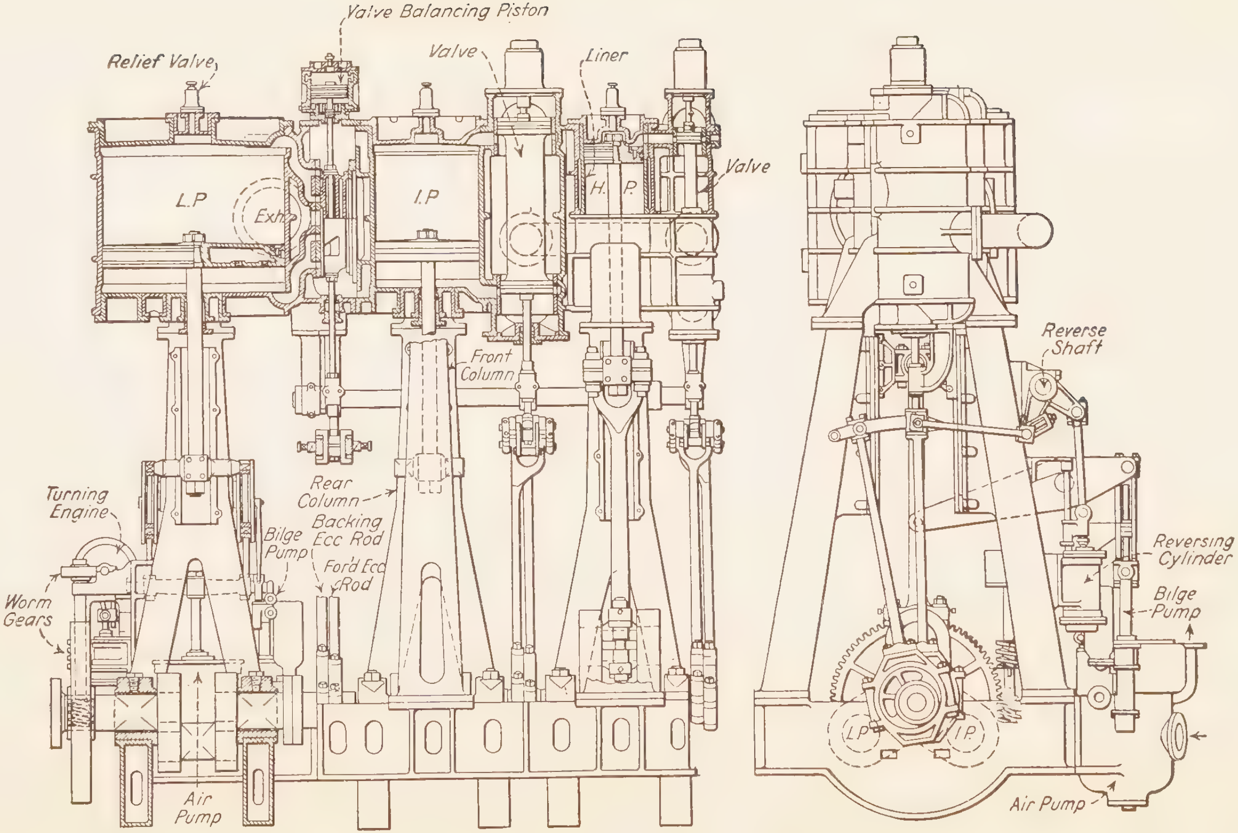

Fig. 223.—Vertical Triple-expansion Marine Engine

hoisting engines, pumping engines, rolling-mill engines, air-compressor engines, steam-hammer engines, etc.

Engines are also classified as stationary, portable, semiportable, and mobile (marine, locomotive, traction, road roller, and automotive engines).

The uses to which some engines are put require that they be capable of being reversed. This is true of marine engines, some rolling-mill engines, hoisting engines, traction engines, etc. They are called "*reversing engines*" and have special valve gears, either of the "link" or "radial" types, which will be discussed later.

235. Conventional Diagram for Simple Engines.—(a) If the actual indicator diagram has been obtained from an engine, the mean effective pressure may be determined by any of the methods discussed in Sect. **96,** and the indicated horsepower of the engine may be obtained by using Eq. (130). In making such computation for a double-acting engine, however, the *area of the piston rod* must be deducted from the area of the piston on one side, and the average of the areas on the two sides of the piston must be used in the formula; or better the power for each side of the piston must be computed separately.

When actual indicator diagrams are not available, it is customary to use a conventional diagram, with proper correction factor, for estimating the probable mean effective pressure.

(b) Before the conventional diagrams can be drawn, however, the *clearance volume* in the cylinder must be known or estimated for an engine that is being designed. For one already built this volume can be determined by pouring a measured quantity of water into the clearance space. It can also be found very roughly from the actual indicator diagram in the following manner (shown in Fig. 224): Select two points 1 and 2 on the expansion line and draw a rectangle with these points as corners and with the sides parallel to the respective PV-axes. Then, the diagonal through the other corners will cut the V-axis at the origin O, assuming that the expansion equation is $PV = \text{constant}$. Then Cl in the figure is the clearance volume to scale. The compression curve may be used in a similar manner to find O. This makes application of the construction shown in Fig. 49.

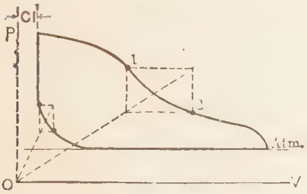

Fig. 224.

The clearance of an engine from which the actual diagram is available may be estimated more accurately by using Clayton's method.[17]

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[17] See Sect. **229.**

By assuming different values of the clearance and then plotting the logarithmic diagram for each assumed value the correct clearance is found by trial and error to be one that gives straight lines for the expansion and compression curves.

The clearance volumes used in practice are about as follows:

Single-valve engine	5 to 15%
Multi-valve engines.	2 to 8%

(c) In constructing conventional diagrams for estimating the probable power of an engine, it is customary to assume that expansion follows the equation PV = constant. This is because the equilateral hyperbola is sufficiently accurate for most purposes, is easier to construct than the polytropic curve with n other than unity, and because the actual value of this exponent may not be known.

The work (A) represented by the area (Fig. 225) under such an expansion line is readily found in the following manner:

$$A = \int_{V_1}^{V_2} P\,dV = P_1V_1\int_{V_1}^{V_2}\frac{dV}{V}$$

$$= P_1V_1 \log_e\frac{V_2}{V_1} = P_1V_1 \log_e r, \quad . \quad . \quad . \quad (304)$$

where r is the ratio of expansion $\left(\frac{V_2}{V_1}\right)$. In Fig. 226 r is $\frac{V_c}{V_b}$.

(d) In the case of an **engine without clearance** the conventional diagram is *abcde* of Fig. 226. The work shown by area A_1 is P_1V_1

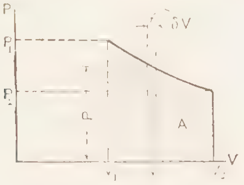

FIG. 225.

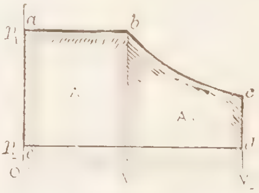

FIG. 226.

foot-pounds, and that represented by A_2 is $P_1V_1 \log_e r$. Hence if the back pressure is P_2, the work shown by the conventional diagram *abcde* is

$$P_1V_1 + P_1V_1 \log_e r - P_2V_2 = P_mV_2,$$

in which P_m is the mean effective pressure in pounds per square foot. Solving this equation for P_m gives

$$P_m = P_1\left[\frac{1 + \log_e r}{r}\right] - P_2.$$

As the mean effective pressure is generally used in pounds per square inch, it is more convenient to divide both sides of this equation by 144, giving

$$p_m = p_1\left[\frac{1 + \log_e r}{r}\right] - p_2. \qquad (305)$$

The values of the bracketed quantity for different values of r are given in Table XIII.

TABLE XIII

r	$\frac{1 + \log_e r}{r}$	r	$\frac{1 + \log_e r}{r}$	r	$\frac{1 + \log_e r}{r}$
1.0	1.000	6.0	0.465	16.0	0.236
1.5	0.937	7.0	0.421	17.0	0.226
2.0	0.847	8.0	0.385	18.0	0.216
2.5	0.766	9.0	0.355	19.0	0.208
3.0	0.700	10.0	0.330	20.0	0.200
3.5	0.644	11 0	0 309	21.0	0 192
4.0	0 597	12.0	0.290	22.0	0.186
4.5	0.556	13.0	0.274	23.0	0.180
5.0	0.522	14.0	0.260	24.0	0.174
5.5	0.492	15.0	0.247	25.0	0.169

(e) The actual indicator diagram of course differs from the computed one drawn by this method. The ratio of the area of the actual to that of the conventional diagram is called the "**Diagram Factor**" (*DF*). Then if the diagram factor is known for engines similar to that which is being considered, the probable mean effective pressure for the new engine is

$$p_m' = DF \times p_m. \qquad (306)$$

It is a somewhat common practice to use Eq. (305) even for engines which have clearance, and to substitute $\left(\frac{1}{\textit{cut-off ratio}}\right)$ for r, thus ignoring the clearance.

The diagram factors to be used for different types of engines in such cases are given in the following table:

TABLE XIV.—DIAGRAM FACTORS

Simple slide-valve engine	55 to 90%
Simple Corliss engine	85 to 90%
Compound slide-valve engine	55 to 80%
Compound Corliss engine	75 to 85%
Triple-expansion engines	55 to 70%

(f) The conventional diagram for an **engine with clearance** is shown by *abcde* in Fig. 227. Here the ratio of expansion is

$$r = (L + l_c) \div (l + l_c). \quad . \quad . \quad . \quad . \quad . \quad (307)$$

using scalar distances to represent volumes.

The net work shown by the area is

$$\begin{aligned} A &= A_1 + A_2 - A_3 \\ &= P_1 l + P_1 (l_c + l) \log_e r - P_2 L \end{aligned}$$

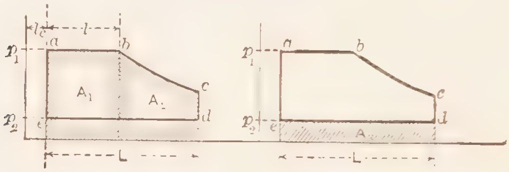

FIG. 227.

Dividing by L and by 144 gives the mean effective pressure for this case as follows:

$$p_m = \frac{A}{144L} = p_1 \left\{ \frac{l + (l_c + l) \log_e r}{L} \right\} - p_2. \quad . \quad . \quad . \quad (308a)$$

To simplify this expression, let $C = \frac{l}{L}$ = cut-off ratio, and $\frac{l_c}{L} = c$ = clearance ratio; then

$$p_m = p_1 \{ C + (c + C) \log_e r \} - p_2. \quad . \quad . \quad . \quad (308b)$$

The diagram factors for this case are 3 or 4 per cent larger than those given in Table XIV. If desired additional corrections for compression and throttling can be made.

236. Conventional Diagrams for Multiple-Expansion Engines.— (a) By referring to Fig. 206 on which diagrams of both the high-pressure and the low-pressure cylinders of a compound engine are drawn to the same scale, it will be seen that if the dividing line at T_R is omitted there results a single indicator diagram of area equal to the sum of the areas H.P. and L.P.; thus, neglecting throttling, leakage, friction and cylinder condensation, a simple engine of the same size as the low-pressure cylinder (total volume = V_2) would give the same amount of power as is obtained with the two cylinders of the compound engine.

Evidently, then, to calculate the i.hp. of the compound engine, it is only necessary to consider the mean effective pressure of this simple (or "combined") diagram as acting on the low-pressure piston, and then to use the proper diagram factor. The i.hp. of triple- and quadruple-expansion engines can be computed in a similar manner.

The mean effective pressure of the combined diagram is usually called the "m.e.p. referred to the low-pressure cylinder," or more briefly the "**referred m.e.p.**" (p_{mR}). Its probable value for an actual engine can be computed by using the proper diagram factor with Eq. (305).

(b) If it is desired to estimate the size of a compound engine that will give a specified amount of power, the referred m.e.p. is first computed; then with the stroke, L (feet), and number n of cycles per minute selected, the *area of the low-pressure piston* in square inches to give any i.hp. can be computed from

$$a_L = \frac{\text{i.hp.} \times 33{,}000}{p_{mR} \times DF \times L \times n}, \quad . \quad . \quad . \quad . \quad . \quad (309)$$

in which DF is the diagram factor (see Table XIV).

Then with the ratio R of low-pressure cylinder volume to that of the high-pressure cylinder known, the *area of the high-pressure piston* is of course $1/R$th of the low-pressure area if the strokes are equal.

The size of the cylinders in triple- and quadruple-expansion engines is found in similar manner. The cylinder ratios to be used are found in Section **240.**

(c) The diagrams of multiple-expansion engines will now be considered more in detail, and to facilitate the discussion the engines will be divided into two groups: (1) the *Woolf type*, without receivers; and (2) *engines with receivers.*

237. Diagrams of Woolf Type of Engine.—(a) The compound engine was patented in 1781 by Jonathan Hornblower, but Watt's broad patents on expansion steam engines delayed its use. In 1804, Woolf reintroduced the compound engine and used an arrangement in which the steam was exhausted from the high-pressure cylinder, directly through very short passages to the low-pressure cylinder. Because there is little or *no receiver* volume or storage volume between the cylinders in such an engine, it is necessary for the pistons to start and finish their strokes together, and the *low-pressure cylinder must receive steam throughout its entire stroke from the high-pressure cylinder.* If the steam were cut off in the low-pressure cylinder, there would be no place into which the high-pressure steam could be exhausted during the remainder of the stroke after this cut-off had occurred. The pistons may move together or in opposite directions.

(**b**) Fig. 228 (*a*) shows a Woolf engine whose pistons move synchronously and in the same direction. This motion would result if both piston rods were connected to the same end of a "walking beam" or to cranks set together. In Fig. 228 (*b*) the indicator diagram *H* is for the head end of the high-pressure cylinder and *L* is for the crank end of the low-pressure cylinder, clearance volume being neglected in both cases. In operation, steam is admitted to the high-pressure cylinder according to line *abc*; it is cut off at *c*; is expanded along *cd*; and it is exhausted from the high-pressure cylinder along line *da*. This steam exhausted from the high-pressure cylinder is received by the low-pressure cylinder along the line *ABC* and is then exhausted along line *CDA*. *BC* and *da* will be called hereafter the **line of transference** or **receiver line.** In Fig. 228 (*b*) the indicator diagrams of both cylinders have the same length, that is, the abscissas are piston positions, and are numbered to correspond with the positions shown in Fig. 228 (*a*).

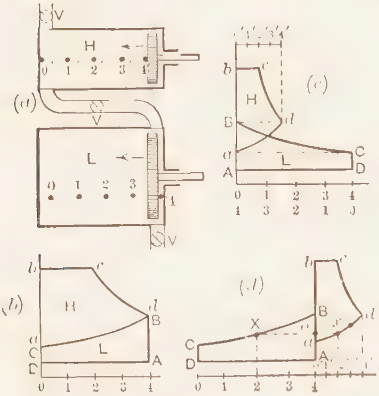

FIG. 228.

In (*c*) of this figure, the diagrams have been "combined," with abscissas representing the respective *volumes* in the two cylinders. In (*d*) the diagrams have been combined in such a way that the volume of the steam during transference from the first to the second cylinder can be scaled directly. Thus, when the pistons have reached simultaneous positions 2 and 2′ the distance 0′–2′ (= ox) is the volume of steam in the high-pressure cylinder, the distance 4–2 (= oX) is the volume it occupies in the low-pressure cylinder, and distance 2′–2 (= Xx) is the total volume of the steam between the two pistons for this position in the stroke. Obviously, the distances between piston positions bearing like numbers in this figure represent the volumes of steam during the period of transference. After these volumes have been determined (by scaling), the pressures at the corresponding piston positions can be found if the expansion is assumed to be hyperbolic, for during expansions *cd*, *da*, and *BC* the product PV remains constant, since there is no change in the quantity of steam involved during these processes. Thus the high- and low-pressure PV-diagrams can be readily constructed.

238. Diagrams for Engines with Infinite Receivers and No Clearance (General).—(**a**) If a receiver of infinite volume is placed between the cylinders of the Woolf engine the curves *da* and *BC*, in Fig. 228, would become horizontal straight lines, and the low-pressure indicator diagram would be a rectangle. Evidently, with finite receiver, the larger the receiver volume the more nearly horizontal and straight will the line of transference become.

With a receiver of considerable volume into which the high-pressure cylinder can exhaust, it is possible to " cut off " in the low-pressure cylinder and thus to expand the steam independently in this cylinder. The pressure in the receiver will vary, because part of the time steam is being received from the high-pressure cylinder, at other times steam is being delivered to the low-pressure cylinder, and during part of the cycle both of these operations may occur simultaneously. Consequently the back-pressure line on the H.P. indicator diagram and the admission line of the L.P. diagram will be irregular. The character of the line of transference will be discussed in detail later.

(**b**) When a receiver of considerable volume is used it is possible to have any angle between the cranks of the two cylinders, whereas in the Woolf engine this angle is limited to zero degrees, or 180 degrees in cases where there is a separate crank for each cylinder.

(**c**) In Fig. 229, *AbcD* is a conventional " combined " diagram for a compound engine with receiver of infinite volume. In it, *AD* is the volume of the low-pressure cylinder, *ad* is that of the high-pressure cylinder, *bc* is the volume of steam admitted to the high-pressure cylinder, and *BC* is that at the time of cut-off in the low-pressure cylinder. Then

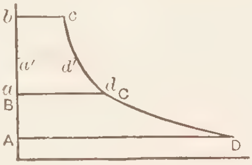

FIG. 229.

$$\frac{ad}{bc} = r_H = \text{ratio of expansion in the high-pressure cylinder;}$$

$$\frac{AD}{BC} = r_L = \text{ratio of expansion in the low-pressure cylinder;}$$

$$\frac{AD}{bc} = r_T = \text{total ratio of expansion.}$$

Since, if hyperbolic expansion is assumed,

$$p_c(bc) = p_d(ad),$$

the receiver pressure is therfore

$$p_d = p_c\left(\frac{bc}{ad}\right) = p_c\frac{1}{r_H}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (310)$$

for the case in which the expansion is complete in both cylinders.

(d) It is evident that the horizontal transfer line obtained with a receiver of infinite volume would correspond to the *mean* transfer pressure if a receiver of finite volume is used, and that indicator diagrams drawn with this horizontal transfer line would have practically the same areas as with the variable line of the small receiver. Hence these diagrams may not only be used for the engine as a whole but also when each cylinder is considered separately.

(e) **Changing the low-pressure cut-off** to make it occur earlier results (1) in raising the receiver line, as shown dotted in Fig. 229, (2) in a *reduction* of the area of the high-pressure diagram and (3) an *increase* in the area of the low-pressure diagram. Making the low-pressure cut-off later reverses these results. Thus the cut-off in the low-pressure cylinder influences the receiver pressure and distribution of work between the cylinders, but does not affect the total work done by the engine.

(f) The **selection of the receiver pressure** is based on the following considerations:

(1) For greatest economy in the use of steam the *temperature ranges* in the two cylinders should probably be equal, although this is not certain. Hence the receiver pressure should probably be such that the corresponding temperature of the steam is midway between the initial and final temperatures of the working fluid. Other considerations may be more important than this, however.

(2) It is sometimes desirable to have the *same cut-off* (that is, the same expansion ratios) in both cylinders. For example, in the tandem-compound engine shown in Fig. 220, the two valves are on the same rod, hence the cut-offs in the two cylinders must change together.

(3) Usually it is desirable to have *equal work* done in the two cylinders. In this case the receiver line should be so drawn that the areas of the high-pressure and low-pressure diagrams are equal. This is especially desirable when the engine is a cross compound.

(4) In some special cases, equal *maximum thrusts* on the piston rods are desirable, and these thrusts are dependent on the receiver pressure.

(5) The *uniformity in turning effort* is dependent on the shape and relative proportions of the indicator diagrams of the two cylinders, and hence is dependent on the receiver pressure.

Usually compound engines are operated to perform equal work in the two cylinders, and this gives about as uniform a crank effort as is possible, and hence, considerations (3) and (5) are satisfied together with sufficient accuracy for ordinary purposes.

239. Receiver Pressures in Compound Engines with Infinite Receivers and No Clearance.—(a) It has just been seen that the distribution of work among the cylinders depends on the receiver pressures, hence the problem is one of determining the mean receiver pressures which will give the desired distribution. The receiver pressure may be determined either graphically or analytically, using the conventional diagram; but only the former method will be considered here. The *receiver volume* will be assumed *infinite* and the *clearance zero.*

Let p_1, p_2, and V_2 in Fig. 230 be given, and assume a terminal pressure p_D such as will give the drop (DE) in pressure due to incomplete expansion. With these data available, the combined PV-diagram, $AbcDE$, can be easily drawn and its work area can be determined. If the high-pressure cylinder is to do $1/n$th of the total work, the area H will be $1/n$th of the total area. The problem then is to find the location of line *ad* which will give this distribution of work. The line *ad* can be drawn tentatively and then the area above it can be integrated by planimeter to see if it has the proper value. If it is not correct, another position of *ad* can be tested, and by repeated trials a proper receiver line can be obtained by this " cut and try " method. This same method can be used when the H.P expansion is incomplete (i.e., when the toe of the H.P. diagram is removed) as in Fig. 231, and can also be applied to multiple-expansion engines with any number of expansion cylinders (whether with or without clearance).

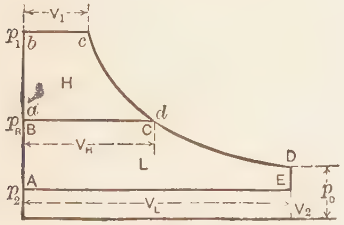

FIG. 230.

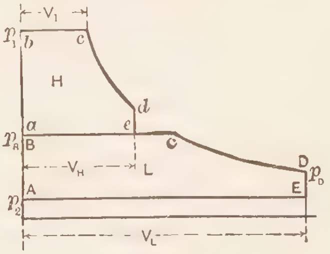

FIG. 231.

In Figs. 230 and 231, V_H is the volume of the high-pressure cylinder, and the corresponding mean effective pressure acting in the high-pressure cylinder is

$$p_{mH} = \frac{\text{area } H}{\text{length } V_H} \times \text{scale of ordinates. . . . .} \qquad (311)$$

Similarly the L.P. mean effective pressure is

$$p_{mL} = \frac{\text{area } L}{\text{length } V_L} \times \text{scale of ordinates. . . . .} \qquad (312)$$

The total m.e.p. "referred" to the low-pressure cylinder is

$$p_{mR} = \frac{\text{area } (H + L)}{\text{length } V_L} \times \text{scale of ordinates.} \quad . \quad . \quad (313)$$

(**b**) By removing the toe from the H.P. diagram, as in Fig. 231, the high-pressure cylinder is decreased in volume in the ratio $\frac{ae}{BC}$ and the cost of the engine is consequently reduced. On account of this saving, and because the expansion should not be to a pressure lower than that which is sufficient to overcome the engine friction, most compound engines are operated with the drop *de* at release in the high-pressure cylinder.

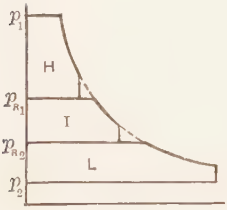

FIG. 232.

The same method is applicable when the expansion takes place in more than two cylinders, as is indicated in Fig. 232.

240. Cylinder and Expansion Ratios Used in Multiple-Expansion Engines.—(**a**) In general the greater the total range of pressures in the engine the larger should be the cylinder ratio and the expansion ratio. Thus high-pressure engines have larger ratios than low-pressure engines, and those condensing have greater ratios than those which operate noncondensing. Practice varies widely and only the average values can be given here.

(**b**) Modern *compound engines* usually operate with steam pressures between 125 pounds and 175 pounds per sq. in. gage. In many instances, however, much higher and lower values have been used. Stationary engines of this type usually have cut-offs in the high-pressure cylinders between 0.25 and 0.4 of the stroke under normal load. With late cut-off a smaller engine can be used for a given power than with early cut-off; but the consequent saving in "first cost" of engine may be more than balanced by loss in efficiency and greater cost of operation. Cylinder ratios customarily used are about as follows:

CYLINDER RATIOS FOR COMPOUND ENGINES

Cylinder ratio	$2\frac{1}{2}$	$3\frac{1}{4}$	4	$4\frac{1}{2}$
Gage pressure, noncondensing	100	120		
Gage pressure, condensing	...	100	120	150

Dividing the cylinder ratio by the H.P. cut-off fraction (0.25 to 0.4) gives the total ratio of expansion. What the best cylinder and expansion ratios are, is still under discussion. Some advocate cylinder ratios even as large as 6 or 7 and remarkable economies have been obtained with such.[18]

(c) The ratio of expansion is sometimes fixed by first assuming the pressure drop at release. If this drop is added to the L.P. exhaust pressure, the pressure (p_D in Fig. 231) at release is obtained. Then, considering the expansion to be hyperbolic, the *total ratio of expansion* for the conventional diagram is

$$r_T = \frac{p_1}{p_D}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (314)$$

which is approximated more or less closely in the actual case. If the expansion ratio (r_H) in the high-pressure cylinder is then selected, the *cylinder ratio* is

$$R = \frac{r_T}{r_H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (315)$$

After the receiver pressure, which will give the proper distribution of work between the cylinders, has been determined, the drop in pressure at H.P. release should be checked.

(d) Modern *triple-expansion engines* usually operate with steam pressures from 150 to 200 pounds per sq. in. gage or even higher. The pressure at L.P. release in condensing marine engines is commonly about 15 pounds per square inch absolute under normal load, and in stationary engines it is about half this value. As before, the total expansion ratio (r_T) can be found approximately by dividing the initial pressure by the L.P. release pressure (considering the expansion to be hyperbolic); or it can be obtained from economy curves of similar engines operating under similar conditions, when ratios have been used as abscissas.

The H.P. cut-off in marine engines is usually from 0.55 to 0.7 of the stroke and in stationary engines is much earlier. The H.P. expansion ratio (r_H) is the reciprocal of this cut-off ratio (neglecting clearance). With r_T and r_H known, the *volume ratio* of high-pressure to low-pressure cylinder is $R = \frac{r_T}{r_H}$. If the strokes are equal, as is almost invariably the case, the *ratio of piston areas* will be the same as the volume ratio.

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[18] $r = 6.25$ Cross-compound Corliss. "Am. Electrician," June, 1903.
$r_T = 7.3$ Fleming Four-valve. Trans. A.S.M.E., Vol. XXV, page 212.
$r_T = 6.4$ Tandem-compound Corliss—Barrus' Engine Tests, page 185.
$r_T = 6.2$ Edison Waterside Station, New York. "Power," July, 1904, page 424. Also see papers in Trans. A.S.M.E.

If the conventional diagrams of the various cylinders have sharp toes, the work will be equally distributed among the cylinders if the cylinder volumes (or piston areas) are such that $\frac{H}{I} = \frac{I}{L}$ (in which the letters refer to the high-, intermediate-, and low-pressure cylinder volumes, or areas). In such a case the intermediate cylinder volume (or piston area) is found from

$$I = \sqrt{H \times L}.$$

In the actual case, because of departure of the real indicator diagrams from the conventional and because of cylinder condensation, cushion steam, etc., the intermediate-pressure cylinder is made a little smaller than this equation would give. Seaton states that in marine practice [19] the intermediate cylinder volume (or piston area) is about as given by the following equation:

$$I = \frac{\sqrt{H \times L}}{1.1}. \quad \quad (316)$$

Marine triple-expansion engines are proportioned about as follows:

Initial pressure, abs	165	175	195
Ratio I to H	2.33	2.4	2.54
Ratio L to H	6.6	7.0	7.8
Total expansion ratio	11.0	11.7	13.0

241. Effects of Changing the Cut-offs in the Respective Cylinders of Multiple-expansion Engines.—(a) In "regulating" the engine to make the power output equal to the demand, the steam distribution to the cylinders can be varied in several ways.

(b) It has already been shown that *the effect of making the L.P. cut-off occur later* in the stroke (other things remaining the same) is (1) to lower the receiver pressure, (2) to increase the H.P. work, (3) to decrease the L.P. work; and *vice versa*. But (4) such change does not affect the total work of the engine if the toes of the diagrams are not lost, hence the engine cannot be regulated by changing merely the L.P. cut-off.

(c) If the **L.P. cut-off is fixed,** and the H.P. cut-off is made to occur later, there results (1) an increase in the receiver pressure (Fig. 233), (2) a greater increase in the L.P. work than in the H.P. work because the increased receiver pressure offsets the extra work that would otherwise be obtained from the H.P. cylinder with its later cut-off. Making H.P. cut-off occur earlier produces the reverse effects. Compound engines can be regulated by having an automatic governor control only the cut-off in the high-pressure cylinder. But in such case, if there is much change in the load on the engine, the L.P. cut-off should be adjusted by hand to equalize the distribution of the load between the cylinders.

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[19] See Seaton's "Manual of Marine Engineering;" Seaton and Rounthwaite's "Pocket Book of Marine Engineering," or any other text book on marine engineering.

(d) If the initial, receiver, and exhaust pressure lines on a *PV*-diagram for a compound engine are extended from one hyperbolic expansion line to another, as from *cD* to *c′D′* in Fig. 234 (*a*), it will be found (1) that the expansion ratios in the cylinders remain unchanged; and that, in consequence, (2) the proportionate distribution of work between the cylinders also remains the same.

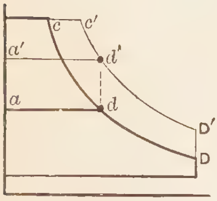

FIG. 233.

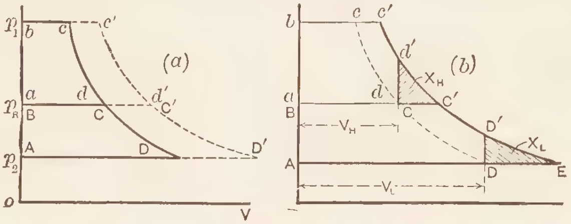

FIG. 234.

In Fig. 234 (*b*) it is seen that the high and low cylinder volumes (V_H and V_L) are such that the expansion lines *cd* and *CD* in the two cylinders are complete and continuous. If the cylinder volumes are related thus, and if the cut-offs are advanced proportionately (so that *c′d′* and *C′D′* are on the same hyperbola), the distribution of work can be shown to be in the same proportion as in the case of complete expansion just discussed; and further (from this), that (3) the toe areas (X_H and X_L) lost will be in this same proportion. These same statements are also true in case the cut-offs are decreased proportionately as in Fig. 235. In this figure, however, it is seen that the diagrams have "loops" X_H and X_L, which represent negative work. Evidently the cut-off should not be earlier than *c*, if good economy is important.

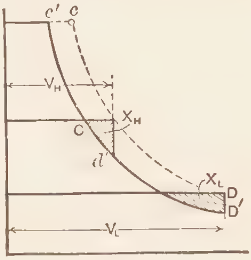

FIG. 235.

With such arrangement the automatic governor can be made to change the cut-off equally in the two cylinders and the proper balance of work will be always automatically maintained. The tandem-compound engine in Fig. 220 is an example of this case.

If the L.P. toe loss is greater than the similar H.P. loss, it can be shown that to maintain the same relative balance of power between the cylinders, the L.P. cut-off must vary more rapidly than the H.P. cut-off; thus as the power is increased the receiver pressure must be raised.

(e) If the cut-offs (or expansion ratios) in the two cylinders remain constant, the power of the engine may be decreased by *throttling* the steam, and in this case the distribution of power between the cylinders remains in substantially the same proportion. In other words, the effect of throttling is substantially equivalent to changing the pressure scale of the diagram.

(f) Because of the effect of clearance, "wire drawing," cylinder condensation, etc., the real diagrams differ greatly from the conventional ones, hence the conclusions just given can be used only in a very general sense in actual cases.

242. The Actual Combined Indicator Diagrams of Multiple-expansion Engines. (a) The actual indicator diagrams depart considerably from the conventional. This is partly because of wire drawing during flow of steam through valves, receivers, and piping, partly because of condensation or reevaporation in cylinders, receiver, and piping, partly from radiation and similar losses, partly because the real expansion line is not hyperbolic, and may also be slightly due to the withdrawal of the condensate collecting in "separating" receivers.

(b) Given the *actual indicator* diagrams obtained from the engine, as h and l in Fig. 236, they can be readily "combined," as shown by H and L, if the cylinder and clearance volumes are known.

The areas of diagrams H and L can then be found and the "referred m.e.p." determined in the usual manner.

After this, the actual diagram factor can be obtained by getting the ratio of these quantities to the area of the conventional diagram.

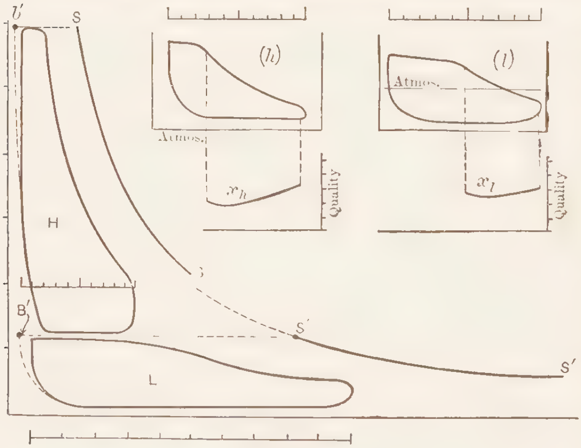

Fig. 236.

(c) In Fig. 236 the **saturation curves**, SS and $S'S'$, have been drawn.

As the weights of cushion steam in the two cylinders are not the same, and because the condensate has been removed from the receiver in this case, there are unequal weights of steam in the two cylinders during the respective expansions, consequently the saturation line $S'S'$ lies to the left of SS.

Fig. 236 also shows the *quality curves* x_h and x_l, which are obtained, after the saturation lines have been drawn, by the same method that was described for simple engines.

243. Applications and Performance of Steam Engines. (a) Reciprocating steam engines are made for use in many different fields of work in which the requirements vary widely. Thus in some steel mill engines the ability to reverse quickly combined with enormous torque when turning slowly in either direction is essential. The locomotive engine must also possess both of these features but also combined with mobility. On the other hand, in large pumping engines the chief requirements are reliability and thermal economy, because these engines must be kept in operation almost continuously. The reciprocating engine is well adapted for driving directly or indirectly (as by belt) machinery that must operate at low speeds. For driving high speed electric generators, centrifugal pumps and centrifugal compressors, it is not so well suited as the steam turbine, because the latter has a more uniform torque and runs at a more appropriate speed in many cases. The lower speed of the engine also makes it occupy a larger floor space than the turbine.

(b) The *first cost* of an engine is an item of importance in all cases, but, as with most engineering equipment, this usually varies inversely with its *operating cost*. In general it may be expected that the steam engine that is selected because of its simplicity and low first cost will not have a very high thermal efficiency, and yet such an engine may be the proper one to install for certain purposes. Small engines that actually run only a small portion of the hours in a year, such as those on steam shovels, derricks, and cranes, are good illustrations of this type. Other examples are engines that are used primarily for their convenience or suitability, the thermal economy being of secondary importance, and those from which the exhaust steam can be applied to some useful purpose.

The kind of service needed from an engine, combined with its manufacturing cost, will usually be the deciding factors in its design. The great variety of valves and the speed regulating mechanisms used on both turbines and reciprocating engines will be treated later in special chapters.

(c) The *method of governing* reciprocating engines is either by (1) throttling the steam before admission to the cylinder, (2) varying the point of cut-off, or (3) combining (1) and (2).

Engines that are governed by *throttling* have a fixed cut-off that takes place sufficiently late in the stroke to meet, with the full steam pressure, the greatest possible load that may occur; hence the expansion of the steam after cut-off is relatively small, which has an influence on the thermal efficiency. Also, the throttling necessary with the smaller demands for power reduces the available energy of the steam, as was shown in Sect. **206**; thus the thermal economy under light loads, of engines governed by this method is comparatively poor. These losses are slightly offset by a reduction in the cylinder condensation, which is brought about in several ways. It is apparent from Fig. 178 that the throttling increases the quality of the steam and decreases its temperature, and thus the cylinder condensation is somewhat reduced.

As the pressure at cut-off becomes lower the loss due to incomplete expansion is also lessened. As a result of all of these factors it has been found by tests of steam engines operating with constant cut-off, and with the speed held constant by throttling, that the total steam consumption per unit of time is almost directly proportional to the indicated horse power. Assuming that this holds exactly there results a straight-line relationship as is indicated by the line *ab* in Fig. 237, which is commonly called the **Willans Law.** When two points on this line, or one point and the slope, are given, the line can at once be drawn. Then, dividing ordinates by corresponding abscissas gives the simultaneous values of the steam rate, and these values can be used for plotting the rate curve (*R*).

For some greater ratio of expansion, resulting from an earlier cut-off, less steam is used for a given output, hence for such cases the curves *a′b′* and *c′d′* in Fig. 237 would lie below the others.

(**d**) When the engine is governed by **varying the cut-off,** the steam rate curve resembles *cfg* in Fig. 238, the reasons for which were made clear in Sect. **230** (**f**). To this figure has been added the curve *cd* of Fig. 237, the point *d* of course coinciding with *g*. Thus it is seen that cut-off governing gives better steam economy than throttle governing except at the maximum load.

The product of abscissas by ordinates gives the total steam consumption, plotting which gives the curved line *hij* as the *curve of total steam consumption* (*TC*) for cut-off governing. Evidently point *i*, where a line drawn from *O* becomes tangent to *hij*, determines the abscissa for the lowest steam rate,—for that point has the smallest ratio of ordinate abscissa.

(**e**) The *y*-intercept *Oy* of the *TC*-curve represents the weight of steam that would be used to overcome the losses from condensation, leakage, and radiation, assuming that they remain the same even though the engine is not running, i.e., though the i.hp. is zero.

Curves similar to those in Figs. 237 and 238 might have B.t.u. as ordinates; and m.e.p., cut-off, ratio of expansion, or d.hp. may be used as abscissas. When abscissas are d.hp., then the y-intercept represents the consumption due to engine friction in addition to the other losses mentioned in the preceding paragraph.

(**f**) If, in Fig. 238, OO' is the i.hp. used in overcoming the engine friction, then $O'Y'$ is the axis from which the d.hp. is measured. If the engine friction is assumed constant for all loads (which is not strictly true), the curve TC in the figure, with origin at O', gives the total consumption for the d.hp. developed. The corresponding curve $e'f'g'$ of the steam rate per d.hp-hr. will of course lie above efg, and the lowest point f' will be farther from O than f. Evidently, on the basis of *delivered power*, the best economy in this case occurs when the i.hp. equals Ok' (corresponding to a d.hp. of $O'k'$), and this should be the power which the engine normally develops (" Normal Power ") if steam economy is of prime importance. This should then be the **rated power,** or power at which the engine is rated to operate normally. When the i.hp. developed is either more or less than this, the engine has poorer economy.

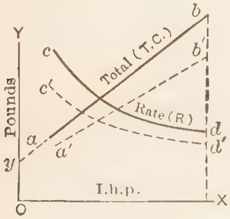

FIG. 237.

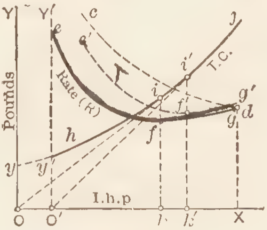

FIG. 238.

(**g**) For the best thermal economy the ratio of the actual load to the rated load should be unity; and, since it is better to overload than to underload a steam engine (see Fig. 238), a load that is a certain amount above the rated value is preferable to one the same amount below. There are, however, other considerations which may make it financially more profitable to rate the engine at output other than that giving best thermal economy, and to operate with some load other than the rated one.

In many instances, the average load of a power plant as a whole is low, but in such cases it is customary, when possible, to have several engines and to place in commission such a number as will cause those in service to operate under the most economical conditions. Some engines give a much flatter steam rate curve than others and this factor may be of the utmost importance in the final selection. The uniflow engine gives a very flat economy curve as indicated in Sect. **232** (**e**).

(**h**) Curves of *steam consumption* for an engine are only useful in determining the best conditions of operation for that particular engine and for comparing it with others that operate under *similar conditions*. When the conditions are widely different the steam rates should not be compared directly as a means of determining their relative thermal economies. The *thermal efficiencies*,[20] or the energy consumptions per unit of output, and the *engine efficiencies* [20] are the only true measures by which the thermal performance of one engine may, in general, be compared with that of another.

(**i**) The actual values of *thermal efficiencies* and of *engine efficiencies* cover a very wide range, due to the use of the great variety of steam pressures, temperatures, and exhaust pressures, and because the means of preventing the losses due to incomplete expansion, leakage, radiation and friction, are not all equally effective. The best reciprocating engines and the best steam turbines, have engine efficiencies referred to the Rankine, of more than 80 per cent while the cheaper and smaller ones of both types may not be much over one-half of this value.[21]

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[20] See Sects. **205**, **207**, and **209**, for method of calculating these efficiencies.

[21] The Am. Soc. of Mech. Engrs. has prepared a Code on the Testing of Reciprocating Engines.

CHAPTER XX

STEAM TURBINES—GENERAL

244. Introductory.—The purpose of this chapter is to describe *very briefly* the general types and methods of operation of the more common forms of steam turbines, and to show how the thermodynamic principles which are given in the preceding chapters may be applied to such units. A more detailed discussion of turbines, the flow of steam, nozzles and other parts, velocity diagrams, losses, performance, etc., will be deferred to subsequent chapters. A brief history of the development of steam turbines is given in Sects. **19** and **26**.

245. Turbine Principles.—(a) Steam turbines are broadly divided into the following classes: (1) **impulse,** (2) **reaction,** and (3) **combination.** In impulse turbines the steam expands through stationary nozzles and acts on blades on the rims of rotating discs, as in the Branca turbine in Fig. 3 (page 17). The simple elements of the first commercially successful impulse turbine, that developed by deLaval, are shown in Fig. 239. In the pure reaction turbine the steam expands through rotating nozzles, or their equivalent, as in the Hero turbine in Fig. 2 (page 17), and the reaction of the issuing jet causes the motion of the nozzles and parts to which they are attached. Modern turbines generally employ neither pure impulse nor pure reaction, but use a combination of both. In so-called "impulse" turbines the expansion of the steam takes place wholly in nozzles that are *stationary*, none occurring between the rotating blades; in "reaction" machines the steam expands through or between rotating elements. The "combination" type of turbine is one which has several stages and does not use the same method of operation in all. There are many other ways of classifying turbines, but they will not be considered at this point.

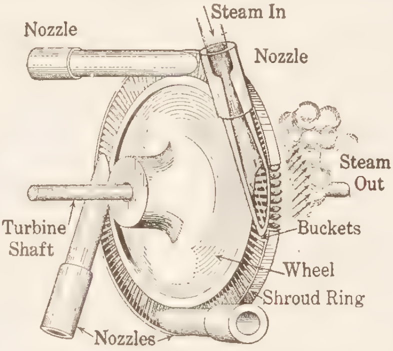

Fig. 239.—Simple Impulse Elements (deLaval).

(**b**) The *thermodynamic problem* encountered in the ideal steam turbine is centered in the *nozzle*, where all the transformation of heat into kinetic energy should occur. Nozzle design will be considered in a later chapter. For the present purposes it is only necessary to know that mechanical energy can be developed within a turbine through the utilization of the " available energy " of the steam, which in the ideal case would have isentropic expansion. After this conversion in the nozzle the problem becomes a *dynamic one*, namely, to use this kinetic energy for the production of power. The nozzles and blades are so arranged as to avoid turbulence as much as possible.

(**c**) The *velocity diagrams* used in designing the blades, or buckets, of the steam turbine are similar in many respects to those used for water turbines. But, despite this resemblance, the problems of design and construction in the former differ greatly from those in the latter. This is principally because, in the steam turbine, (1) the jet velocities are enormously greater (in some cases they exceed 3500 feet per second, or 41 miles per minute), (2) the bucket speeds are very much higher, (3) the working substance is elastic and tends to expand as fully as the surrounding media will allow, and (4) the kinetic energy of the jet is obtained primarily from *heat* conveyed by the working substance and little, or none, from gravity head.

(**d**) The steam turbine differs as much from the steam engine as to its mechanism and method of operation as does the water turbine. Although both of these steam-actuated prime movers use the available energy of the steam, the turbine utilizes it by means of the kinetic energy of the steam jet, whereas in the steam engine the transformation is brought about by the controlled expansion within the cylinder.

(**e**) In turbines, there is a certain definite ratio, called the " *velocity ratio*," of bucket velocity to jet velocity that will give the best steam economy in any given case. In practice, however, if the full expansion from initial to final pressure takes place in a *single* set of nozzles, the ideal bucket velocity, with a single row of blades, is usually greater than the structure of the rotor will stand, because of the enormous stresses produced by high rotative speeds. Also, these speeds commonly prohibit the direct connection of the driven machinery to the turbine shaft. Hence, with this arrangement, it is generally necessary to adopt lower bucket velocities than those which would give the highest economy, and also to use gearing of some kind between the turbine and the machine it drives. This difficulty may be overcome by " staging."

246. Pressure Staging.—(**a**) In order to obtain lower jet and bucket velocities, most turbines are of the " **multi-stage** " type in one kind of which " pressure staging " is used. Fig. 240 shows diagrammatically an *impulse turbine* of the latter type. In such turbines each stage by itself constitutes a simple turbine, in the nozzle of which the steam expands through a small range and therefore acquires relatively low velocity. The stages are usually arranged in series with diaphragms between and with all rotors mounted on the same shaft.

In Fig. 240 the sections of the turbine casing and the diaphragms are shown by crosshatching, and the nozzle and turbine wheel sections are black. Steam enters at the left, expands through the first nozzle (or ring of nozzles) N_1, in which it acquires a relatively low velocity, and discharges so as to act properly on the buckets on the wheel in the first-stage casing, in which the pressure is but little lower than the initial. The steam then expands through the nozzle (or ring of nozzles) N_2 in the diaphragm between the first and second stages, and acts on the buckets of the wheel in the second chamber, where the steam pressure is somewhat lower than it is in the first. In similar manner the process is continued in a third stage; and so on in other stages until the exhaust pressure is reached in the last one. Small turbines may have two or three pressure stages and large ones twenty or more.

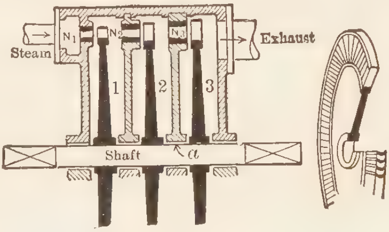

FIG. 240.

(b) Multi-stage impulse turbines with nozzles and blades arranged as in Fig. 240, are commonly said to be of the **Rateau type,** being named after the originator who developed this arrangement in 1897. They are also sometimes called the "*Zoelly*" *type*, after another inventor. As will be seen later, the pressure stages may have other arrangements of blades and nozzles, and such turbines are given other names. All commercial reaction turbines of necessity use pressure staging.

(c) The nozzles in all the stages of multi-stage turbines must all deliver simultaneously the *same weight* of working substance per second.[1] They may be designed to do this with *equal velocities*, in which case the bucket velocities in all stages would be the same and the mean diameters of the wheels would be equal; or the *jet velocities* may be *varied* and the bucket velocities and wheel diameters be made to correspond.

As the steam traverses the turbine it expands by increments in the successive nozzles, and increases in volume, hence the nozzle areas must increase in like manner through the series, as is illustrated in Fig. 240.

By properly proportioning the cross-sectional areas of the inlet and outlet nozzles of any stage, the designer can fix at any desired value the pressure that will be maintained in that stage. When there are many nozzles used in a stage they are usually made rectangular in cross-section, are curved, and are arranged closely together, with thin partitions between them, as in Fig. 241.

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[1] Except for turbines from which steam is extracted from certain stages.

247. Velocity Staging.—(a) In some designs of impulse turbines the so-called "*Velocity Staging*" or "*Compounding*" is used. In this case, the "velocity ratio" is less than the ideal value for a simple impulse element, and, in consequence, the steam has a rather high velocity as it leaves the row of blades on which the jet from the nozzle impinges. However, by using a row of stationary blades, S in Fig. 241, the steam can be redirected so as to act on a second row of rotating blades, M_2, which can absorb much of the kinetic energy that is still available; and the process could be continued, if desired, with other pairs of stationary and rotating blades. Usually, however, it is not economical thermodynamically or commercially to use velocity-

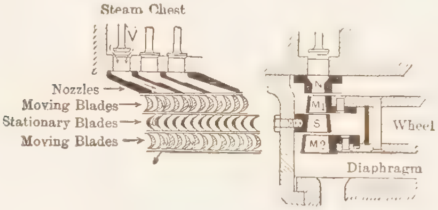

FIG. 241.—Curtis Staging.

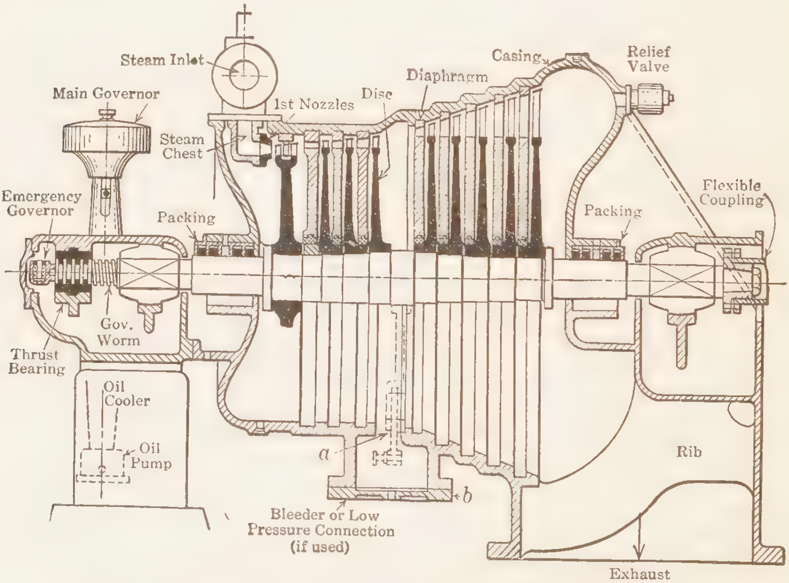

FIG. 242.—Elements of a Curtis-Rateau Turbine.

compounding more than once. The arrangement described is generally called a **Curtis stage**, after its inventor.

Multi-stage turbines may have Curtis velocity-compounding in each of the pressure stages, or only in the first ones, the others having either simple impulse or reaction elements. Such a turbine is of the "combination" type. The principal elements of a Curtis-Rateau turbine are shown in Fig. 242, in which the parts are labeled.

(b) *Velocity-compounding* can also be accomplished with a single row of rotating buckets. In one arrangement the steam is conducted in a serpentine path so as to act repeatedly on the same blades, as indicated in Fig. 243. This is known as the **re-entry** type of turbine. As the velocity becomes reduced the transverse areas of the guide passages must be increased in the manner illustrated. In another arrangement the steam is conducted in a helical path as in Fig. 244. Many small turbines having a single pressure stage employ one or the other of these simple methods as a means of reducing their rotative speeds.

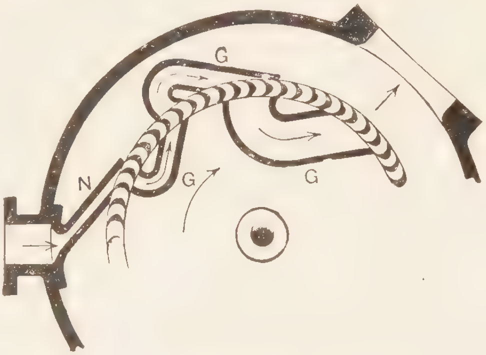

FIG. 243.

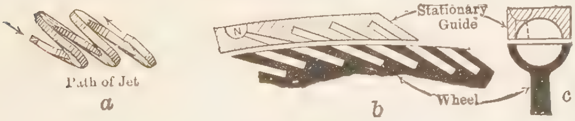

FIG. 244.

248. The Reaction Turbine. (a) When economical performance is important, the *reaction turbine* is necessarily always made *multi-stage*, the reasons for which will be given in a later section. However, in order to obtain a clear conception of the principles of the reaction element, the single stage arrangement will first be considered. Fig. 245 shows an arrangement similar to Hero's, the black portion *a* showing the rotating nozzles through which the pressure drops from P_1 to P_2. Another simple reaction turbine is shown in Fig. 246 with blades mounted on the periphery of a disc which is arranged to rotate about the axis XX. It is seen that the space *b* between these blades has the same form as the nozzle *a* in Fig. 245, hence, there are as many rotating nozzles in this case as there are spaces between blades.

In this arrangement, which effects compactness with high capacity, there is a "**full peripheral discharge**" of steam around the entire circumference, and it is important to note that there is a difference between the pressures P_1 and P_2 on the two sides of the disc, a condition contrary to that present in the impulse type of turbine.

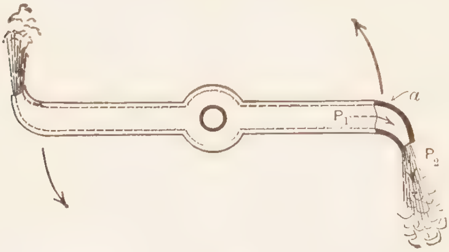

Fig. 245.

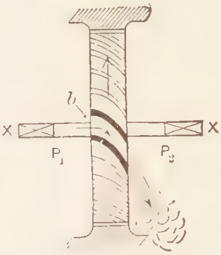

Fig. 246.

(**b**) Fig. 247 may be used to show certain elements of the modern type of reaction turbine. Between the casing and the tips of the blades on the drum, there is necessarily a *radial clearance* space, and because of the inequality between the pressures P_1 and P_2 *leakage* occurs through this space. This clearance is of course always made the minimum practicable. The relative amount of leakage is evidently dependent on the ratio of this annular space to the passage area between blades; thus, the longer the blades are, the less the leakage, with the same clearance. If the peripheral diameter is decreased, not only is the annular space reduced, but the blades must be lengthened to maintain the same passage area between them; hence there is a twofold reduction in the leakage accompanying such change.

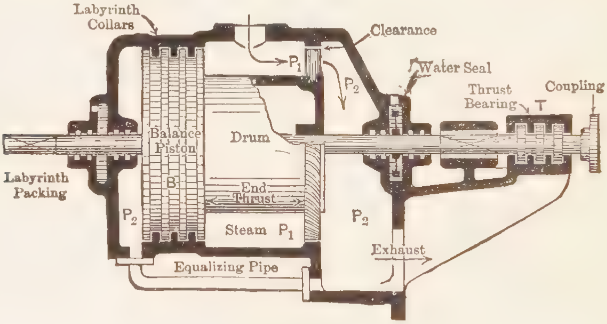

Fig. 247.

(**c**) It is apparent that the difference between pressures P_1 and P_2 in Fig. 246 causes an end thrust on the shaft. The same is true of the arrangement in Fig. 247. This thrust may be resisted (1) by the thrust bearing T in this figure; or (2) by the balance piston B, which presents to the pressures P_1 and P_2 areas equal to those exposed by the blades and drum end; or (3) by using a "double-flow" arrangement of drum wherein there are similar rows of blades having discharges which are equal but opposite in direction and hence give opposite end thrusts; or (4) by using a combination of the foregoing. In any case there must be a thrust bearing to maintain the rotor in its proper position.

(d) The leakage between the piston B and the shell is usually reduced by employing mating collars, as in Fig. 247, or rings which form a "labyrinth passage" which becomes more or less sealed by the moisture present in the vapor. Where the shaft passes through the casing the leakage is counteracted by similar labyrinth packings, or by packing rings, or by water seals.

(e) The so-called multi-stage **"reaction" turbine,** invented by **Parsons,** besides having the rotating reaction blades similar to those in Fig. 247, has stationary guide blades which act as nozzles, the jets from which impinge on the rotating blades. Hence such turbines combine the impulse and the reaction principles, although commercially known as "reaction" turbines.

Fig. 248 shows such an arrangement, S and M being respectively stationary and moving blades. It is seen that not only is there leakage at the tips L_1 of the moving blades, but also at the ends L_2 of the stationary ones. These turbines have many stages, with stationary and rotating blades alternating. The steam expands through the passages between the stationary blades as through nozzles. The action of the steam on the moving blades is twofold: (1) The direction of the jet is changed, and if no other action took place the conditions would be the same as in the impulse element; and (2) the steam *expands* while passing through the moving blades and acquires velocity, so that when discharged rearwards there is a *reaction* effect on the blade. The residual velocity of the jet leaving the rotating blade is redirected and increased by the next stationary blades, then discharged against the next row

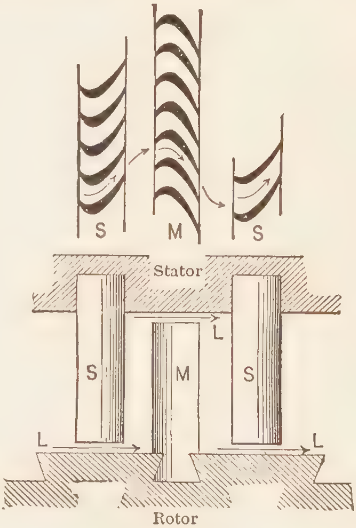

Fig. 248.

of moving blades, and so on from one end of the turbine to the other.

As the bucket velocities used are relatively low, and as the ideal "velocity ratios" are large, it is necessary to have small "heat drops" per stage, hence a large number of stages must be used. Reaction turbines are not usually made in sizes below 300 kw.

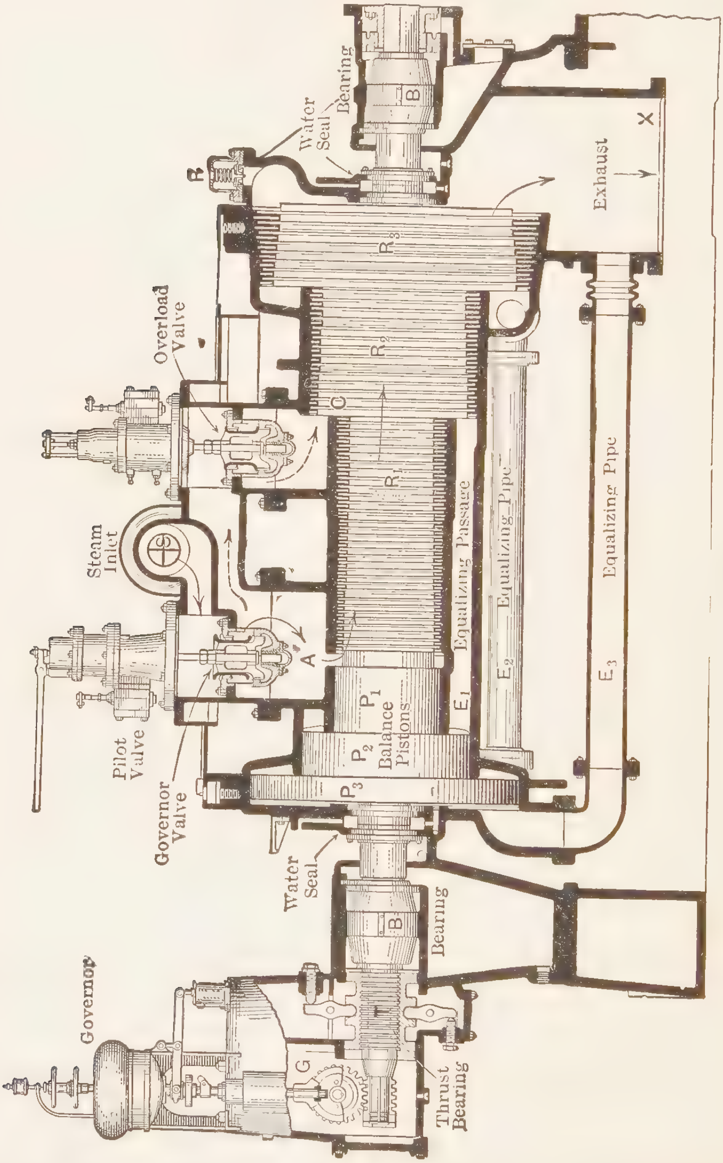

FIG. 249.

One arrangement which is typical of a medium size reaction turbine is shown in Fig. 249, with the more important parts labeled. Many of the smaller turbines are of the " combination type," using a Curtis element followed by reaction stages. Large reaction turbines are often made " double-flow " or " semi-double-flow"; and very large ones are sometimes " cross-compounded," having separate shaft for the high and low pressure elements. These arrangements will be treated later.

249. Special Forms of Steam Turbines.—As the primary patents on the various arrangements of nozzles and blades have expired, manufacturers are now at liberty to use any type, or any combination, which will give the best thermodynamic and commercial results for the conditions under which a turbine must operate. Hence there are a great variety of arrangements used.

The structure of a turbine is of such form that the design can be easily modified to meet special conditions. Thus the casing can be provided with outlets, as in Fig. 242, for bleeding steam from one or more stages to use in a regenerative cycle (discussed in Sect. **216**), or for heating buildings, or for industrial applications. Such a machine is called a **Bleeder,** or **Extraction Turbine.** When the demand for bled steam is variable, but at constant pressure, the turbine can be equipped with a device to effect the necessary regulation automatically. If all the steam is continuously bled at a high pressure, the low-pressure elements may be omitted. Such a machine is a **Back-pressure Turbine.**

Sometimes the turbine is designed to use only low-pressure steam which may be received from a high-pressure engine, or from industrial processes, etc. In this case it is known as a **Low-pressure Turbine.** As turbines are more economical than reciprocating engines in the use of steam at low pressures, this application is often a desirable one when the cost of the equipment involved is not prohibitive. In some cases, the supply of steam to such a turbine may be intermittent or fluctuating, and it may then be necessary to use " live steam " during these periods of deficiency. For such conditions a **Mixed-pressure Turbine** applies. This machine has a high-pressure element which is added to the low-pressure one, but which is normally not used. The change from low- to high-pressure operation, or vice versa, may be effected automatically by means of a suitable regulating device. When the periods of deficiency are short, Steam Regenerators or Accumulators may be used with a low-pressure turbine, as will be discussed in a later chapter.

A turbine can be easily constructed to provide for the withdrawal of steam for its reheating before use in the subsequent stages, in which case it is a **Reheating Turbine.** Occasionally two or more of the special provisions mentioned are combined in the same turbine; thus, there are *reheating-regenerative*[2] turbines, combined bleeder and mixed pressure arrangements, etc.

250. The Performance of Steam Turbines.—(a) Without going into details as to how the available energy of the steam is transferred into mechanical energy within a turbine, the unit may be considered thermodynamically as a whole, and its overall performance may be determined and compared with that of the corresponding ideal engine cycle, the analysis of which has already been discussed. Later it will be seen that each stage of a multi-stage turbine can be analyzed independently as a turbine by itself, the whole unit being considered as composed of a series of these elements.

(**b**) As the steam passes continuously and with extreme rapidity through the turbine, in one direction, and is always in contact with surfaces that are at its own temperature, and that are of relatively small area, nothing equivalent to cylinder condensation occurs within the turbine. The principal losses, in addition to the ones in the ideal machine, are those due to imperfect expansion of the steam through the nozzles; leakage; improper action of the jet on the blades; friction that occurs between the fluid and the guiding surfaces of the nozzles and blades; the " windage " or " rotational " loss due to the rotation of the discs, drums and blades in the more or less stagnant vapor; and energy lost in mechanical friction and in driving oil pumps and the governing mechanism.

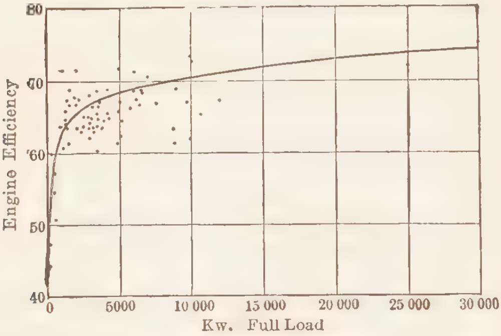

Fig. 250.—Combined Engine Efficiency of Turbine-generator Units of Various Sizes.

(**c**) With small turbines it is usually either not possible, or else inexpedient, to use the ratio of bucket speed to jet velocity that gives the maximum efficiency; or to use the high rotative speeds, or large number of stages, that would be required to obtain the best economy. Hence the engine and thermal efficiencies of these small units are generally much poorer than those of the larger turbines which can be designed to conform more closely to the ideal conditions. The effect of size on the engine efficiency of the combined turbine-generator units which are operated without bleeding or reheating, is indicated in Fig. 250.

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[2] The reheating and the reheating-regenerative cycles are discussed in Sects. **219** to **212** incl.

Large units of the latest designs, using high superheat, and reheating, have efficiencies considerably higher than those shown in the figure, some exceeding 80 per cent. For a fixed throttle temperature, the engine efficiencies are generally lower for turbines having high initial steam pressures and extreme vacuums than when these quantities are smaller; but, nevertheless, the thermal efficiency under the former conditions may be better than for the latter, because the increase in cycle efficiency may be greater in proportion than the reduction in engine efficiency.

(d) The B.t.u. used per kw-hr. delivered by turbine-generators ranges from 9000 to 14,000 in the larger turbines, to 40,000 or more in the smaller ones, the corresponding thermal efficiencies being roughly from 38 to 24 per cent in the former, to 8 per cent in the latter.

(e) Typical **performance curves** for a turbine-generator are shown in Fig. 251, in which the portion to the right of OO' is for overload conditions. It is seen that the *curve of total steam consumption* (TC-curve) to the left of OO' is substantially a straight line; and this is a characteristic of such curves for practically all types of turbines when operating without overloading. If this TC-curve is extended to intersect the Y-axis, the intercept (Y_o) represents the steam required to operate the turbine when delivering no power. It is the amount needed to overcome the friction of the turbine and the windage, that required for driving the governor, oil pumps, etc., and that for meeting the losses due to turbulence, leakage and radiation under no-load conditions.

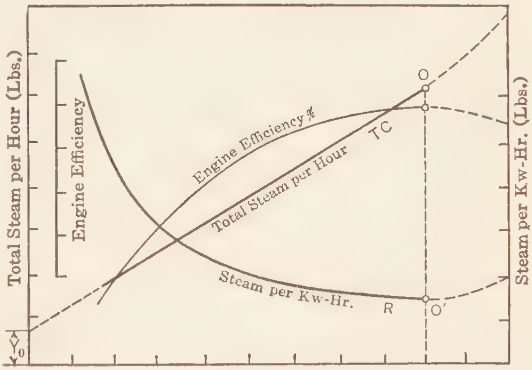


FIG. 251.

The *steam-rate curve* (R), or curve of steam used per kilowatt hour, is also shown in Fig. 251. If the TC-curve passed through the origin, as it would in the ideal case, the R-curve would be a horizontal straight line, and the steam-rate of the turbine would be the same at all loads. The greater the Y-intercept of the TC-curve the more curvature does the R-curve have, and the greater are the relative consumptions of steam under light loads. Obviously the best economy is obtained when the turbine is operated at a load corresponding to O'. When wide fluctuations of load may occur, a flat steam-rate curve is especially desirable.

The *curve of engine efficiency* shown in the figure is based on the available energy from the steam having its initial state as it is in the actual pipe line just before it reaches the throttle valve. Thus this efficiency includes the losses entailed by the throttling of the steam by the governor valve, which is the principal reason for the decrease of this efficiency when the turbine output is diminished.

Many turbines have an auxiliary "*overload valve*" which admits an extra quantity of steam to the turbine to meet emergency overloads. When this valve opens the performance curves change character, as shown by the dotted portions to the right of OO'.

(f) The **energy stream** shown at the bottom of Plate V in the Appendix is typical of a modern steam turbine. Just as in the case of the reciprocating engine (see Sect. **226** (**h**)), the magnitude of the various losses are expressed in terms of the energy, E_s, supplied to the turbine and also in terms of the available energy E_a.

The American Society of Mechanical Engineers has prepared a Power Test Code for Steam Turbines. Further discussion of the design, construction and performance of various types of turbines will be given in greater detail in Part II.

PROBLEMS

CHAPTER I

1. Find the number of B.t.u. equivalent to 33,000 ft-lb. Ans. 42.42.

2. Assume yourself called upon to investigate the claims made for a piece of mechanism with the following characteristics. It receives no energy of any kind excepting that given it by a driving belt which supplies 7.5 hp. It is claimed that the mechanism gives out or makes available 400 B.t.u. per minute. Would you make the investigation? Why?

3. (*a*) What is the largest amount of energy in B.t.u. which the mechanism operating as in problem 2 could make available per minute neglecting friction, radiation, conduction and similar losses. (*b*) Could it do this in practice? Why?

Ans. (*a*) 318.

4. In a certain waterfall, the water drops perpendicularly 1000 ft. Assuming the specific heat of water is unity and neglecting evaporation and all losses of heat, what would be the rise in temperature of the falling water after striking the rocks at the bottom? Ans. 1.29° F.

5. A winch is used in lowering a load of two tons a vertical distance of 50 ft. The load is lowered by means of a friction brake which prevents the attainment of too high a speed and which brings the load to rest just as it reaches the end of the 50-ft. drop. It takes one minute to lower the load. (*a*) Neglecting friction of bearings and similar losses, how much heat must be radiated by the mechanism of the brake and winch? (*b*) How many horse power must be absorbed by the brake?

Ans. (*a*) 257 B.t.u. per min. (*b*) 6.06 hp.

6. (*a*) If heating a house with coal could be carried out so 60 per cent of the energy in the fuel was usefully applied and the same house could be heated with electricity without wasting any of the energy supplied, to what price could coal rise per ton (of 2000 lb.) before it would become more economical to heat with electricity costing 5 cents per kw-hr.? Assume the burning of each pound of coal will liberate 13,500 B.t.u. and ignore first cost and maintenance. (*b*) What would the price of coal have to become before it would be economical to use electricity if 80 per cent of the energy in the coal could be usefully applied? Ans. (*a*) $237.50. (*b*) $316.50.

7. Electric motors receive electrical energy, convert part of it into heat within themselves and deliver the remainder at the pulleys as available mechanical energy. A certain motor delivers, in this way, 20 horsepower and converts into heat 15 per cent of all the energy supplied it. (*a*) How much heat must this electric motor dissipate per hour? (*b*) How many ft-lbs. of energy must be supplied it per minute? (*c*) How many killowatt hours of energy must be supplied if the motor runs under these conditions for 24 hours? Ans. (*a*) 8982 B.t.u. (*b*) 776,500. (*c*) 421.

8. A factory building is being designed. Calculations from the radiating surface of the building, character of that surface, location, direction of winds, etc., indicate that about 400,000 B.t.u. per hour must be liberated within the building to keep the temperature up to 65 F°. The heating engineer desires to keep the cost of the heating equipment down to a low figure and believes that he can do so by allowing for heat generated by the friction of moving mechanisms within the factory. He discovers that 100 horsepower are to be continuously supplied the factory by means of an electric motor within the building and that all of this power will be consumed therein. The motor has an efficiency of 85 per cent. What allowance can the heating engineer make? Ans. 299,400 B.t.u. per hr.

9. In the manufacture of a certain chemical compound it is necessary to stir and mix a rather heavy liquid in a large vat. If the temperature of the liquid rises above a certain value it may cause a violent explosion. The formation of the compound causes the absorption of 20,000 B.t.u. per hour and the vat is so arranged that 25,000 B.t.u. can be carried away per hour under all conditions by means of a water jacket and loss to the surrounding atmosphere. What is the maximum horsepower that can be supplied the stirring apparatus without causing a rise in temperature? Ans. 17.68 hp.

10. Find the torque of a steam turbine shaft which is transmitting 26,000 kw. at 1800 r.p.m. Ans. 101,700 lb-ft.

11. An automobile fitted with 30 × 5 in. tires will run 50 miles per hour on level when 25 hp. are supplied to the rear axle. Find the torque of the rear axle, assuming that 30 in. is the effective diameter of the rear wheels. Ans. 234 lb.-ft.

12. If the car in Prob. 11 develops maximum torque when climbing a grade that requires 22 hp. to be delivered to the rear axle to maintain a speed of 20 miles per hour, find the twisting moment in pound-feet, to which the rear axle is subjected. Ans. 516 lb-ft.

13. An inventor claims that his gasoline engine will deliver continuously 4 hp. when supplied with fuel at the rate of 2 gal. per 24 hours, it being understood that burning the gasoline will yield 122,160 B.t.u. per gallon. Is the performance possible? Why?

14. If an ideal heat engine has 240,000 B.t.u. per hour supplied to it when delivering continuously 15 hp., how much heat energy does the engine reject? Ans. 201,800 B.t.u. per hr.

15. During a steam-turbine test, the average gage pressure in the steam pipe leading to the turbine was observed to be 245 lb. per sq. in., and from a vacuum gage, the average pressure of the steam leaving the turbine was observed to be 28 in. Hg. If the barometer read 29.7 in. Hg, the temperature of the mercury in both cases being 32° F., find the absolute pressures, in pounds per square inch, of the steam entering and leaving the turbine. Ans. 259.6. 0.835.

16. If the steam leaves a small turbine with a velocity of 1000 ft. per sec., what is the energy loss, in B.t.u. per pound of steam, represented by this leaving velocity? Ans. 19.95.

17. A certain substance has a pressure of 174 lb. per sq. in. abs. and a volume of 1.724 cu. ft. per lb. In this state, the internal energy is 576.2 B.t.u. per lb., measured above an arbitrarily chosen zero. (*a*) Find the specific heat content of the substance. (*b*) What is the datum above which this heat content is measured? Ans. (*a*) 631.7 B.t.u. per lb.

18. A pound of a certain substance undergoes a change from the state defined by a pressure of 8.7 lb. per sq. in. abs. and a volume of 50 cu. ft., to the state defined by a pressure of 150 lb. per sq. in. abs. and a volume of 3 cu. ft. If there has been no change in the internal energy of the substance, how much has the heat content increased? Ans. 2.78 B.t.u. per lb.

CHAPTER IV

1. Suppose a certain weight of a substance is confined under a pressure of 175 lb. per sq. in. abs. in a cylinder whose volume is 170 cu. in. How much work in B.t.u. will be done *on the piston* if, with the pressure maintained constant, more substance is added to that in the cylinder until the volume has increased to 2000 cu. in.? Ans. 34.3.

2. A working substance, existing initially in the state defined by a pressure of 75 lb. per sq. in. abs. and a volume of 20 cu. ft., has heat abstracted at constant pressure until the volume is halved. (*a*) How much work has been done *by* the working substance? (*b*) If the *gain* in internal energy during this period is −347 B.t.u., how much heat is added *to* the working substance? Ans. (*a*) −138.8 B.t.u. (*b*) −485.8 B.t.u.

3. A certain substance, initially in state *a*, with $p_a = 350$ lb. per sq. in. abs. and $V_a = 0.5$ cu. ft., expands to state *b* with $V_b = 1.5$ cu. ft., according to the law $p = 50V + 325$. Find Wk_{ab} and show the area representing it. Ans. 54,000 ft-lb.

4. Suppose the substance of the previous problem, initially in state *a*, is expanded at constant pressure to state *c*, such that $V_c = 1.5$ cu. ft., and then is heated at constant volume until state *b*, defined in Prob. 3, is reached. (*a*) Find Wk_{acb}, and with PV-coordinates, show the area representing it. (*b*) Noting that the initial and final states in this problem are the same as in Prob. 3, why are the results different? Ans. (*a*) 50,400 ft-lb.

5. For the condition as given in Prob. 4, suppose the weight of substance is 0.45 lb., $\Delta I_{a,\,b} = 226$ B.t.u., $\Delta I_{a,\,c} = 178$ B.t.u., $t_a = 600°$ F., $t_b = 3175°$ F., and $t_c = 2720°$ F. (*a*) Find the average specific heat during the process *ab*. (*b*) Same during the process *ac*. (*c*) Same for process *cb*. Ans. (*a*) 0.255. (*b*) 0.255. (*c*) 0.234.

6. Assuming the constant pressure specific heat of water to be constant and equal to unity, how much heat must be added to raise the temperature of 1 lb. of water from 32° F. to 212° F., the pressure being kept constant? Ans. 180 B.t.u.

7. (*a*) Assuming no loss by radiation, how much electrical energy in British thermal units would be required to raise the temperature of a copper wire one mile long and weighing 0.3 lb. per foot, through a range of 10° F. The specific heat of copper is 0.095. (*b*) In problems of this kind is it assumed that the specific heat used is that at constant pressure? Why? Ans. (*a*) 1505.

8. 130 B.t.u. raise the temperature of 10 lbs. of cast iron 100° F., what must be the specific heat of this material? Ans. 0.13.

9. Assume the specific heat of wrought iron as 0.113, the specific heat of water as 1.0 and the weight of water as 62.5 lb. per cu. ft. Find the increase in temperature of 2 cu. ft. of water when a common temperature of 45° results from putting into the water a piece of iron weighing 10 lbs. and at a temperature of 1000° F. Ans. 8.64° F.

10. Assuming the constant pressure specific heat of a particular substance to be 0.503, how much heat must be added to raise the temperature of 3 lb. of this substance from 281° F. to 371° F., the pressure remaining constant? Ans. 135.8 B.t.u

11. If 282 B.t.u. are required to raise the temperature of 2 lb. of a substance from 70° F. to 300° F. while the pressure of the substance is maintained constant, what must be the average constant pressure specific heat of the substance?

Ans. 0.613.

12. Assume that the value of the variable specific heat c of a substance is given for a temperature of t by the equation

$$c = 0.5 + 0.02\, t.$$

Find the heat that must be added to raise the temperature of 12 lb. of the material from 50° to 100° F. Ans. 1200 B.t.u.

13. The constant pressure specific heat of a certain substance varies with the absolute temperature according to the law $c_p = 0.248 + 4.28(10)^{-9}T^2$. (*a*) Find the heat that must be abstracted to cool 10 lb. of the substance from 1500° F. to 100° F. (*b*) What per cent error would have been introduced if this specific heat had been considered constant at 0.248? Ans. (*a*) 3577 B.t.u. (*b*) 2.94.

14. A machine has working substance supplied to it at a rate of 288,000 lb. per hour. This substance enters the machine with a heat content of 1284 B.t.u. per lb. and a velocity of 5000 ft. per min. at a point 10 ft. above the exit where it leaves with a heat content of 949 B.t.u. per lb. and a velocity of 10,000 ft. per min. The machine delivers 36,500 hp. There is intentionally no heat added to or abstracted from the machine. Find the radiation and conduction losses in B.t.u. per hour.

Ans. 3,463,000.

CHAPTER V

1. Discuss the meaning of the term *equilibrium* as applied to mechanics and also as applied to thermodynamics.

2. Distinguish between the terms *uniform* and *constant* as applied to any property of a substance.

3. Give an illustration of an irreversible mechanical process. Explain why it is irreversible and how it may be made to approach reversibility.

4. What are the conditions to be fulfilled in order that heat may be transferred from one body to another, (*a*) in a reversible manner, and (*b*) in an irreversible manner?

5. (*a*) What is meant by turbulence in a working substance? (*b*) Give an illustration of extreme turbulence.

6. Indicate how heat may be transferred from a hot body to a working substance in an irreversible manner and yet the working substance may have only a very slight degree of turbulence.

7. When heat is to be transferred from a hot body to a working substance what advantage is gained by having them at nearly the same temperature? What disadvantage is involved?

8. (*a*) Can the state of a working substance be completely specified if it is in equilibrium? Why? (*b*) Is this also true for a turbulent state? Why?

9. What are some ways by which energy may be dissipated in irreversible processes?

10. Explain why reversible processes are desirable (*a*) when heat is to be transferred, without loss of available energy, from one body to another, and (*b*) when it is desired to *transform* heat into mechanical energy with the least loss possible.

CHAPTER VI

1. During the reversible process *ab* the heat added to the working substance is expressed by the symbol Q_{ab} and the initial absolute temperature by T_a. Why would it not be true in general, then to express $\Delta\phi_{ab}$ as equal to $\frac{Q_{ab}}{T_a}$?

2. Under what particular condition does $\Delta\phi_{ab} = \frac{Q_{ab}}{T_a}$?

3. Why does the area under a reversible path drawn on the temperature entropy diagram represent the heat transferred to the working substance? Is it necessary that the temperature shall be plotted from absolute zero? Why?

4. If 10 lb. of ice are melted at a temperature of 32° F. by the absorption of 1433 B.t.u. find the gain in entropy of this substance during this process. (*a*) In B.t.u. per deg. Fahr. (*b*) In lb. °C. calories per deg. Cent. (*c*) In kg. calories per deg. Cent. Note: 1 B.t.u. = 0.5556 lb. ° C. cal. = 0.252 Kg. cal., 1 lb. = 0.4536 kg.

Ans. (*a*) 2.91. (*b*) 2.91. (*c*) 1.323.

5. Distinguish between the change in entropy of a system of bodies and the change in entropy of one or more bodies in this system.

CHAPTER VII

1. The pressure and absolute temperature of a given mass of gas are doubled, the final volume being the same as the initial and *nothing whatever is known concerning the process* the gas undergoes. Can Charles' Law for constant volume be applied?

2. A gas has its volume halved by an increase of pressure at constant temperature. The initial pressure was 20 lbs. per sq. in. abs.; what is the final pressure?

Ans. 40 lb. per sq. in. abs.

3. The products of combustion from a boiler furnace reach the base of the stack at a temperature of 500° F. At the top of the stack their temperature is only 200° F. Neglecting the slight pressure change which would occur during the ascension and assuming that the products of combustion obey the gas laws, determine the relative values of the cross-sectional areas at top and bottom of the stack to give equal gas velocities at the two points.

Ans. 0.687.

4. A closed metal tank is designed to be safe when subjected to an internal pressure of 100 lb. per sq. in. abs. It is used to hold compressed air and is filled with this material at a temperature of 60° F. and a pressure of 80 lb. per sq. in. abs. The tank stands in the sun and its contents may attain a temperature of 125° F. (*a*) Assuming that the tank does not expand with temperature and pressure changes, will the designed pressure be exceeded? (*b*) What temperature would have to be attained to raise the pressure of the air to the value for which the tank was designed?

Ans. (*b*) 190° F.

5. A submarine boat is closed at the surface, with air content at a temperature of 80° F. and a pressure of 14.5 lb. per sq. in. abs. After sinking beneath the surface the temperature of the air drops to 40° F. (*a*) If the hull has not changed size during the temperature change what must be the pressure of the air under the submerged conditions? (*b*) Assuming that the men and machinery in the boat radiate enough heat to maintain a temperature of 60° within the boat what will the air pressure be?

Ans. (*a*) 13.42 lb. per sq. in. (*b*) 13.94 lb. per sq. in.

6. Using the two parts of Charles' Law, develop the characteristic equation of ideal gases.

7. A certain gas is collected over mercury and measured. It is found to have a volume of 10 cu. in. at a pressure of 14.6 lb. per sq. in. abs. and a temperature of 60° F. The gas is then passed through a reagent which absorbs part of it and the remainder is collected over mercury and measured. It measures 6 cu. in. at the same pressure as before but the temperature has changed to 70° F. between the two measurements. What percentage of the original volume was absorbed by the reagent? Ans. 41.4.

8. A diving bell is to be used for executing certain work under water. It is made in the form of a flat-ended cylinder open at the bottom. The inside diameter is 12 ft. and the inside height is 14 ft. The men and tools accommodated within the bell occupy a cubical content of 120 cu. ft. If the bell is lowered into the water when atmospheric pressure is 14.7 lb. per sq. in. and temperature is 60° F., how far below the surface can the bottom of the bell be lowered if the water has a temperature of 40° F., weighs 62.5 lb. per cu. ft., and is not to rise to a height of more than 4 ft. from the bottom of the bell? Assume that men and tools remain entirely within the air space, that they do not change volume with pressure change, and that the air within the bell acquires the same temperature as the surrounding water. Ans. 17.3 ft.

9. The inner tube of a certain tire has a capacity of 854 cu. in. (*a*) How many pounds of air will it contain when filled with air at an abs. pressure of 70 lb. per sq. in. and a temperature of 32° F.? (*b*) What will be the increase of pressure of air if the temperature rises to 70° F. and the tire does not stretch during the process?

Ans. (*a*) 0.19. (*b*) 5.4 lb. per sq. in.

10. Ten pounds of air are contained in a receiver at a temperature of 55° F. and a pressure of 100 lbs. per sq. in. abs. Air leaks out until at a later time the pressure in the receiver is found to be only 40 lb. per sq. in abs. with a temperature of 50° F. What weight has leaked out? Ans. 5.96 lb.

11. A water pump running at 50 strokes per minute delivers 1 cu. ft. of water per stroke. An air chamber is to be fitted to this pump of such size that the discharge pressure on the pump shall vary from 100 lb. per sq. in. abs. at the beginning of the stroke to 150 lb. per sq. in abs. at the end of the stroke if all the water delivered during one stroke must be accommodated in the air chamber. The temperature of the water and of the air in the chamber remains constant at 60° F. and no air is absorbed by the water. (*a*) What must be the volume of the air chamber? (*b*) Would there be any economic advantage in using a gas with $R = 96$? Ans. (*a*) 3 cu. ft.

12. A balloon is filled with hydrogen gas at a pressure of 14.7 lb. per sq. in abs. and a temperature of 60° F. The balloon is spherical in shape and has an internal diameter of 25 ft. At a later time it is found that the pressure of the gas within the balloon is only 0.95 of the original value but that the temperature is the same as before. (*a*) What fraction of the original weight of gas must have escaped if the dimensions of the balloon have not changed? (*b*) Assuming the specific heat to be constant, how much heat would have had to be removed to cause the pressure to fall to the same extent if no leakage occurred and the volume had remained constant? (*c*) What would be the final temperature under the conditions of part (*b*)?

Ans. (*a*) 0.05. (*b*) 2423 B.t.u. (*c*) 34° F.

13. A large mining company has provided 109,000 cu. ft. of compressed air storage. The pressure of the air in storage drops from 100 lb. per sq. in. abs. to 25 lb. per sq. in abs., the temperature remaining constant at 80° F. (*a*) What is the weight of air stored initially? (*b*) By what percentage has the weight of air in storage been reduced? Ans. (*a*) 54,500 lb. (*b*) 75 per cent.

14. A certain gas with molecular weight equal to 28 occupies a volume of 12.8 cu. ft. per lb. What volume will another gas with molecular weight of 32 theoretically occupy when at the same temperature and pressure?

Ans. 11.2 cu. ft. per lb.

15. A certain gas with molecular weight of 44 weighs 0.1224 lb. per cu. ft. at standard conditions. Another gas has a molecular weight of 26. What is its theoretical density under the same conditions of temperature and pressure?

Ans. 0.0723 lb. per cu. ft.

16. It is desired to ship a certain compressed gas whose molecular weight is known to be 83 but whose gas constants are unavailable. Determine the approximate density of the compressed gas if it is to be shipped under a pressure of 100 lb. per sq. in. abs. and a temperature of 60° F. Ans. 1.489 lb. per cu. ft.

17. Five pounds of air have a temperature of 40° F. and a pressure of 100 lb. per sq. in. abs. in state *1*. In *2*, for the same weight of air, the pressure is 200 lb. per sq. in. abs. and the temperature is 140° F. Considering this air to be an ideal gas: (*a*) Find $I_2 - I_1$. (*b*) Find $V_2 - V_1$. (*c*) Can $Q_{1,2}$ be determined without additional data? Why? Ans. (*a*) 85.75 B.t.u. (*b*) − 3.7 cu. ft.

18. Three pounds of oxygen, considered as an ideal gas, initially at a pressure of 100 lb. per sq. in abs. and a temperature of 500° F., undergoes a constant pressure reduction of volume until the volume is halved. (*a*) Find the heat added to the oxygen during this process. (*b*) Find the gain in the internal energy of the oxygen during this process. Ans. (*a*) −313 B.t.u. (*b*) −223.5 B.t.u.

19. For the gas defined in Problem 16, suppose it is known that $\gamma = 1.68$. (*a*) Determine the approximate value of the specific heat at constant pressure; and (*b*) that at constant volume. Ans. (*a*) 0.0591. (*b*) 0.0351.

20. Two pounds of a certain ideal gas, with $R = 77.1$ and $c_p = 0.254$, initially under a pressure of 50 lb. per sq. in. abs. and occupying a volume of 12 cu. ft., undergo a change of state during which the pressure and volume are both doubled. Find the gain in internal energy. Ans. 520 B.t.u.

21. Three pounds of air, considered as an ideal gas, are contained in a cylinder under a pressure of 100 lb. per sq. in. abs. and a temperature of 1000° F. The state of this air is changed at constant pressure until the volume is doubled and then at constant volume until the pressure is halved. (*a*) Would the same amount of heat be added to the air if its state had been changed from the same initial conditions to the same final conditions by a constant volume process followed by a constant pressure process? Why? (*b*) Would there be the same gain in the internal energy of the air?

22. (*a*) Prove that the gain in the heat content during a constant pressure process equals the heat added during the process. (*b*) Is the gain in the heat content during a constant volume process equal to the heat added? Why?

23. An air preheater raises the temperature of the air supplied to a furnace from 60° F. to 400° F., the pressure remaining constant. (*a*) Considering the specific heat variable, find the gain in the internal energy of each pound of air. (*b*) Find the heat added to each pound of air. Ans. (*a*) 58.7 B.t.u. (*b*) 82 B.t.u.

24. A boiler flue gas analysis shows the composition by weight to be as follows: $CO_2 = 19.5\%$, $CO = 0.1\%$, $O_2 = 6.2\%$ and $N_2 = 74.2\%$. Find the value of c_p for this gas at a temperature of 500° F. Ans. 0.2466.

25. Air and water vapor are mixed together in a steam condenser having a volume of 1200 cu. ft. and a temperature of 80° F. If the partial pressure of the air is 0.25 in. Hg abs. measured at 32° F., find the weight of air in the condenser. Ans. 0.737 lb.

26. Dry exhaust gas from a gasoline engine shows by analysis the following composition by weight: $CO_2 = 13.5\%$, $O_2 = 0.4\%$, $CO = 0.4\%$, $H_2 = 0.1\%$ and $N_2 = 85.6\%$. For this gas mixture at 2000° F. find (*a*) c_p, (*b*) c_v, (*c*) γ, and (*d*) R.
Ans. (*a*) 0.2807. (*b*) 0.2118. (*c*) 1.325. (*d*) 53.6.

27. At a temperature of −60° F. and a pressure of 1.73 lb. per sq. in. abs., the composition of air by volume at an elevation of 50,000 ft. is approximately, $Ar. = 0.77\%$, $N_2 = 79.55\%$ and $O_2 = 19.68\%$. Find the value of R for air under these conditions. Ans. 53.6.

CHAPTER VIII

Note.—The *PV*- and $T\phi$-diagrams should be drawn for each of these problems.

When not otherwise specified the specific heats of an ideal gas may be taken from table II (page 80) for the corresponding real gas at 32° F.

1. A vessel with a capacity of 5 cu. ft. is filled with air at a pressure of 125 lb. per sq. in. abs. when at a temperature of 600° F. It is desirable to lower the pressure to 60 lb. per sq. in. abs. (*a*) What amount of heat will have to be abstracted and what will be the final temperature of the gas, assuming that the vessel does not change in size with change of temperature, and that the air may be treated as an ideal gas? (*b*) Find the gain in entropy during this process.
Ans. (*a*) 150.5 B.t.u.; 49° F. (*b*) −0.2.

2. (*a*) Find how much heat would be required to heat 4 lb. of ideal air at constant volume so that it would experience an entropy gain of 0.468, its initial temperature being 60° F. (*b*) Find the gain in internal energy of the air.
Ans. (*a*) 350 B.t.u. (*b*) 350 B.t.u.

3. Five pounds of nitrogen are heated at constant volume from 50° F. to 4000° F. Taking account of the variation in the specific heats, find (*a*) the heat added to the nitrogen and (*b*) its gain in entropy. Ans. (*a*) 4121 B.t.u. (*b*) 2.125.

4. Twenty-five pounds of hydrogen are compressed from a volume of 50 cu. ft. to a volume of 10 cu. ft., the pressure remaining constant. Considering hydrogen to be an ideal gas, find the gain in entropy. Ans. −126.5.

5. Ten pounds of ideal air are heated at constant pressure until $t_2 = 2\,t_1$. If the heat added to the gas is 1185 B.t.u., find the gain of entropy. Ans. 1.00.

6. Three pounds of methane are cooled at constant pressure from 5000 to 2000° F. Taking account of the variation in the specific heats, find, (*a*) the gain in the internal energy of the methane; (*b*) its gain in entropy.
Ans. (*a*) −24,288 B.t.u. (*b*) −6.447.

7. A given weight of ideal gas occupies 3.09 cu. ft. under a pressure of 200 lb. per sq. in. abs. It expands isothermally, the ratio of expansion being 3. Find (*a*) final volume, (*b*) ft-lb. of work done, and (*c*) the heat added in B.t.u.

Ans. (*a*) 9.27. (*b*) 97,800. (*c*) 125.6.

8. Five pounds of ideal air expand isothermally from a pressure of 120 lb. per sq. in. abs. to a final pressure of 20 lb. per sq. in. abs., the work done being 248,500 ft.-lb. Find (*a*) initial volume, (*b*) final volume, (*c*) initial temperature, and (*d*) heat supplied, and (*e*) the gain in entropy.

Ans. (*a*) 8.05 cu. ft. (*b*) 48.3 cu. ft. (*c*) 62° F. (*d*) 320 B.t.u. (*e*) 0.613.

9. Find the gain in entropy of an ideal gas which is compressed isothermally from a pressure of 14.7 lb. per sq. in. abs., and a volume of 60 cu. ft. to a pressure of 75 lb. per sq. in. abs. The gas is maintained at a temperature of 80° F. Ans. −0.494.

10. Solve the equation $P_2 = 150\left(\frac{520}{860}\right)^{\frac{1.3}{0.3}}$ by means of a log-log slide rule (if available) and by means of common logarithms. Ans. $P_2 = 17.01$.

11. (*a*) How much work will be required to compress 2 lb. of an ideal gas from $V_1 = 25$ cu. ft. and $p_1 = 13.5$ lb. per sq. in. abs.; to $p_2 = 75$ lb. per sq. in. abs. according to the equation $PV^{1.35} = \text{const.}$? (*b*) What will be the final temperature if the initial temperature is 55° F.? Ans. (*a*) 77,900 ft-lb. (*b*) 344° F.

12. Two pounds of ideal air expand according to the law $PV^{1.2} = K$, from a temperature 600° F. to a temperature of 50° F. Find (*a*) the heat added to the air, (*b*) the gain in internal energy of the air, and (*c*) the gain in entropy of the air.

Ans. (*a*) 188.7 B.t.u. (*b*) −188.7 B.t.u. (*c*) 0.2505.

13. Five pounds of oxygen are compressed according to the law $PV^{1.25} = K$, from a pressure of 12 lb. per sq. in. abs. and a temperature of 50° F. until the temperature rises to 300° F. Considering oxygen as an ideal gas find (*a*) the final pressure, (*b*) the initial volume, (*c*) the final volume calculated from the TV-relationship, (*d*) the final volume calculated from the law $PV = wRT$, and (*e*) the gain in the internal energy.

Ans. (*a*) 88 lb. in. abs. (*b*) 71.4 cu. ft. (*c*) and (*d*) 14.52 cu. ft. (*e*) 194 B.t.u.

14. Ten cu. ft. of hydrogen under a pressure of 20 lb. per sq. in. abs. and at a temperature of 60° F. are compressed according to the law $PV^{1.15} = K$ until the volume becomes 2 cu. ft. Considering this hydrogen to be an ideal gas, find (*a*) the heat added to the hydrogen, and (*b*) the gain in the entropy of the hydrogen.

Ans. (*a*) − 46.1 B.t.u. (*b*) − 0.078.

15. One cu. ft of air under a pressure of 500 lb. per sq. in. abs. and at a temperature of 3500° F. expands according to the law $PV^{1.3} = K$ until the temperature drops to 500° F. Taking account of the variation in the specific heat, find (*a*) the heat added to the air, and (*b*) the gain in entropy of the air.

Ans. (*a*) 31 B.t.u. (*b*) 0.0173.

16. One-quarter of a pound of an ideal gas with $c_p = 0.238$ and $c_v = 0.169$ is expanded isentropically from $V_1 = 0.2$ cu. ft. and $p_1 = 300$ lb. per sq. in. abs. to $p_2 = 150$ lb. per sq. in. abs. (*a*) What are the values of R, T_1, V_2, T_2? (*b*) How much work is done by the gas?

Ans. (*a*) 53.6; 643° F. abs.; 0.327 cu. ft.; 527° F. abs. (*b*) 3870 ft.-lb.

17. Air is drawn into an air compressor at a temperature of 61° F. and at an atmospheric pressure of 14.7 lb. per sq. in. abs. The flash point of the oil used to lubricate the compressor piston is 350° F. (*a*) If compression is adiabatic, what pressure could be attained in the compressor if the maximum allowable temperature is 50 degrees below the flash point of the oil? (*b*) If the compression follows the law $PV^{1.30} = K$, what is the maximum allowable pressure?

Ans. (*a*) 55 lbs. per sq. in. abs. (*b*) 75.4 lbs. per sq. in. abs.

18. Imagine 0.4 lb. of an ideal gas having $c_p = 0.124$ and $c_v = 0.093$ to expand in a cylinder which prevents any heat flow to or from the gas, the initial pressure being 100 lb. per sq. in. abs. and the initial volume $\frac{1}{2}$ cu. ft. (*a*) Find the work done when its volume has become 3 cu. ft. (*b*) Find the drop in temperature. Find the gain in entropy. ($c_p = 0.124$. $c_v = 0.093$.)

Ans. (*a*) 9730 ft.-lb. (*b*) 336° F. (*c*) 0.

19. What will be the difference in the amounts of work required to compress 5 cu. ft. of ideal air at 60° F. and 14.7 lb. per sq. in. abs. to a pressure of 90 lb. per sq. in. abs. (*a*) when the compression is adiabatic and when it is isothermal? (*b*) What is the gain in internal energy in each case.

Ans. (*a*) 1230 ft.-lb. (*b*) 22.95 B.t.u., 0 B.t.u.

20. Two pounds of an ideal gas with $R = 50$ and $\gamma = 1.4$ are compressed from an initial pressure of 22 lb. per sq. in. abs. and an initial temperature of 60° F. to a pressure of 88 lb. per sq. in. abs., according to the law $PV^{1.25} = K$. (*a*) Find the final temperature. (*b*) Find the gain in entropy of the gas.

Ans. (*a*) 226° F. (*b*) −0.0535.

21. One pound of air is compressed isentropically from a temperature of 200° F., to a temperature of 2000° F. Taking account of the variation in the specific heat. (*a*) find the work done by the air and (*b*) find the ratio of the initial volume to the final volume. Ans. (*a*) −326.6 B.t.u. (*b*) 31.6.

22. Four pounds of oxygen are compressed from state *1* to state *2* isothermally, and from state *2* to state *3* at constant pressure. Consider oxygen to be an ideal gas. If $t_1 = 140°$ F., $p_1 = 15$ lb. per sq. in. abs., and $p_2 = p_3 = 90$ lb. per sq. in abs. and $V_3 = 0.8 V_2$. (*a*) Find $\Delta I_{1,\,2}$, $\Delta I_{2,\,3}$, and $\Delta I_{1,\,3}$; and (*b*) $Q_{1,\,2}$ and $Q_{2,\,3}$.

Ans. (*a*) 0, −74.6 B.t.u., −74.6 B.t.u. (*b*) −267.5 B.t.u., −104.3 B.t.u.

23. One quarter of a pound of ideal air is compressed isentropically from 13 lb. per sq. in. abs. and 60° F. to a pressure of 100 lb. per sq. in. abs. After compression its temperature is decreased to 60° F. while the volume is maintained constant. (*a*) How much heat will have to be abstracted to bring this about? (*b*) What will be the final pressure? (*c*) If the gas is now allowed to expand isentropically to a pressure of 13 lb. per sq. in. abs., how much work can it do? (*d*) What will be the temperature and volume at end of expansion in (*c*) above? Ans. (*a*) 17.6 B.t.u.

(*b*) 55.8 lb. per sq. in. abs. (*c*) 7.58 B.t.u. (*d*) −117° F., 2.44 cu. ft.

24. Two cu. ft. of hydrogen under a pressure of 100 lb. per sq. in abs. and at a temperature of 300° F., are expanded isentropically to 20 lb. per sq. in. abs. This gas then undergoes a constant pressure process, after which, the hydrogen is returned to its initial condition by a polytropic compression with $n = 1.15$. Considering hydrogen as an ideal gas, (*a*) find the heat added to the gas during the constant pressure process, and (*b*) draw, approximately to scale, the PV and $T\phi$-diagrams of these processes. Ans. (*a*) 24.55 B.t.u.

25. It is desired to have air delivered from a nozzle at a velocity of 1800 ft. per sec., a pressure of 15 lb. per sq. in. abs., and a temperature of 40° F. It is known that the nozzle coefficient is 98 per cent and the velocity of the air entering the nozzle is so low that it may be ignored. Considering the air to be an ideal gas, find (*a*) the initial temperature of the air (*b*) the initial pressure.

Ans. (*a*) 310° F. (*b*) 73.3 lb. per sq. in. abs.

26. If the air in Prob. 25 is taken from an air line in which the pressure is 100 lb. per sq. in. abs. and throttled to the required pressure entering the nozzle, (*a*) what temperature must be maintained in this air line assuming the throttling to be adiabatic? (*b*) Find the gain in entropy per pound of air during this throttling process.

Ans. (*a*) 310° F. (*b*) 0.0213.

27. Consider 4 lb. of an ideal gas, for which $R = 50$ and $\gamma = 1.4$, changing from state *1* to state *2*. $p_1 = 40$ lb. per sq. in. abs., $t_1 = 500°$ F., $p_2 = 120$ lb. per sq. in. abs., and $t_2 = 1000°$ F. (*a*) Find the gain in entropy during this change of state (*b*) Find the gain in internal energy during this change of state. (*c*) Find the gain in heat content during this change of state.

Ans. (*a*) 0.0945. (*b*) 321.6 B.t.u. (*c*) 450 B.t.u.

28. Prove that Eq. (127) reduces to Eq. (99) if the specific heat is constant and a known polytropic process is involved.

29. Using logarithmic cross-section paper determine the pressure exerted by an ideal gas for each cubic foot of volume increase when expanding according to the law $PV^{1.35}$ = constant, from an initial pressure of 100 lb. per sq. in. abs. and an initial volume of $1\frac{1}{2}$ cu. ft. to a terminal pressure of 15 lb. per sq. in. abs.

Ans.

V (cu. ft.)	*p* (lb. per sq. in. abs.)
2.5	50.2
3.5	31.8
4.5	22.7
5.5	17.3

CHAPTER IX

1. Find the indicated horsepower of a single-cylinder, double-acting engine having a 10-in. bore and a 12-in. stroke (known as a " 10 × 12 engine ") running at 200 r.p.m. and operating on a two-stroke cycle. The mean effective pressure for the head and crank ends of the engine are respectively 28 and 32 lb. per sq. in. and the diameter of the piston rod may be ignored. Ans. 28.55.

2. Suppose the engine of Prob. 1 has given indicator diagrams having an area of 2.6 sq. in. with a length of 3 in. for the head end, and an area of 2.8 sq. in. with a length of 3 in. for the crank end. If the spring scale of the indicator is 50 lb. per sq. in. find the indicated horsepower. Ans. 42.8.

3. An 8 × 12 in. air compressor, which operates on a two-stroke cycle, while running 200 r.p.m. gives an indicator card having an area of 1.8 sq. in. for the head end, and 2 sq. in. for the crank end. The length of each card is 2 in. Scale of the spring is 60 lb. per sq. in. per in. The piston rod is $1\frac{1}{2}$ in. in diameter. Find the i.hp.

Ans. 34.1.

4. A 12 × 18-in. engine has an indicator motion arranged to give diagrams $3\frac{1}{2}$ in. long. Ignoring the piston rod, if an indicator is used with a spring scale of 80, find the value of the work represented by each square inch of card area, and express it, (*a*) in ft.-lb., and (*b*) in B.t.u. Ans. (*a*) 3875 (*b*) 4.98.

CHAPTER X

Note.—In the following air compressor and air-engine problems, it is assumed that there is no throttling or leakage, that valve operations are instantaneous, and that there is no change in the temperature of the air during suction, delivery, or exhaust.

1. The delivery pressure in a reciprocating compressor having isentropic compression and handling 0.1 lb. of air per cycle, is 50 lb. per sq. in. abs. The temperature of the air drawn in is 60° F., and the average atmospheric pressure where the compressor is located may be taken as 14.7 lb. per sq. in. abs. Determine: (*a*) The net work of the compressor cycle, if the compressor has no clearance. (*b*) The net work of the compressor cycle if the compressor has 5 per cent clearance. (*c*) If the compressor has 5 per cent clearance, find the weight of air in the cylinder during the compression. (*d*) What per cent increase in the required piston displacement results from the 5 per cent clearance when compared to the compressor of the same capacity with no clearance?

Ans. (*a*) 4055 ft.-lb. (*b*) 4055 ft.-lb. (*c*) 0.11285 lb. (*d*) 7.5.

2. Determine the capacity of either compressor given in Prob. 1 in terms of the volume of free air handled per minute if the compressor operates at a speed of 180 r.p.m. and is double acting. Ans. 471 cu. ft./min.

3. Assume three compressor cylinders without clearance and with a piston displacement of 1 cu. ft., one cylinder so arranged as to give adiabatic compression; one arranged to give isothermal compression; and one arranged to give a compression curve with exponent equal to 1.25. (*a*) Determine the work done during the compression process alone in each cylinder and the final temperature in each case if air with an initial temperature of 60° F. and at an initial pressure of 14.7 lb. per sq. in. abs. is compressed to 100 lb. per sq. in. abs. (*b*) Express the second and third cases as a percentage of the compression work in the case of the adiabatic process. (*c*) Determine the net work per cycle in each case, assuming discharge to occur at the constant pressure of 100 lb. per sq. in. abs. (*d*) Express results as percentages as in (*b*).

Ans. (*a*) 3860 ft.-lb.; 440° F. 4055 ft.-lb.; 60° F. 3950 ft.-lb.; 303° F. (*b*) 105.3; 102.5. (*c*) 5400 ft.-lb.; 4055 ft.-lb.; 4940 ft.-lb. (*d*) 75.3; 91.5.

4. What is the volumetric efficiency of a compressor having 5 per cent clearance and operating as in Prob. 1? Ans. 93.03%.

5. An air compressor is delivering 80 lb. of air per min. at a pressure of 150 lb. per sq. in. abs. Atmospheric pressure is 14.7 lb. per sq. in. abs., and the temperature of the entering air is 60° F. Ignoring frictional losses; (*a*) Find the horsepower required to operate the compressor for the following cases: 1. If it is a single stage machine with $n = 1.35$. 2. If it is a two-stage machine with the air going to the second stage cylinder at a pressure of 48 lb. per sq. in. abs. and temperature of 65° F. (*b*) Find the temperature of the air going to the receiver in each case. (*c*) If the allowable temperature rise of the circulating water in the intercooler is 15° F., find the weight of water required per minute.

Ans. (*a*) 214; 182. (*b*) 490° F.; 245° F. (*c*) 231 lb.

6. If, in Prob. 5, the single stage compressor cylinder has a clearance of 5 per cent in both ends, which is the same clearance as in the ends of the low pressure cylinder of the two-stage machine, and if both compressors are double acting, and run at a speed of 180 r.p.m., find (*a*) the weight of the clearance air in the single-stage machine and in the low-pressure stage of the two-stage machine, and (*b*) the piston displacement required in both cases of (*a*).

Ans. (*a*) 0.0287 lb.; 0.0803 lb. (*b*) 3.12 cu. ft.; 3.77 cu. ft.

7. A centrifugal air compressor draws in 12,000 cu. ft. of air per minute at a pressure of 14 lb. per sq. in. abs. and a temperature of 60° F. This air is delivered from the compressor at a pressure of 70 lb. per sq. in. abs. and a temperature of 164° F. The area of the suction pipe is 2.1 sq. ft., the area of the discharge pipe is 0.4 sq. ft., and the discharge pipe is located 20 ft. above the suction pipe. The weight of jacket water, which enters at 60° F. and leaves at 110° F., is 677 lb. per min. Find the horsepower required to drive this compressor assuming no loss due to radiation.

Ans. 1315.

8. Power is obtained by expanding air adiabatically in an engine cylinder. It is found that when the temperature of the air drops below 32° F. the moisture which is carried by the air freezes and impairs the action of the engine. Air is received by the engine at a temperature of 60° F. and a pressure of 100 lb. per sq. in. abs. (*a*) What is the lowest pressure to which the air can expand if its temperature is not to drop below 32° F.? (*b*) To what initial temperature would it be necessary to heat the air in order that it may be possible to expand to 15 lb. per sq. in. abs. without dropping below the minimum allowable temperature?

Ans. (*a*) 82.6 lb. per sq. in. abs. (*b*) 387° F.

9. Air is available for use in a compressed air engine at a temperature of 65° F. and at a pressure of 125 lb. per sq. in. abs. It is desired to preheat it at constant pressure to such a temperature that it will not drop below a temperature of 35° F. when expanded to a final pressure of 16 lbs. per sq. in. abs. according to the equation $PV^{1.35} =$ const. How much heat will be required per pound? Ans. 76.8 B.t.u.

10. Suppose compressed air at 110 lb. per sq. in. abs. and 60° F. is used in air turbine tools designed to make use of the complete isentropic expansion of the air. If atmospheric pressure is 14 lb. per sq. in. abs., and the air is preheated before use to 350° F., what per cent increase in the energy that may be delivered to the turbine per pound of air can be credited to the preheating? Ans. 55.5.

11. (*a*) Air is supplied to an air engine at a pressure of 195 lb. per sq. in. abs. and a temperature of 275° F. Atmospheric pressure where the engine is located is 14 lb. per sq. in. abs. Find the piston displacement required by a double-acting engine which develops 20 i.hp. when running 100 r.p.m. and allowing the air to expand to four times the volume at cut off within the cylinder. Consider the expansion isentropic and ignore clearance and compression. (*b*) What will be the temperature of the air at the end of the isentropic expansion? (*c*) How much coal must be provided per hour for the preheater, if the temperature rise of the air in the preheater is 215° F. and if 6000 B.t.u. of heat energy is added to the air per pound of coal burned?

Ans. (*a*) 0.2635 cu. ft. (*b*) −38° F. (*c*) 4.85 lb.

12. It is desired to compress 50 lb. of air per minute to a pressure of 85 lb. per sq. in. abs. Consider that, for a compressor of this size, the isothermal compression efficiency is 77 per cent. If, at the place where the compressor is located, the average atmospheric temperature is 50° F. and the average atmospheric pressure is 13.8 lb. per sq. in. abs., what will be the indicated horsepower of the air compressor?

Ans. 97.2.

CHAPTER XI

1. An ideal gas is used as the working substance in a heat engine, the gas being carried through the following processes making up a closed cycle:

process *a–b*, a constant volume addition of heat;
process *b–c*, an isentropic expansion;
process *c–d*, a constant pressure compression;
process *d–a*, an isentropic compression.

(*a*) Show on an appropriate diagram the ratio of areas representing the efficiency of this cycle. (*b*) Develop an expression for the efficiency of this cycle in terms of temperatures and other factors.

2. An engine using ½ lb. of an ideal gas ($\gamma = 1.4$, $R = 53.3$) as a working substance receives heat from a hot body at a temperature of 1000° F. and rejects heat to a cold body at a temperature of 100° F. This engine operates on a cycle composed of an isothermal expansion, an isentropic expansion, an isothermal compression, and a constant volume process. If the maximum and minimum temperatures of the working substance during the cycle are, respectively, 1000° F. and 100° F., and if the volume doubles during the isothermal expansion, determine: (*a*) the heat added to the gas during each process, (*b*) the work done by the gas during each process, and (*c*) the efficiency of the cycle.

Ans. (*a*) 34.7; 0; −59.2; 77.1 B.t.u. (*b*) 34.7; 77.1; −59.2; 0 B.t.u. (*c*) 47.1 per cent.

3. One-half of a pound of ideal gas ($\gamma = 1.4$, $R = 53.3$) is enclosed in a cylinder fitted with a movable piston. The gas occupies a volume of 3 cu. ft. and exerts a pressure of 100 lb. per sq. in. abs. The gas is expanded at constant pressure to a volume of 6 cu. ft.; the pressure is then dropped to 15 lb. per sq. in. abs. at constant volume. The gas is next compressed at constant pressure to a volume of 3 cu. ft.; and lastly the pressure is raised to 100 lb. per sq. in. abs. at constant volume. (*a*) Find the net work of this cycle by the easiest method. (*b*) Why is the chosen method easiest? (*c*) Check the net work obtained in (*a*) by a different method.

Ans. (*a*) 36,750 ft-lb.

4. An engine operating on the Carnot cycle uses an ideal gas as a working substance. The temperature of the hot body is 1000° F. and that of the cold body is 60° F. If the heat supplied per cycle is 10 B.t.u., what is the net work delivered per cycle?

Ans. 6.44 B.t.u.

5. In the case of the Carnot cycle in Prob. 4 with the higher temperature 1000° F. and the lower one 60° F., how much net work would be done per cycle if the heat rejected per cycle is 10 B.t.u.?

Ans. 18.08 B.t.u.

6. Imagine an engine operating on the Carnot cycle. It is desired to obtain 100 ft-lb. of work per cycle. The heat supplied per cycle equals 0.3 B.t.u. and the temperature of the hot body is 500° F. What must be the temperature of the cold body?

Ans. 89° F.

7. (*a*) Determine the efficiencies of Carnot engines when the hot body and the cold body have the following temperatures respectively:

Hot body temperature	Cold body temperature
(1) 3000° F.	500° F.
(2) 1500° F.	0° F.
(3) 1500° F.	500° F.

(*b*) Which is the more effective method of increasing the efficiency, raising T_1, or lowering T_2? Why? Ans. (*a*) 72.2 per cent, 76.5 per cent, 51 per cent.

8. If the maximum and minimum temperatures and pressures are both given, develop an expression for the ratio of isothermal expansion in a Carnot engine using an ideal gas with a known value of γ as the working substance.

Ans. $\frac{P_{\text{max.}}}{P_{\text{min.}}}\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$.

9. (*a*) Draw the Carnot cycle to PV and $T\phi$ coordinates for the following conditions. Two pounds of an ideal gas ($\gamma = 1.4$, $R = 55.3$) are used as the working substance. The maximum temperature is 1500° F. and the maximum pressure is 200 lb. per sq. in. abs. The ratio of isothermal expansion is 2. The minimum temperature is 50° F. (*b*) Find the heat supplied, the heat rejected, and the net work per cycle. (*c*) Find the efficiency of the cycle.

Ans. (*b*) 193.3 B.t.u., 50.2 B.t.u., 143.1 B.t.u. (*c*) 73.9 per cent.

10. A heat pump, operating on the Carnot reversed cycle, is used to remove 10 B.t.u. per cycle from a body at a temperature of 32° F., and to discharge heat to a body at a temperature of 100° F. (*a*) How much energy will be required per cycle to operate this heat pump? (*b*) How much will be required if the upper temperature is 200° F.? (*c*) How much heat will be discharged to the hot body in each case? (*d*) What is the ratio of the first result in part (*c*) to the result in part (*a*) and what is the significance of this ratio?

Ans. (*a*) 1.383 B.t.u., (*b*) 3.43 B.t.u., (*c*) 11.383 B.t.u., 13.43 B.t.u. (*d*) 8.22.

11. A Rider hot-air engine, operating on the Stirling cycle, uses 0.066 lb. of air (consider $\gamma = 1.4$, $R = 53.3$) as the working substance. Heat is supplied when the working substance is at a temperature of 1500° F. and rejected when at 200° F., the initial volume being 0.8 cu. ft. and the final volume 1 cu. ft. Find: (*a*) the pressures at the beginning and end of expansion, and at the beginning and end of compression; (*b*) the net work of the cycle; (*c*) the mean effective pressure; and (*d*) the efficiency of the cycle.

Ans. (*a*) 59.9, 47.9, 16.15 and 20.2 lb. per sq. in. abs. (*b*) 1020 ft.-lb. (*c*) 35.45 lb. per sq. in. (*d*) 66.3 per cent.

12. If an internal combustion engine working on the Otto cycle has 25 per cent clearance, find the efficiency of the cycle (*a*) on the cold-air standard; and (*b*) on the hot-air standard considering $\gamma = 1.3$. Ans. (*a*) 47.5 per cent. (*b*) 38.4 per cent.

13. A real engine working on the Otto cycle has a pressure at the beginning of compression of 13 lb. per sq. in. abs. (*a*) Find the clearance to give a compression of 91 lb. per sq. in. abs., assuming the exponent for compression to be 1.35 and the working substance to be an ideal gas. (*b*) If the initial temperature is 60° F., find the temperature at the end of compression.

Ans. (*a*) 31 per cent. (*b*) 400° F.

14. (*a*) Calculate the cyclic efficiencies of Otto engines for the cold-air standard using the following compression ratios: 3, 4, 5, 6, and 7. (*b*) Same as (*a*) except on the hot-air basis with $\gamma = 1.3$. (*c*) Plot the results of parts (*a*) and (*b*) on cross-section paper, thus showing the variation in the cyclic efficiency with the compression ratio. (*d*) What limits the compression pressure in real Otto engines?

Ans. (*a*) 35.6, 42.6, 47.5, 51.2, 54.1 per cent. (*b*) 28.1, 34.0, 38.3, 41.6, 44.2 per cent.

15. Suppose the ratio of compression in an Otto engine is 5. Consider warm air as the working substance with $\gamma = 1.3$ for both the compression and expansion processes. The pressure at the beginning of compression is 14.7 lb. per sq. in. abs., and the maximum pressure is 240 lb. per sq. in abs. (*a*) Find the ratio of the mean effective pressure to the maximum pressure. (*b*) Of what significance is this ratio?

Ans. (*a*) 0.1613.

16. If a Diesel engine has a ratio of compression of 14 and the corresponding ideal cycle has a cut-off ratio of 2, find the efficiency of this ideal cycle (*a*) for the cold-air standard and (*b*) for the hot-air standard considering $\gamma = 1.37$.

Ans. (*a*) 59.2 per cent. (*b*) 56.5 per cent.

17. (*a*) Calculate on the cold-air basis the ideal Diesel cycle efficiencies for a 15-to-1 compression ratio and for the following cut-off ratios: 1.5, 2, 2.5, and 3. (*b*) Same as (*a*) except for a 13-to-1 compression ratio. (*c*) Plot the results of (*a*) and (*b*) on cross-section paper and discuss the significance of the resulting curves.

Ans.

Cut-off ratio	Efficiencies with $r = 15$	Efficiencies with $r = 13$
1.5	0.631	0.609
2	0.604	0.58
2.5	0.58	0.555
3	0.558	0.532

18. Suppose the ratio of compression in a Diesel engine is $13\frac{1}{2}$. Consider warm air as the working substance with $\gamma = 1.37$ for both the compression and expansion processes. The pressure at the beginning of compression is 14.7 lb. per sq. in. abs., and the cut-off ratio for the corresponding ideal cycle is 1.75. (*a*) Find the mean effective pressure for such a case. (*b*) The mean effective pressure is what per cent of the maximum pressure. Ans. (*a*) 65.5 lb. per sq. in. (*b*) 12.6 per cent.

19. Suppose a Diesel engine with a compression ratio of 15 is operating with a fuel having a heating value of 19,300 B.t.u. per lb. At half load this engine requires a 70-to-1 ratio of air to fuel. Assume that the weight of the burnt gases remaining in the clearance is negligible. (*a*) What value of the cut-off ratio should be used in the corresponding ideal cycle for the hot-air standard if γ be taken as 1.38 during all parts of the cycle and the temperature at the beginning of compression is 200° F.? (*b*) What is the ideal cyclic efficiency? Ans. (*a*) 1.592. (*b*) 60.6 per cent.

20. Find the efficiency of the Dual Combustion cycle with the following ratios, using the hot-air standard with $\gamma = 1.33$. The ratio of compression is 10, the constant volume pressure ratio = the cut-off ratio = 1.75. Ans. 49.7 per cent.

21. The ratio of compression in an ideal Brayton cycle is 5. Consider warm air as the working substance with $\gamma = 1.35$ for both the compression and expansion processes. The pressure at the beginning of compression is 14.7 lb. per sq. in. abs., and the cut-off ratio is 2. (*a*) Find the mean effective pressure. (*b*) What is the cyclic efficiency? (*c*) Discuss the significance of the result of part (*a*) compared to the mean effective pressures found in problems 15 and 18.

Ans. (*a*) 23.8 lb. per sq. in. (*b*) 43 per cent.

CHAPTER XII

1. A heat engine shows on test a brake thermal efficiency of 24 per cent and a mechanical efficiency of 75 per cent. The corresponding ideal cycle efficiency is 39 per cent. Find (*a*) the indicated thermal efficiency, (*b*) the brake engine efficiency, and (*c*) the indicated engine efficiency.

Ans. (*a*) 32 per cent. (*b*) 61.5 per cent. (*c*) 82 per cent.

2. At a given load a heat engine shows the following performance: brake thermal efficiency = 16 per cent, mechanical efficiency = 89 per cent, and indicated engine efficiency = 75 per cent. What is the efficiency of the ideal cycle on which the engine operates? Ans. 24 per cent.

3. A certain heat engine requires 16.2 lb. of working substance per brake hp.-hr. provided each pound supplies the engine with 1107 B.t.u. of heat energy. The indicated thermal efficiency is 15.7 per cent and the conditions of operation are such that the corresponding ideal cycle efficiency is 26.2 per cent. Find (*a*) the brake thermal efficiency, (*b*) the brake engine efficiency and (*c*) the mechanical efficiency. Ans. (*a*) 14.2 per cent. (*b*) 54.2 per cent. (*c*) 90.4 per cent.

4. A heat engine is directly coupled to a generator which delivers electrical energy at a constant rate of 27,350 kw. when steadily supplied with 275,000 lb. of working substance per hour. Each pound of substance supplies the engine with 1257 B.t.u. of heat energy and the conditions of operation are such that the ideal cycle efficiency is 35.8 per cent. Find (*a*) the combined thermal efficiency, and (*b*) the combined engine efficiency. Ans. (*a*) 27 per cent. (*b*) 75.4 per cent.

5. An air compressor is direct-connected to a heat engine. The compressor handles 1000 cu. ft. of free air per minute, atmospheric pressure being 14 lb. per sq. in. and atmospheric temperature being 60° F. The air is discharged at a pressure of 175 lb. per sq. in abs. and the overall isothermal efficiency of the compressor is 70 per cent. The brake thermal efficiency of the heat engine is 25 per cent and all the heat supplied to the engine comes from an oil, the combustion of each gallon of which liberates 147,000 B.t.u. (*a*) Find the gallons of oil per day required by the engine if run continuously at the given load. (*b*) If the compression efficiency is 77 per cent, find the combined thermal efficiency based on the indicated hp. of the compressor cylinders. Ans. (*a*) 366. (*b*) 22.7 per cent.

6. A fuel having a higher heating value of 19,300 B.t.u. per lb. is burned in a central station to supply energy for the heat engines driving electric generators. During one week, an average of 0.8 lb. of oil is consumed per kw.-hr. output of the station. Find (*a*) the plant thermal efficiency, and (*b*) the B.t.u. supplied per net kw.-hr. for the station. Ans. (*a*) 22.1 per cent. (*b*) 15,420.

CHAPTER XIV

1. An automobile engine, having a ratio of compression of 4.6, operates under certain conditions with a brake thermal efficiency of 23 per cent. Find the brake engine efficiency based on (*a*) the cold air standard, (*b*) the hot air standard with $\gamma = 1.29$. (*c*) Discuss the significance of the results of (*a*) and (*b*).

Ans. (*a*) 50.3 per cent. (*b*) 64.4 per cent.

2. An airplane engine with a clearance of 22.7 per cent shows an economy of 0.48 lb. of gasoline per b.h.p. hr. under specified conditions. If the gasoline has

a higher heating value of 20,320 B.t.u. per lb. and a lower heating value of 18,940 B.t.u. per lb., find (*a*) the brake thermal efficiency, and (*b*) the brake engine efficiency on the hot air basis with $\gamma = 1.30$. (*c*) If the jacket loss is 13 per cent and the exhaust loss is 40 per cent, find the radiation and the unaccounted for losses in per cent. (*d*) If the load on the engine is 400 b.h.p., give the heat balance of the engine.

Ans. (*a*) 26.1 per cent. (*b*) 65.7 per cent. (*c*) 20.9 per cent. (*d*) B.h.p. = 16,970 B.t.u. per min., Jacket loss = 8,440 B.t.u. per min., Exhaust loss = 26,000 B.t.u. per min., Radiation and unaccounted for loss = 13,590 B.t.u. per min.

3. A blast furnace gas engine (Otto type) shows by test a gas consumption of 109.3 cu. ft. per b.h.p. hr., the gas having a higher heating value of 90 B.t.u. per cu. ft. The mechanical efficiency is 86.3 per cent and the ratio of compression is 7. Find (*a*) the brake thermal efficiency, (*b*) the indicated thermal efficiency and (*c*) the brake engine efficiency on the hot air standard with $\gamma = 1.32$.

Ans. (*a*) 25.85 per cent. (*b*) 29.95 per cent. (*c*) 55.7 per cent.

4. A certain engine using blast furnace gas as a fuel, operates under such conditions that its corresponding ideal cycle may be taken as the Otto using hot air with an average value of $\gamma = 1.33$. At the start of compression this air has a temperature of 200° F. and a pressure of 14.7 lb. per sq. in. abs. The maximum pressure in this ideal cycle is 500 lb. per sq. in. abs. If the engine has a ratio of compression of 7 and the cards taken under the specified load conditions have a mean effective pressure of 50 lb. per sq. in., find the indicated engine efficiency.

Ans. 68.6 per cent.

5. A four cylinder, $3'' \times 5''$ automobile engine (four stroke cycle) with a compression ratio of 5 : 1 shows the following performance when tested with wide open throttle at various speeds:

R.p.m.	B.h.p.	Lb. fuel per b. hp-hr.
1000	18	0.56
1500	28	0.52
2000	38	0.53
2500	48	0.55

The higher heating value of the fuel is 20,000 B.t.u. per lb. Using a tabulated scheme of calculation, find, for these observations (*a*) the brake thermal efficiency, (*b*) the brake engine efficiency using a hot air standard with $\gamma = 1.3$, (*c*) the brake torque and (*d*) the brake m.e.p. (*e*) Plot the results of parts (*a*), (*b*), (*c*), and (*d*) against the speed of the engine.

Ans. (*a*) 22.7, 24.45, 24, 23.15 per cent. (*b*) 59.3, 63.9, 62.7, 60.5 per cent. (*c*) 94.5, 98.1, 99.7, 100.9 lb. ft. (*d*) 100.8, 104.6, 106.2, 107.4 lb. per sq. in.

6. A Diesel engine operating under a constant load uses 4 bbls. of crude oil per day of 24 hours. (1 bbl. = 336 lb.) The higher heating value of the oil is 19,000 B.t.u. per lb. The mechanical efficiency is 73 per cent and the brake thermal efficiency is 29 per cent. Find (*a*) the indicated h.p., (*b*) the brake h.p., and (*c*) the amount of oil required per b.h.p. hr. (*d*) If the ratio of compression is 14, the mixture ratio is 30 : 1, the temperature at the beginning of compression is 200° F., and γ for the hot air standard may be assumed as 1.37, find the brake engine efficiency. Ans. (*a*) 166. (*b*) 121.1. (*c*) 0.462. (*d*) 53.1 per cent.

7. The two-cycle Diesel engines driving generators in a power plant operate with an average indicated thermal efficiency of 42.6 per cent. The oil burned has a higher heating value of 19,100 B.t.u. per lb. Of the heat supplied the engines, 3.4 per cent is used to drive the injection air compressors, 3.0 per cent to drive the scavenging pumps, and 6.2 per cent is lost due to friction and turbulence in the engine. The generator efficiency is 92 per cent and 3.5 per cent of the generator output is required for the auxiliaries of the plant, such as the water circulating pumps, the fuel oil handling pumps, lighting, etc. Find (*a*) the brake thermal efficiency, (*b*) the combined thermal efficiency, (*c*) the plant thermal efficiency, and (*d*) the lb. of oil that must be supplied the engines per kw. hr. delivered by the plant. Ans. (*a*) 30 per cent. (*b*) 27.6 per cent. (*c*) 26.6 per cent. (*d*) 0.672.

8. Suppose a 1500 kw. steam power plant can be built for $320 per kw. installed, and suppose a Diesel plant of the same capacity costs $355 per kw. The average yearly plant thermal efficiency of the steam plant is 15 per cent, using a coal with a heating value of 13,500 B.t.u. per lb. and costing $5.00 a ton of 2000 lb. at the plant. The corresponding plant thermal efficiency of the Diesel plant is 25 per cent when using an oil with a higher heating value of 19,000 B.t.u. per lb. and costing $2.00 per barrel of 336 lb. Twenty per cent of the first cost of each plant in addition to the fuel cost, must be expended annually. (*a*) What is the cost of the coal per million B.t.u. supplied? (*b*) What is the cost of the oil per million B.t.u. supplied? (*c*) What is the cost of fuel per year in the steam plant if 6,570,000 kw. hrs. of energy are sold annually? (*d*) What is the corresponding cost of fuel in the Diesel plant if the same amount of energy is sold? (*e*) What is the average yearly cost of energy from the steam plant? (*f*) What is the average yearly cost of energy from the Diesel plant?

Ans. (*a*) 18.5c. (*b*) 31.35c. (*c*) $27,650. (*d*) $28,040. (*e*) 1.882 c. per kw-hr. (*f*) 2.05 c. per kw-hr.

9. A 300 h.p. oil engine operating on the Dual-Combustion cycle uses 0.45 lb. of oil per b.h.p. hr. at rated load. The mechanical efficiency of the engine is 80 per cent and the higher heating value of the fuel is 19,000 B.t.u. per lb. Find (*a*) the brake thermal efficiency, (*b*) the indicated thermal efficiency, (*c*) the lb. of oil per i.h.p. hr., and (*d*) the lb. of oil required per hour at this load. Ans. (*a*) 29.8 per cent. (*b*) 37.25 per cent. (*c*) 0.36. (*d*) 135.

10. A four cylinder $21\frac{1}{2}'' \times 22''$ Diesel engine (4 stroke cycle) with a compression ratio of 10 shows the following performance when tested at a constant speed of 300 r.p.m.:

B.hp.	Lbs. of Oil per B. hp-Hr.	Mechanical Efficiency Per Cent	Lbs. of Air per Lb. of Oil
1000	0.402	76.5	33.3
750	0.406	72.6	38.8
500	0.435	63.8	55.9
250	0.59	42.8	93.4

The higher heating value of the fuel is 19,120 B.t.u. per lb. Using a tabular scheme of calculation, find for these observations (*a*) lb. of oil per i.h.p. hr, (*b*) brake thermal efficiency, (*c*) cycle efficiency considering $\gamma = 1.37$, and the temperature at the beginning of compression to be 200° F., (*d*) brake engine efficiency and (*e*) the brake mean effective pressure. (*f*) Plot the results of (*a*), (*b*), (*c*), (*d*) and (*e*) against the b.h.p. Ans. (*a*) 0.308, 0.295, 0.278, 0.252. (*b*) 33.1, 32.8, 30.6, 22.6 per cent. (*c*) 48.4, 49.5, 51.5, 53.6 per cent. (*d*) 68.4, 66.2, 59.4, 42.2 per cent. (*e*) 82.5, 61.8, 41.3, 20.6 lb. per sq. in.

CHAPTER XV

Note.—Table XI is intended to be used with these problems whenever necessary.

1. A pound of liquid with $\bar{c}_p = 0.5$ solidifies at a temperature of 2° F. and vaporizes at a temperature of 150° F. when its pressure is kept at 20 lb. per sq. in. abs. For this pressure, find (*a*) the heat of the liquid measured above the fusion temperature, (*b*) the corresponding increase in entropy, and (*c*) the corresponding increase in the internal energy of the liquid if its volume increases 0.005 cu. ft.

Ans. (*a*) 74 B.t.u. (*b*) 0.1388. (*c*) 73.98 B.t.u.

2. A certain liquid has a variable specific heat at atmospheric pressure given by the following equation, $c_p = 0.7 + 0.003t - 0.0001t^2$. At this pressure, the saturation temperature is 100° F. and the temperature of fusion is −35° F. What is the value of the heat of the liquid per pound measured above the temperature of fusion?

Ans. 72.9 B.t.u.

3. When 1 lb. of a certain liquid vaporizes under a pressure of 35 lb. per sq. in. abs., there is a volume increase of 50 cu. ft. If the corresponding latent heat of vaporization is 900 B.t.u., and the saturation temperature is 250° F., find (*a*) the entropy of vaporization, (*b*) the external latent heat of vaporization, and (*c*) the internal latent heat of vaporization. Ans. (*a*) 1.267. (*b*) 324 B.t.u. (*c*) 576 B.t.u.

4. Twenty pounds of a certain liquid, for which $c_p = 1.13$, are heated at a constant pressure of 200 lbs. per sq. in. abs. from an initial temperature of 40° F. and a volume of 0.5 cu. ft. until it is completely vaporized. Under this pressure, the saturation temperature is 96° F., the volume of the saturated liquid is 0.027 cu. ft. per lb., the volume of the saturated vapor is 1.5 cu. ft. per lb., and the latent heat is 480 B.t.u. per lb. Find (*a*) the quantity of heat transferred to the substance during the entire process, (*b*) the corresponding gain in the entropy of the substance, (*c*) the increase in the internal energy of the liquid while being heated to the saturation temperature, and (*d*) the increase in the internal energy of the substance during vaporization.

Ans. (*a*) 10,866 B.t.u. (*b*) 19.64. (*c*) 1264 B.t.u. (*d*) 8510 B.t.u.

5. Under a pressure of 100 lb. per sq. in. abs., a certain substance has a saturation temperature of 328° F. The dry saturated vapor has a specific volume of 4.43 cu. ft. per lb. When superheated at constant pressure from the saturated vapor state until the temperature increases 100° F., the volume increases to 5.15 cu. ft. per lb. and $\bar{c}_p = 0.53$. Find, for one pound of the substance, (*a*) the heat of superheating, (*b*) the gain in internal energy during superheating, and (*c*) the entropy of superheating. Ans. (*a*) 53 B.t.u. (*b*) 39.7 B.t.u. (*c*) 0.0629.

6. (*a*) Sketch typical saturation curves on *PV* and $T\Phi$ coordinates. (*b*) Show a datum state at which the internal energy and entropy are assumed zero. (*c*) By an appropriate area, show the heat content of the substance at the datum state.

7. If, in problem 1, the volume of the liquid at the fusion temperature is 0.03 cu. ft., and if the internal energy of the saturated liquid at this temperature be taken at zero, (*a*) find the heat content of the saturated liquid at the specified pressure, (*b*) show an area on a *PV*-diagram that represents the difference between the heat content of the liquid and the heat of the liquid, and (*c*) determine the heat content of this liquid at a temperature of 150° F. and a pressure of 100 lb. per sq. in. abs. Ans. (*a*) 74.111 B.t.u. (*c*) 74.63 B.t.u.

8. The latent heat of vaporization of a certain substance at a pressure of 110 lb. per sq. in. abs. is 525 B.t.u. Starting with one pound of this substance as a saturated liquid at 110 lb. per sq. in. abs., an amount of heat equal to 300 B.t.u. is transferred to the substance at constant pressure and the corresponding gain in internal energy is 270 B.t.u. Find (*a*) the resulting quality of the mixture, (*b*) the internal latent heat of vaporization of the substance, and (*c*) *u*.

Ans. (*a*) 57.1 per cent. (*b*) 472 B.t.u. (*c*) 2.6 cu. ft.

9. When the saturation temperature of a certain vapor is 100° F., its saturation pressure is 100 lb. per sq. in. abs. The corresponding value of the internal latent heat of vaporization is 700 B.t.u./lb., the external latent heat of vaporization is 60 B.t.u./lb., and the mean value of the constant pressure specific heat when the saturated vapor is superheated 100° F. is 0.4. When the quality is 35 per cent, the specific heat content of the substance is 420 B.t.u./lb. and the specific entropy is 1.00 measured above the arbitrary datum. Find (*a*) the specific heat content of the substance when superheated 100° F., and (*b*) the corresponding specific entropy of the substance. Ans. (*a*) 954 B.t.u. (*b*) 1.9472.

10. In a certain boiler test the partial pressure of the CO_2 in the flue gas was 2.25 lb. per sq. in. abs. and its temperature was 300° F. Under these conditions why may CO_2 be considered as a gas rather than as a superheated vapor?

11. Should steam under the same pressure and temperature as specified for CO_2 in problem 10 be considered a gas rather than a superheated vapor? Why?

12. Should ammonia under the following conditions of pressure and temperature be considered as a gas or a superheated vapor: (*a*) 1700 lb. per sq. in. abs., 300° F., (*b*) 200 lb. per sq. in. abs., 100° F.? Before answering the questions, show approximately to scale on the $T\Phi$-diagram the states specified.

13. A vapor with a quality of 100 per cent and at a temperature of 100° F. is supplied to suitable coils for special heating purposes, and the condensate leaves the coils as a saturated liquid at this same temperature. Assuming that the heat given up to the coils amounts to 1,000,000 B.t.u. per hour, find the weight of vapor that must be supplied the coils, in pounds per hour, if the fluid used is (*a*) steam, (*b*) ammonia, and (*c*) sulphur dioxide. Ans. (*a*) 966. (*b*) 2,093. (*c*) 7,100.

14. Find the heat content of 100 lb. of water under an abs. pressure of 1574 lb. per sq. in. abs. and a temperature of 100° F. Ans. 7266 B.t.u.

15. Assuming that for each case in problem 13 the velocity of the vapor in the pipe line leading to the coils is 6000 ft. per minute and that of the condensate in the pipe line leading from the coils is 5 ft. per second, find the respective cross sectional areas of the pipes in square inches.

Ans. (*a*) 135.6 for vapor, and 0.125 for liquid. (*b*) 1.19 for vapor, and 0.46 for liquid. (*c*) 2.75 for vapor, and 0.625 for liquid.

16. Using Eq. (212) compute the value of $\frac{dP}{dT}$ for ammonia under a saturation temperature of (*a*) 0° F., (*b*) 125° F. Ans. (*a*) 105.8. (*b*) 632.

17. Saturated steam at 99° F. has a pressure of 0.918 lb. per sq. in. abs. and at 101° F., has a pressure of 0.975 lb. per sq. in. abs. Using Eq. (212) check, approximately, the latent heat of vaporization given in Table XI corresponding to a saturation temperature of 100° F., assuming the value of u indicated in this table.

18. Two pounds of carbon dioxide under a pressure of 650 lbs. per sq. in. abs., occupy a volume of 0.2 cu. ft. For this mixture, find (*a*) the quality, (*b*) the specific entropy, and (*c*) the specific internal energy.

Ans. (*a*) 80.2 per cent. (*b*) 0.1553. (*c*) 69.6 B.t.u./lb.

19. A tank having a volume of 175 cu. ft. contains a mixture of sulphur dioxide liquid and vapor at 75° F. The liquid occupies half the volume of the tank. What is the quality of the mixture? Ans. 0.743 per cent.

CHAPTER XVI

Note.—Most of the problems in this chapter may be solved by the aid of Table XI and Fig. 151, but vapor charts may be also employed to great advantage after the student has become thoroughly proficient in the use of the tables.

1. Determine the amount of heat that must be added to one pound of steam in order to change its state at a constant pressure of 678.7 lb. per sq. in. abs. from a volume of 0.5 cu. ft. until it is superheated 150° F., and show the area representing this heat. Ans. 311.9 B.t.u.

2. One pound of ammonia is heated at constant pressure from state 1 with $x_1 = 30$ per cent and $t_1 = 50°$ F., to state 2 such that $D_2 = 75°$ F. $\overline{C}_{p2} = 0.6$ and $V_2 = 3.3$ cu. ft. Find in B.t.u. (*a*) $Q_{1,2}$; (*b*) $Wk_{1,2}$; (*c*) $\Delta I_{1,2}$, and (*d*) I_2. (*e*) Show the areas roughly to scale representing (*a*) and (*b*).

Ans. (*a*) 414. (*b*) 37.8. (*c*) 376.2 (*d*) 615.7.

3. A 1000-hp. steam turbine uses 12 lb. of steam per hp-hr. when running under full load. The operating conditions are such that the steam is exhausted from the turbine into a heating system at a temperature of 200° F. and a quality of 95 per cent. (*a*) Assuming that there is no leakage and that the condensate is removed from the heating system as a saturated liquid at 200° F., find the amount of heat given up by the steam in B.t.u. per hour. (*b*) The same as part (*a*) except the condensate is removed at a temperature of 100° F. and a pressure of 11.52 lb. per sq. in. abs. (*c*) Would it be good engineering to use the heat content of the saturated liquid at 100° F. in solving part (*b*) of this problem? Why? (*d*) Under what conditions is there an important distinction between the heat content of the saturated and the non-saturated liquid at a given temperature? Ans. (*a*) 11,130,000. (*b*) 12,330,000.

4. One pound of steam at a pressure of 247.1 lb. per sq. in. abs. with a quality of 100 per cent has heat abstracted from it at constant volume until the temperature becomes 100° F. Find (*a*) the final quality, and (*b*) the amount of heat abstracted from the steam during the process, and show the approximate area representing it.

Ans. (*a*) 0.527 per cent. (*b*) 1042.8 B.t.u.

5. Ten pounds of sulphur dioxide are heated at constant volume from a temperature of 25° F. and a quality of 10 per cent until the temperature becomes 100° F. Find (*a*) the final quality, (*b*) the gain in internal energy of the substance.

Ans. (*a*) 41 per cent. (*b*) 624 B.t.u.

6. Two pounds of steam under a pressure of 678.7 lb. per sq. in. abs. and a temperature of 650° F. occupy a volume of 1.76 cu. ft. If this steam is cooled at constant volume until its pressure becomes 67 lb. per sq. in. abs., find, in B.t.u., (*a*) its initial internal energy, (*b*) its final internal energy, and (*c*) the heat abstracted from it. Ans. (*a*) 2430. (*b*) 761.2. (*c*) 1668.8.

7. A container having a volume of 10 cu. ft. is filled with a mixture of liquid and vapor ammonia, the liquid occupying 7.5 cu. ft. The initial temperature of the ammonia is 50° F. The container is carelessly placed in the storeroom alongside heating coils and in the course of a day its temperature finally rises to 125° F. Assuming that the tank does not explode, find (*a*) the initial quality, (*b*) the resultant increase of pressure, and (*c*) the heat absorbed by the ammonia.

Ans. (*a*) 0.259 per cent. (*b*) 218.6 lb. per sq. in. (*c*) 25,495 B.t.u.

8. One pound of dry saturated ammonia at a pressure of 307.8 lb. per sq. in. abs. is expanded isentropically until its pressure becomes 53.73 lb. per sq. in. abs. Find (*a*) the final quality, (*b*) the final volume, and (*c*) the work done by the ammonia during this expansion, and show the area representing it.

Ans. (*a*) 86.6 per cent. (*b*) 4.615 cu. ft. (*c*) 79 B.t.u.

9. Three pounds of carbon dioxide under a pressure of 505 lb. per sq. in. abs. and with a quality of 75 per cent are compressed isentropically until the pressure becomes 906 lb. per sq. in. abs. Find (*a*) the final quality and (*b*) the gain in internal energy of the carbon dioxide. Ans. (*a*) 90.1 per cent. (*b*) 21.75 B.t.u.

10. One pound of steam is expanded from state 1 to 2 isentropically. If $t_1 =$ 525° F., $V_1 = 2.28$ cu. ft., $p_1 = 247.1$ and $p_2 = 11.52$ lb. per sq. in. abs., find (*a*) V_2/V_1, (*b*) $Wk_{1,2}$, and show the area representing it. Ans. (*a*) 13.09. (*b*) 202.4 B.t.u.

11. If saturated liquid ammonia in state 1 with $t_1 = 75°$ F. is expanded isentropically until state 2 is reached such that $t_2 = 0°$ F., find (*a*) x_2, (*b*) $\overline{V}_2$, (*c*) $\overline{V}_2/\overline{V}_1$, (*d*) $Wk_{1,2}$.

Ans. (*a*) 13.49 per cent. (*b*) 1.2512 cu. ft. (*c*) 47.3 (*d*) 13.15 B.t.u. per lb.

12. The same problem as 11 except that the fluid is CO_2.

Ans. (*a*) 33.75 per cent. (*b*) 0.1091 cu. ft. (*c*) 4.95. (*d*) 7.54 B.t.u. per lb.

13. The same problem as 11 except that the fluid is SO_2.

Ans. (*a*) 13.63 per cent. (*b*) 0.973 cu. ft. (*c*) 88.5 (*d*) 4 B.t.u. per lb.

14. One pound of steam at a pressure of 67 lb. per sq. in. abs. and a temperature of 500° F. is cooled isothermally until its quality becomes 35 per cent. The initial volume is 8.45 cu. ft. Find, in B.t.u., (*a*) the amount of heat abstracted from the steam, (*b*) the gain in its internal energy, and (*c*) the work done on it. Show the areas representing (*a*) and (*c*) respectively. Ans. (*a*) 771. (*b*) −470.4 (*c*) 300.6.

15. One pound of ammonia having a quality of 80 per cent and a temperature of 125° F. is expanded according to the law $PV =$ const. until the pressure becomes 53.73 lb. per sq. in. abs. Find (*a*) the work done by the ammonia, (*b*) the gain in its internal energy, and (*c*) the heat added to it.

Ans. (*a*) 60,500 ft.-lb. (*b*) −12.9 B.t.u. (*c*) 64.9 B.t.u.

16. One pound of steam at a pressure of 11.52 lb. per sq. in. abs. and a temperature of 300° F. is expanded according to the law $PV^{1.2} = const.$ until the temperature becomes 100° F. The initial volume is 39 cu. ft. Find (*a*) the final quality, (*b*) the work done by the steam, (*c*) the gain in its internal energy, and (*d*) the heat transferred to it.

Ans. (*a*) 88.9 per cent. (*b*) ~~161,700 ft~~.-lb. (*c*) −176.1 B.t.u. (*d*) ~~31.4~~ B.t.u.

17. Steam is throttled through a valve from a pressure of 247.1 lb. per sq. in. abs. and a quality of 95 per cent to a pressure of 67 lb. per sq. in. abs. Find (*a*) the final quality, (*b*) the gain in the specific entropy of the steam and (*c*) the increase in the specific volume of the steam.

Ans. (*a*) 97.8 per cent. (*b*) 0.1278. (*c*) 4.545 cu. ft.

18. A throttling calorimeter is used to determine the quality of steam in a steam pipe. The pressure of the steam in the sampling pipe is 247.1 lb. per sq. in. abs., and the pressure and temperature of the steam in the calorimeter indicate a heat content of 1168.1 B.t.u. per lb. Making the usual assumptions regarding this type of calorimeter, find the quality of the steam in the sampling pipe. Ans. 96 per cent.

19. If, for a given throttling calorimeter installation, the minimum value of the heat content for which the calorimeter will give satisfactory readings is 1155 B.t.u. per lb. when used with steam, find the minimum quality in the steam main for which the calorimeter may be used if the pressure in the main is (*a*) 67 lb. per sq. in. abs., and (*b*) 678.7 lb. per sq. in. abs. Ans. (*a*) 97.4 per cent. (*b*) 92.5 per cent.

20. A throttling calorimeter is used to determine the quality of steam in a main in which the pressure is 67 lb. per sq. in. abs. The exhaust from the calorimeter is connected to a condenser so that the pressure in the calorimeter is 11.52 lb. per sq. in. abs. and the temperature is 225° F. (*a*) Ignoring radiation from the calorimeter and differences in velocities and elevations, find the quality of the steam in the main. (*b*) If radiation from this calorimeter amounts to 36 B.t.u. per hour with a flow of 72 lb. per hour, neglecting differences in velocities and elevations, find how much the specific heat content in the main should be modified from that leaving the calorimeter. Ans. (*a*) 97.6 per cent. (*b*) + 0.5 B.t.u.

21. A universal calorimeter of the type described in Sect. **196**(f) is used to determine the quality of steam in a main in which the absolute pressure is 67 lb. per sq. in. while that in the calorimeter is 14.7 lb. per sq. in. Under these conditions the flow of steam through the calibrated nozzle of the calorimeter is 27 lb. per hour. If 0.22 lb. of water collects in the separating chamber of the calorimeter every ten minutes, find the quality of the steam in the main, assuming that a fair sample flows through the calorimeter. Ans. 91.5 per cent.

22. If saturated liquid ammonia in state 1 with $t_1 = 75°$ F., is throttled adiabatically until state 2 is reached such that $t_2 = 0°$ F., find (*a*) x_2, (*b*) p_2, (*c*) $\overline{V}_2$, (*d*) $\overline{V}_2/\overline{V}_1$, (*e*) ϕ_2, (*f*) ϕ_2/ϕ_1.

Ans. (*a*) 14.63 per cent. (*b*) 30.42. (*c*) 1.354 cu. ft. (*d*) 51.10. (*e*) 0.2786. (*f*) 1.053.

23. The same as problem 22, except that the fluid is CO_2.

Ans. (*a*) 38 per cent. (*b*) 309. (*c*) 0.1209 cu. ft. (*d*) 5.49. (*e*) 0.065. (*f*) 1.201.

24. The same as problem 22, except that the fluid is SO_2.

Ans. (*a*) 15 per cent. (*b*) 10.4. (*c*) 10.7 cu. ft. (*d*) 97.4. (*e*) 0.0328. (*f*) 1.17.

25. Suppose that each of the respective fluids used in problems 22, 23, and 24, after throttling to state 2, is then heated isothermally from 2 to 3, such that $x_3 = 100$ per cent. Find $Q_{2,3}$ in B.t.u. per lb. and show the area representing it.

Ans. 485, 72.5, 137.5.

Note.—It is recommended that most of the following problems in this chapter and those in the succeeding ones be solved with the aid of **suitable vapor charts.**

26. Five pounds of steam under a pressure of 110 lb. per sq. in. abs. and occupying a volume of 25 cu. ft., undergo a constant volume process until the pressure becomes 80 lb. per sq. in. abs. Find (*a*) the final quality, and (*b*) the heat abstracted from the steam. Ans. (*a*) 91.3 per cent. (*b*) 671 B.t.u.

27. Steam, initially under a pressure of 200 lb. per sq. in. abs. and with a superheat of 200° F., expands isentropically until its pressure becomes 1 in. Hg abs. Find (*a*) the work done by each pound of steam during this expansion, and (*b*) the ratio of the final to the initial volume. Ans. (*a*) 350.6 B.t.u. (*b*) 173.9.

28. Suppose a pound of steam having an absolute pressure of 150 lb. per sq. in. and a temperature of 400° F. is compressed isothermally until its quality becomes 98 per cent. Find (*a*) the quantity of heat added to the steam, and (*b*) the external work done by it. Ans. (*a*) $-$ 75.6 B.t.u. (*b*) $-$ 48.1 B.t.u.

29. Starting with the same initial condition as in Prob. 28, suppose the steam is compressed according to the law $PV^{1.4}$ = const. until the pressure becomes 250 lb. per sq. in. abs. Find for this process (*a*) the work done by the steam and (*b*) the quantity of heat added to it. Ans. (*a*) $-$ 33.8 B.t.u. (*b*) 8.7 B.t.u.

30. Steam is flowing steadily through a valve which reduces its pressure from 200 to 20 lb. per sq. in. abs. Assuming that the superheat before passing this valve is 130° F., that the velocity is the same on each side of the valve, and that there is no loss of energy by radiation or conduction, find (*a*) the gain in the specific volume of the steam, (*b*) the gain in its specific entropy, (*c*) the drop in its temperature. Ans. (*a*) 24.82 cu ft. (*b*) 0.248. (*c*) 37 deg. F.

31. Suppose an apparatus could be devised to permit steam to expand from state 1 to 2 along the saturated vapor line. Assume that this steam follows the law $PV^n = k$. If $p_1 = 200$ and $p_2 = 100$ lb. per sq. in. abs., (*a*) find the value of n such that $P_1V_1{}^n = P_2V_2{}^n$. (*b*) Using this value of n, find the mean value of the specific heat of the steam for this process. (*c*) The same as (*a*) and (*b*) except that $p_1 = 15$ and $p_2 = 1$ lb. per sq. in. abs.

Ans. (*a*) 1.043. (*b*) $-$ 0.857. (*c*) 1.068, $-$ 1.13.

32. On a certain day the relative humidity is 76 per cent, the barometric pressure is 14.2 lb. per sq. in., and the temperature of the air is 65° F. Find (*a*) the pressure of the water vapor, (*b*) the pressure of the dry air, (*c*) the density of the water vapor, and (*d*) the dew point.

Ans. (*a*) 0.232 lb. per sq. in. abs. (*b*) 13.968 lb. per sq. in. abs. (*c*) 0.00074 lb. per cu. ft. (*d*) 57.1° F.

33. What is the ratio of the density of dry air at a temperature of 100° F. and an absolute pressure of 14.7 lb. per sq. in. to that of the so-called "saturated air" for the same temperature and pressure. Ans. 1.023.

34. (*a*) What is the ratio of the density of dry air at a temperature of 80° F. and an absolute pressure of 14.7 lb. per sq. in. to the density of the atmosphere having a relative humidity of 80 per cent and the same total pressure and temperature? (*b*) What is the dew point of the atmosphere in this condition?

Ans. (*a*) 1.01. (*b*) 73.2° F.

35. For certain industrial purposes a mixture of air and water vapor is heated at the constant pressure of 13.5 lb. per sq. in. abs. from a temperature of 45° to 100° F. (*a*) If the relative humidity of the mixture entering the heater is 85 per cent, find its value upon leaving the heater, assuming that no water was added in the heater. (*b*) How much heat would be absorbed by this mixture, assuming that 100 lb. per min. are passed through the heater.

Ans. (*a*) 15.83 per cent. (*b*) 1207 B.t.u. per min.

36. At a certain load a boiler requires 174,000 lb. of moist air per hour to support combustion. If this air is supplied the boiler at a temperature of 90° F., an absolute pressure of 14.5 lb. per sq. in., and a relative humidity of 85 per cent, find the weight of water vapor that enters the furnace per hour with the air. Ans. 4520 lb.

CHAPTER XVII

1. Consider the following cycle: Starting with steam under an absolute pressure of 100 lb. per sq. in. and a volume of 7.5 cu. ft. per lb., heat is abstracted at constant volume until the pressure is reduced 50 per cent. The steam is then compressed isentropically to the initial pressure, and a constant pressure addition of heat returns it to the initial state. Find, for this cycle, (*a*) Q_1, (*b*) Q_2, (*c*) net work, and (*d*) the efficiency.

Ans. (*a*) 305 B.t.u. per lb. (*b*) 285.7 B.t.u. per lb. (*c*) 19.3 B.t.u. per lb. (*d*) 6.32 per cent.

Note.—In the following problems on the Carnot cycle, unless otherwise specified, it is understood that the working substance at the beginning of the isothermal expansion is a *saturated* liquid.

2. Consider a heat engine that operates on the Carnot cycle with steam as the working substance. At the start of the adiabatic expansion, the pressure of the steam is 247.1 lb. per sq. in. abs. and the quality is 97 per cent. The lowest temperature in the cycle is 100° F. Find (*a*) the heat supplied, (*b*) the heat rejected, and (*c*) the efficiency of the cycle.

Ans. (*a*) 803 B.t.u. per lb. (*b*) 523 B.t.u. per lb. (*c*) 34.9 per cent.

3. A heat engine operating on the Carnot cycle and using steam as the working substance has the adiabatic expansion starting with a pressure of 60 lb. per sq. in. abs. and a temperature of 400° F. This expansion continues until the temperature becomes 200° F. (*a*) Draw this cycle carefully on both PV- and $T\phi$-diagrams. Find (*b*) the work done by each pound of the steam during the adiabatic expansion, (*c*) the net work of the cycle of each pound of steam, and (*d*) the efficiency of the cycle. Ans. (*b*) 104 B.t.u. (*c*) 229 B.t.u. (*d*) 23.25 per cent.

4. (*a*) Why, in general, are the efficiencies of ideal cycles likely to be less for the steam plant than for the internal combustion engine? (*b*) What determines the lowest temperature in a steam plant in which every possible refinement is introduced?

5. Steam is supplied to a turbine at an absolute pressure of 185 lb. per sq. in. and with a superheat of 140° F. If a condenser maintains an exhaust pressure on the turbine of 1.5 in. Hg abs., (*a*) what is the value of $(\Delta h)_\phi$? (*b*) What is the significance of this quantity? (*c*) Find the exhaust pressure to give only half this amount of available energy. (*d*) By what portion would the available energy be increased if the condenser pressure were reduced from 1.5 to 1 in. Hg abs.?

Ans. (*a*) 374.7 B.t.u. (*c*) 18.6 lb. per sq. in. abs. (*d*) 5.17 per cent.

6. In a certain factory, steam is available from the boilers at an absolute pressure of 160 lb. per sq. in. and with a superheat of 70° F. A small oven, requiring a steam temperature of at least 400° F. is installed. (*a*) If the steam is condensed in the coils of this oven, what pressure must be maintained in them? It is proposed that the small amount of steam thus required be taken from the boilers and pumped up to the required pressure. (*b*) In the ideal case, with no heat transfer during the pumping, how much mechanical energy per pound of steam pumped is required to operate the compressor? (*c*) This mechanical energy is what portion of the heat energy made available from each pound of steam in the oven, if the steam is condensed at constant pressure to a saturated liquid at 400° F.? (*d*) If the oven condenses 3000 lb. of steam per hour and the actual compressor requires 25 per cent more power than the ideal, what will be the horsepower required to drive the compressor? (*e*) Under what conditions would this be a commercially feasible idea?

Ans. (*a*) 247 lb. per sq. in. abs. (*b*) 42.2 B.t.u. (*c*) 4.67 per cent. (*d*) 62.1.

7. If steam at a pressure of 200 lb. per sq. in abs. and a temperature of 500° F. is supplied to a turbine, (*a*) what per cent increase in the efficiency of the Rankine engine cycle would result from lowering the back pressure from 15 lb. per sq. in. abs. to 1 in. Hg abs.? (*b*) Find the per cent increase in the net work of the ideal turbine for the same conditions. (*c*) If the circulating water is to enter the condenser in this plant at a temperature 10 degrees less than the saturation temperature of the steam exhausted from the turbine, what would this temperature have to be when the absolute pressure of the exhaust steam is 1.2″ Hg? When 0.6″ Hg?

Ans. (*a*) 71.6; (*b*) 92.7; (*c*) 74.7° F.; 54° F.

8. If the turbine described in problem 7 operates with the 1 in. back pressure, find the per cent increase in the cycle efficiency that would result if the throttle temperature were increased from 500° to 600° F. Ans. 1.85.

9. Steam at a pressure of 200 lb. per sq. in. abs. and with a superheat of 200° F. is supplied to a turbine which has its back pressure maintained at 1 lb. per sq. in. abs. The steam rate of this turbine is 14 lb. per b.hp-hr. A second turbine is supplied with steam at a pressure of 180 lb. per sq. in. abs. and with a superheat of 100° F.; it has a back pressure of 15 lb. per sq. in. abs. and its steam rate is 16 lb. per b.hp-hr. Find the brake thermal efficiency of (*a*) the first turbine and (*b*) the second turbine. (*c*) explain how the turbine with the lower steam rate can have the poorer thermal economy. Ans. (*a*) 14.7 per cent. (*b*) 14.82 per cent.

10. A 20,000-kw. turbine generator is supplied with steam at a pressure of 250 lb. per sq. in. abs. and a temperature of 650° F. Under test at the most economical load the combined steam rate is found to be 10 lb. per kw-hr. If the back pressure is 0.9 in. Hg abs., find (*a*) the combined thermal efficiency, (*b*) the combined engine efficiency, and (*c*) the ideal steam rate.

Ans. (*a*) 26.35 per cent. (*b*) 76.2 per cent. (*c*) 7.62 lb. per kw-hr.

11. A centrifugal boiler feed pump is direct-connected to a small steam turbine. During a certain test, the power delivered to the water averaged 64 hp. The turbine was supplied with steam, superheated 104° F., and having a pressure of 215 lb. per sq. in. abs. With an exhaust pressure of 5.6 lb. per sq. in. abs., the steam consumption of the turbine amounted to 930 lb. per hr. Find (*a*) the combined steam rate, (*b*) the ideal steam rate, (*c*) the combined thermal efficiency, and (*d*) the combined engine efficiency.

Ans. (*a*) 14.53 lb. per hp-hr. (*b*) 9.44 lb. per hp-hr. (*c*) 15.57 per cent. (*d*) 65 per cent.

12. A turbine-generator, delivering 30,000 kw., has a combined engine efficiency of 77 per cent when the steam is supplied at a pressure of 400 lb. per sq. in. abs. and a temperature of 700° F. and when the exhaust pressure is 1 in. Hg abs., (*a*) Find the weight of steam that must be supplied the turbine per hour. (*b*) Assuming a generator efficiency of 96 per cent and, for the purpose of this problem, that all of the turbine losses appear in the exhaust steam, find the amount of heat that must be abstracted in the condenser from each pound of steam. If the steam velocity, in both the steam main and in the exhaust opening be taken as 12,000 ft. per min., what would be the required area in square feet of (*c*) the piping leading to the turbine and (*d*) the exhaust connection between the turbine and the condenser.

Ans (*a*) 278,000 lb. (*b*) 926.5 B.t.u. (*c*) 0.657. (*d*) 224.

13. A 30,000-kw. turbine generator shows the following performance when tested at various loads:

Generator Load, Kw.	Throttle Pressure, Lb./In.^{2}, Abs.	Steam Temperature at Throttle, Deg. F.	Exhaust Pressure, In. Hg., Abs.	Time of Run, Hours	Total Steam Condensed During Run
0	234	499	1.85	0.5	8,900
13,900	237	551	1.00	3	493,000
18,200	227	546	1.04	3	617,000
22,200	232	552	1.00	3	737,000
26,200	231	540	1.08	3	865,000
34,500	224	498	1.04	3	1,167,000

Using a tabulated scheme of calculations, find (*a*) the combined steam rate, (*b*) the combined thermal efficiency, (*c*) the Rankine engine cycle efficiency, and (*d*) the combined engine efficiency. (*e*) Plot the results of parts (*a*) to (*d*), inclusive, with the generator load as abscissa.

Ans. (*a*) ∞, 11.82, 11.30, 11.08, 11.00, 11.27 lb. per kw-hr. (*b*) 0, 23.2, 24.35, 24.75, 25.1, 24.95 per cent. (*c*) 31.5, 33.35, 33.0, 33.3, 33.0, 32.7 per cent. (*d*) 0, 69.5, 73.8, 74.3, 76.0, 76.3 per cent.

14. Suppose that a steam-pipe line is selected of such a size that it gives a 10 per cent drop in pressure between the boiler and the engine. What is the corresponding percentage loss of mechanical energy available assuming an exhaust pressure of 2 in. of Hg abs. and that the steam leaves the boiler with a temperature of 575° F. and an absolute pressure of 340 lb. per sq. in.

Ans. 1.4.

15. In a power plant operating on the Rankine cycle the pressure in the steam main as it leaves the boiler is 417 lb. per sq. in. abs. and its temperature is 700° F. The pipe line causes a drop in pressure of 17 lb. per sq. in. and the throttle valve further reduces the pressure to 385 lb. per sq. in. abs. The exhaust pressure on the turbine is maintained at 1 in. Hg abs. Assuming no radiation loss, find the per cent loss of energy available in the turbine due to throttling (*a*) in the pipe line and (*b*) in the valve.

Ans. (*a*) 0.45. (*b*) 0.34.

16. In a plant operated on the Rankine cycle steam is supplied the prime mover at a pressure of 600 lb. per sq. in. abs. and with a superheat of 280° F. The condenser pressure is 1.2 in. Hg abs. and the volume of saturated water at this pressure is 0.016 cu. ft. per lb. Find (*a*) the energy that could be delivered by each pound of steam passing through an ideal prime mover; (*b*) the difference between the result of (*a*) and the net work of this Rankine steam cycle; (*c*) the efficiency of this steam cycle. Ans. (*a*) 505.7 B.t.u. (*b*) 1.78 B.t.u. (*c*) 37.8 per cent.

17. Suppose a boiler supplies dry saturated steam at 247.1 lb. per sq. in. abs. to an engine which exhausts the steam at 0.946 lb. per sq. in. abs. The volume of the saturated liquid corresponding to this exhaust pressure is 0.016 cu. ft. per lb. If this plant operates on a Rankine vapor cycle, find (*a*) the net work of the cycle per pound of steam and (*b*) the efficiency of the cycle. (*c*) The same as (*a*) and (*b*) except the plant is supposed to operate on the Carnot cycle. (*d*) What advantage has each cycle as shown by these results.

Ans. (*a*) 349.8 B.t.u. (*b*) 30.85 per cent. (*c*) 288.5 B.t.u; 34.9 per cent.

18. A certain steam-power plant operating on the Rankine vapor cycle averages for one year a kilowatt-hour at the switchboard for every 1.52 lb. of coal which has a higher heating value of 12,900 B.t.u. per lb. The average steam pressure is 235 lb. per sq. in. abs. and the average steam temperature is 600° F. The exhaust pressure averages 1.2 in. Hg abs. Find (*a*) the plant thermal efficiency, and (*b*) the ratio of this efficiency to the ideal vapor cycle efficiency, ignoring the feed pump work.

Ans. (*a*) 17.4 per cent. (*b*) 0.527.

19. For the throttle steam as specified in problem 7, and assuming that the steam is used non-expansively, (*a*) find what per cent increase in the corresponding engine cycle efficiency would result if the back pressure were reduced from 15 lb. per sq. in. abs. to 1 in. Hg abs.? (*b*) Find the per cent increase in the net work of the ideal engine for the same conditions. (*c*) Comparing the results of this problem with those of problem 7, in which type of plant would the investment required to bring about this reduction in back pressure be the more justified?

Ans. (*a*) − 3.62. (*b*) 8.15.

20. A reciprocating engine operating on the non-expansion cycle is supplied with steam at a pressure of 117 lb. per sq. in. abs. and with a superheat of 63° F. The back pressure is 16 lb. per sq. in. abs. During a test at constant load the average indicated horsepower is 27.6 and the total consumption of steam is 1250 lb. per hr. If the mechanical efficiency is 85 per cent, find (*a*) the indicated thermal efficiency, (*b*) the brake thermal efficiency, (*c*) the ideal steam rate, and (*d*) the indicated engine efficiency.

Ans. (*a*) 5.4 per cent. (*b*) 4.59 per cent. (*c*) 32.3 lb. per hp-hr. (*d*) 71.5 per cent.

21. (*a*) For the data in problem 20 what would be the indicated engine efficiency if the Rankine engine cycle were used as the standard? (*b*) If you were a manufacturer of steam engines, which manner of calculating the indicated engine efficiency would you prefer? Why? Ans. (*a*) 37 per cent.

22. Steam is taken by three different pipes from a main in which the pressure is 200 lb. per sq. in. abs. and the superheat is 100° F. The first pipe leads to a steam turbine, the second to a non-expansion engine, and the third to a throttling valve. The discharge from each device is at a pressure of 14.7 lb. per sq. in. abs. The brake engine efficiency of the turbine is 75 per cent, and for the reciprocating engine it is also 75 per cent when referred to the non-expansion ideal. Assuming no loss due to radiation or conduction, find the volume, the entropy, and the heat content of each pound of steam as it is discharged from (*a*) the turbine, (*b*) the reciprocating engine, and (*c*) the throttling valve.

Ans. (*a*) 25.5 cu ft., 1.688, 1104.6 B.t.u. (*b*) 30.2 cu. ft., 1.809, 1188.5 B.t.u. (*c*) 36.4 cu. ft., 1.893, 1257 B.t.u.

23. In a power plant, steam having a pressure of 135 lb. per sq. in. abs. and a superheat of 100° F., is supplied to an engine. The steam is used non-expansively and is exhausted at 5 lb. per sq. in. abs. At this pressure the specific volume of the saturated liquid is 0.0164 cu. ft. per lb. For the corresponding ideal vapor cycle find (*a*) the net work per pound of steam, (*b*) the efficiency, (*c*) the feed pump work per pound of water, and (*d*) the specific volume of the steam at the end of the free expansion. (*e*) In this ideal cycle the feed pump work is what proportion of the net work? (*f*) If this plant were operated on the ideal Rankine vapor cycle, the feed pump work would be what portion of the net work?

Ans. (*a*) 92.7 B.t.u. (*b*) 8.31 per cent. (*c*) 0.394 B.t.u. (*d*) 79.5 cu. ft. (*e*) 0.425 per cent. (*f*) 0.164 per cent.

24. An 8 × 18 in. single-cylinder double-acting steam engine is tested with dry saturated steam supplied at 115 lb. per sq. in. abs. and with the back pressure maintained at 14.5 lb. per sq. in. abs. This engine runs 100 cycles per minute per cylinder end and, when delivering 23 hp., the indicator diagrams show a ratio of expansion of 3.5, and an average mean effective pressure of 55 lb. per sq. in. The steam consumption is 575 lb. per hr. Find (*a*) the indicated horsepower, (*b*) the pounds of steam per indicated horsepower-hour, (*c*) the pounds of steam per brake horsepower-hour, (*d*) the mechanical efficiency, (*e*) the brake thermal efficiency, (*f*) the brake engine efficiency referred to the Rankine and also to the incomplete expansion cycle.

Ans. (*a*) 25.2. (*b*) 22.8. (*c*) 25. (*d*) 91.3 per cent. (*e*) 10.1 per cent. (*f*) 67.4 per cent; 72.5 per cent.

25. An engine operating on the incomplete expansion cycle has steam supplied at a pressure of 400 lb. per sq. in. abs. and with a superheat of 160° F. The back pressure is 60 lb. per sq. in. abs., the engine exhausting into process heating mains. The ratio of expansion in the engine is 3 and the brake engine efficiency is 65 per cent. For the purpose of this problem it may be assumed without serious error that all of the engine losses appear in the exhaust steam. If the process steam in condensing to a saturated liquid at 60 lb. per sq. in. abs. must give up 3,000,000 B.t.u. per hour, find (*a*) the brake horsepower of the engine, and (*b*) the area of the exhaust steam main in square inches to give an average velocity of 1000 ft. per min. to this process steam. Ans. (*a*) 132.7. (*b*) 59.2.

26. A four-cylinder, 14 × 16 in. double-acting steam engine is supplied with saturated steam at 140 lb. per sq. in. abs., and exhausts against a back pressure of 14.5 lb. per sq. in. abs. This engine is direct-connected to an electric generator. When tested under various loads, but at the constant speed of 300 r.p.m., which is maintained by the governor's regulation of the point of "cut off," this combined unit shows the following performance:

Steam Used, Pounds per Hour	Indicated Horsepower	Output of Generator, Kilowatts	Ratio of Expansion of Steam
4,360	212	100	4.7
7,140	351	200	4.2
9,910	492	300	3.7
13,120	635	400	3.1

Using a tabulated scheme of calculation, find the steam rate, (*a*) in pounds per indicated horsepower-hour, (*b*) in pounds per kilowatt-hour of generator, (*c*) of the corresponding ideal engine. Also find (*d*) the engine cycle efficiency, (*e*) the combined thermal efficiency, and (*f*) the combined engine efficiency. (*g*) Plot on one sheet the results of parts (*a*) to (*f*), inclusive, with the output of the generator as the abscissa.

Ans. (*a*) 20.55, 20.3, 20.1, 20.7 lb. per i.hp-hr. (*b*) 43.6, 35.7, 33.0, 32.8 lb. perk w-hr. (*c*) 16.03, 16.4, 16.9, 17.7 lb. per hp-hr. (*d*)15.63, 15.3, 14.85, 14.15 per cent. (*e*) 7.72, 9.43, 10.2, 10.26 per cent. (*f*) 49.3, 61.6, 68.7, 72.5 per cent.

27. The steam in an ideal reciprocating engine operating on the incomplete expansion cycle has a pressure of 6 lb. per sq. in. abs. and a quality of 92 per cent at release. (*a*) If the condenser pressure be assumed constant at 2 in. Hg abs., find the heat removed by the condenser from each pound of steam coming from this ideal engine. (*b*) Would the condenser of a real engine operating with the same throttle conditions, the same ratio of expansion, and the same condenser pressure, have to remove a greater or smaller amount of heat per pound than in (*a*)? Why?

Ans. (*a*) 931.7 B.t.u.

28. In a power plant steam at a pressure of 200 lb. per sq. in. abs. and with a superheat of 125° F. is supplied to an engine in which it expands to a release pressure of 12 lb. per sq. in. abs. The condenser maintains a back pressure of 2 lb. per sq. in. abs. Neglecting the feedpump work, find, for the corresponding ideal incomplete expansion cycle, (*a*) the net work, (*b*) the efficiency, (*c*) the ratio of isentropic expansion, (*d*) the loss of the net work due to incomplete expansion, and (*e*) the quality or superheat of the steam at the end of the free expansion.

Ans. (*a*) 275.7 B.t.u per lb. (*b*) 23.4 per cent. (*c*) 10.53. (*d*) 51.5 B.t.u. per lb. (*e*) 88.3 per cent.

29. Steam at a pressure of 185 lb. per sq. in. abs. and superheated 104° F. is used in a power plant so arranged that the ratio of expansion in the engine cylinder is 7. The exhaust pressure is 5 lb. per sq. in. abs. Neglecting the feedpump work, find, for the corresponding ideal incomplete expansion cycle, (*a*) the net work, (*b*) the efficiency, and (*c*) the gain in the specific entropy of the steam during the free expansion.

Ans. (*a*) 233.6 B.t.u. per lb. (*b*) 20.75 per cent. (*c*) 0.05.

30. In a certain industrial plant saturated steam is available from a boiler at a pressure of 200 lb. per sq. in. abs., and a condenser maintains a pressure of 4 lb. per sq. in. abs. Assume that an ideal engine is interposed between this boiler and condenser, and that it uses $\frac{1}{4}$ lb. of steam per cycle and makes 300 cycles per minute. Neglecting the feedpump work, draw the PV- and $T\phi$-diagrams to scale for the Rankine vapor cycle for these conditions, and find (*a*) the cycle efficiency, (*b*) the ideal steam rate, (*c*) the ideal horsepower, and (*d*) the horsepower per cubic foot of piston displacement for ideal conditions.

Ans. (*a*) 24.55 per cent. (*b*) 9.62 lb. per hp-hr. (*c*) 467. (*d*) 25.55.

31. The same as problem 30, except that the ideal engine is now assumed to operate on the incomplete expansion cycle with a ratio of expansion of 5.

Ans. (*a*) 18.37 per cent. (*b*) 12.88 lb./hp-hr. (*c*) 350. (*d*) 122.7.

32. The same as problem 30, except that the ideal engine now uses the steam non-expansively. What is the significance of problems, 30, 31 and 32?

Ans. (*a*) 7.65 per cent. (*b*) 30.85 lb. per hp-hr. (*c*) 145.8. (*d*) 255.5.

33. Suppose that in each of the three preceding problems the actual engine has a brake engine efficiency of 65 per cent for problem 30, 67 per cent for problem 31, and 68 per cent for problem 32. Find for each case (*a*) the actual steam required per brake horsepower-hour, and (*b*) the brake thermal efficiency.

Ans. (*a*) 14.78, 19.21, 45.4 lb. per b.hp-hr. (*b*) 15.97, 12.28, 5.2 per cent.

CHAPTER XVIII

1. In a power plant, steam at a pressure of 400 lb. per sq. in. abs. and a temperature of 700° F. is supplied by the main boilers. After expansion in the turbine until the pressure is reduced to 80 lb. per sq. in. abs., the steam is reheated to 700° F. Upon completing the expansion in the turbine, the steam is exhausted at 1 in. Hg abs. (*a*) For the corresponding ideal vapor cycle, find the cycle efficiency. (*b*) If the steam were not reheated, what would be the corresponding Rankine vapor cycle efficiency? (*c*) What is the chief advantage due to the reheating?

Ans. (*a*) 37.4 per cent. (*b*) 36.5 per cent.

2. A 35,000 kw. turbine-generator is designed to operate on the reheating cycle with one reheat. The steam is supplied at the turbine throttle with an absolute pressure of 550 lb. per sq. in. and with a temperature of 700° F. When operating at the rated load the steam leaves the turbine for reheating at an absolute pressure of 135 lb. per sq. in. and a temperature of 426° F. The steam is returned to the turbine at an absolute pressure of 125 lb. per sq. in. and a temperature of 700° F. The back pressure is 1 in. of Hg abs. Under these conditions, the turbine-generator shows a steam rate of 7.9 lb. per kw-hr. Find, for the corresponding ideal unit, (*a*) the ideal steam rate, and (*b*) the cycle efficiency. For the actual unit, find (*c*) the combined thermal efficiency, and (*d*) the combined engine efficiency. (*e*) Why is the combined steam rate considerably lower than for a turbine-generator operating on the Rankine cycle with the same throttle conditions and same back pressure? (*f*) Is the thermal efficiency of this unit to be compared with that of the Rankine by their ratio of steam rates? Why?

Ans. (*a*) 6.06 lb. per kw-hr. (*b*) 38.3 per cent. (*c*) 29.9 per cent. (*d*) 78 per cent.

3. In a paper presented to the American Society of Mechanical Engineers at the annual meeting in 1925, C. D. Zimmerman gave the following data from a test conducted at the Lake Shore Station of the Cleveland Electric Illuminating Co. The turbine-generator delivered 29,760 kw. with a throttle flow of 307,590 lb. of steam per hr. under the following conditions: abs. pressure at the throttle = 245 lb. per sq. in.; superheat at the throttle = 252 deg. fahr.; abs. pressure at exhaust = 0.964 in. Hg; abs. pressure at the one bleeding point, = 28.73 in. Hg; temperature of the feedwater leaving the bleeder heater = 163° F. For the corresponding ideal unit find (*a*) the per cent of the throttle steam that is bled at the one stage, (*b*) the net work from each pound of throttle steam, (*c*) the ideal steam rate, and (*d*) the cycle efficiency. For the actual unit find (*e*) the combined steam rate, (*f*) the combined thermal efficiency, and (*g*) the combined engine efficiency.

Ans. (*a*) 12.62. (*b*) 419.1 B.t.u. (*c*) 8.14 per kw-hr. (*d*) 36.05 per cent. (*e*) 10.34 lb. per kw-hr. (*f*) 27.3 per cent. (*g*) 75.7 per cent.

4. The unit described in Problem 3, when tested without bleeding, showed the following performance when delivering 29,850 kw. with a throttle flow of 298,510 lb. of steam per hr.: abs. pressure at the throttle = 248 lb. per sq. in.; superheat at the throttle = 259 deg. fahr.; abs. pressure at exhaust = 1 in. Hg. Find (*a*) the ideal steam rate, (*b*) the ideal cycle efficiency, (*c*) the combined steam rate, (*d*) the combined thermal efficiency, and (*e*) the combined engine efficiency. (*f*) What effect does bleeding have on the combined thermal efficiency and also on the combined engine efficiency? Discuss.

Ans. (*a*) 7.7 lb. per kw-hr. (*b*) 34.2 per cent. (*c*) 10.0 lb. per kw-hr. (*d*) 26.3 per cent. (*e*) 76.9 per cent.

5. A turbine-generator rated at 30,000 kw. is arranged for bleeding at four points during the expansion of the steam. When the generator is delivering 24,725 kw., 267,500 lb. of steam per hour are required at a throttle pressure of 380 lb. per sq. in. abs. and at a temperature of 700° F., the back pressure being 1.5 in. Hg abs. The steam pressure at the bleeding nozzles are respectively 90, 46.4, 19.6, and 6.1 lb. per sq. in. abs. Sufficient steam is bled to make the temperature of the feedwater leaving the high pressure heater equal to 310° F. For the *corresponding ideal unit*, find (*a*) the per cent of the throttle steam that is bled at each point in the expansion, (*b*) the net work from each pound of throttle steam, (*c*) the ideal steam rate, and (*d*) the cycle efficiency. For the *actual unit*, find (*e*) the combined steam rate, (*f*) the combined thermal efficiency, and (*g*) the combined engine efficiency.

Ans. (*a*) 4.7, 4.95, 5.4, 6.95. (*b*) 409.8 B.t.u. (*c*) 8.32 lb. per kw-hr. (*d*) 38.4 per cent. (*e*) 10.8 lb. per kw-hr. (*f*) 29.3 per cent. (*g*) 76.2 per cent.

6. Suppose the reheating unit described in Problem 2 has incorporated in it three regenerative heaters one of which takes steam at the point where it leaves the turbine for reheating. At the rated load, it may be assumed that the condition of the steam at the turbine after reheating and also the exhaust pressure are the same as in Problem 2. At this load, the absolute pressures at the last two bleeding nozzles are respectively 25 and 5 lb. per sq. in. The combined steam rate is now 9.0 lb. per kw-hr. and the weight of steam bled before reheating is 8.8 per cent of the throttle flow. This results in the feedwater leaving the last heater at an actual temperature of 330° F. Find, for the corresponding *ideal unit* (*a*) the portion of the throttle steam that is bled to each of the three heaters, (*b*) the ideal steam rate, and (*c*) the cycle efficiency. For the *actual unit*, find (*d*) the combined thermal efficiency and (*e*) the combined engine efficiency.

Ans. (*a*) 11.28, 6.48, and 6.57 per cent. (*b*) 6.96 lb. per kw-hr. (*c*) 41.75 per cent. (*d*) 32.2 per cent. (*e*) 77.1 per cent.

CHAPTER XIX

Note.—In all the problems of this chapter the valve events are designated by the per cent of stroke measured from the admission end of the cylinder.

1. An 8″ × 10″ double acting steam engine running 150 r.p.m. has a clearance of 7 per cent and the compression begins 25 per cent before the end of the stroke. If the pressure at the start of compression is 6 lb. per sq. in. abs., find the weight of cushion steam. Ans. 0.0015 lb.

2. A 16″ × 24″ double acting steam engine running 120 r.p.m. has a clearance of 4 per cent and cut-off occurs at 15 per cent of the stroke. Find the quality of the steam at cut-off if the pressure at that point is 118 lb. per sq. in. abs., and if the weight of steam present during expansion is 0.21 lb. Ans. 66.4 per cent.

3. An 8″ × 10″ double acting steam engine running 300 r.p.m. with cut-off taking place at 15 per cent of the stroke, and at a pressure of 120 lb. per sq. in. abs., requires 35 lb. of steam per i.hp-hr. when supplied with dry saturated steam. Compression begins 40 per cent before the end of the stroke at a pressure of 5 lb. per sq. in. abs. The clearance is 10 per cent. If this engine delivers 27 hp. and has a mechanical efficiency of 90 per cent, find the quality of the steam at cut-off.

Ans. 62.7 per cent.

4. A 12″ × 24″ single cylinder double-acting steam engine is tested when delivering 120 hp. The speed of the engine is 150 r.p.m. The clearance is 6 per cent, cut-off occurs at 47 per cent of the stroke and compression begins 15 per cent before the end of the stroke. (*a*) If the absolute pressure at cut-off is 109 lb. per sq. in., the quality at cut-off is 94 per cent and the absolute pressure at the beginning of compression is 15 lb. per sq. in., find the brake steam rate of the engine. (*b*) In an engine test why cannot this method of finding the steam rate ordinarily be used?

Ans. (*a*) 30.7 lb. per b.hp-hr.

5. A four cylinder, double acting, 14″ × 16″ uniflow engine gives by test the following data: absolute pressure of saturated steam at throttle = 140 lb. per sq. in., weight of steam = 15,200 lb. per hour, speed = 300 r.p.m., indicated hp. = 650, absolute back pressure = 20 lb. per sq. in., per cent clearance = 22, per cent release = per cent compression = 89, per cent cut-off = 17.5, absolute pressure at compression = 20.4 lb. per sq. in., absolute pressure at cut-off = 130 lb. per sq. in., absolute pressure at release = 47.6 lb. per sq. in., and during the expansion period the absolute pressures at 30, 50 and 70 per cent of the stroke are, respectively, 100, 74 and 58 lb. per sq. in. Find (*a*) the weight of cushion steam, (*b*) the weight of cylinder feed, (*c*) the quality at cut-off, release, and at the three intermediate points. (*d*) Plot the quality curve using the per cent of piston displacement as the abscissa.

Ans. (*a*) 0.08 lb. (*b*) 0.1054 lb. (*c*) 87.9, 95.5, 90.1, 93.6, 95.2 per cent.

6. Give the chief advantage and the chief disadvantage of compounding the steam engine.

7. What effect does superheated steam have on the rate of heat transfer between the steam and cylinder walls? Why?

8. What is the purpose of modifying the uniflow engine by the introduction of auxiliary exhaust valves?

9. How does the floor space of a uniflow engine compare with (*a*) counterflow simple engine of the same capacity? (*b*) a counterflow cross-compound engine of the same capacity?

10. Why are condensing uniflow engines often equipped with automatically controlled or hand controlled auxiliary clearance volumes?

11. It is desired to build a single cylinder, double acting, simple engine, to develop 75 i.hp. when supplied with steam at an absolute pressure of 140 lb. per sq. in. and operating with an absolute back pressure of 2 lb. per sq. in. The speed is 200 r.p.m. and the per cent cut-off at rated load is 20. Assuming a diagram factor of 0.85 and a ratio of l/d of about 1.25, find the size of the engine.

Ans. $10\frac{3}{4}'' \times 13\frac{1}{2}''$.

12. If the engine described in Problem 11 is governed by throttling the steam without changing the point of cut-off, find the pressure to which the steam must be throttled if the load is reduced to 40 i.hp., assuming that the other factors are unchanged.

Ans. 76.4 lb. per sq. in. abs.

13. It is desired to build a compound engine with its low- and high-pressure cylinders double acting. This engine is to develop 600 i.hp. when using steam having an absolute pressure of 150 lb. per sq. in. and an absolute back pressure of 2 lb. per sq. in. The speed of the engine is 150 r.p.m. and the piston speed in each cylinder is 750 ft. per min. If the cylinder ratio is to be 4 and the total ratio of expansion 12, find the size of the engine assuming a diagram factor of 0.8. Ans. 16″, 32″ × 30″.

APPENDIX.

USE OF COMMON LOGARITHMS FOR SPECIAL CASES.

CASE I. TO DETERMINE THE nTH POWER OF A NUMBER LESS THAN UNITY.

Example: Find $0.5^{1.55}$ by logs.

In general $\log_{10} V^n = n \log_{10} V$; and in this case $V = 0.5$ and $n = 1.55$.

From the tables $\log_{10} 0.5 = 9.6990 - 10$,

Then, $1.55 \log_{10} 0.5 = 1.55\,(9.6990 - 10) = 15.033450 - 15.5$

Subtract 5.5 to reduce the negative part of the characteristic to 10, $\underline{5.5 \qquad - \;\; 5.5}$

$$\text{Log. of answer} = 9.533450 - 10.0$$

$$\text{Corresponding number} = 0.342 = 0.5^{1.55}.$$

(Note that a fraction raised to a power greater than unity gives a result less than the original fraction.)

CASE II. TO DETERMINE THE nTH ROOT OF A FRACTION.

Example: Given $V^{1.5} = 0.75$; Find V. Evidently, —

$$\log_{10} V = \log_{10} \left(\sqrt[1.5]{0.75}\right) = \log_{10} \left(0.75^{\frac{1}{1.5}}\right) = (\log_{10} 0.75) \div 1.5,$$

which is in the general form of $\log_{10} V = (\log_{10} C) \div n$,

where $C = 0.75$ and $n = 1.5$.

From the tables $\log_{10} 0.75 = 9.8751 - 10$.

Then $$\frac{(\log 0.75)}{1.5} = \frac{(9.8751 - 10)}{1.5} = 6.5833 - 6.666.$$

Add 3.334 to raise the negative part of the characteristic to 10, $\underline{3.3340 - \;\; 3.334}$

$$\text{Log. of } V = 9.9173 - 10.000$$

The corresponding number is 0.8266 which is $\sqrt[1.5]{0.75}$.

TABLE A. — COMMON LOGARITHMS ($\log_{10}$).

No.	0	1	2	3	4	5	6	7	8	9	Diff.
0	0	0000	3010	4771	6021	6990	7782	8451	9031	9542	
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	42
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	38
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	35
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	32
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	30
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	28
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	26
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	25
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	24
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	22
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	21
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	20
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	19
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	19
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	18
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	17
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	16
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	16
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	15
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	15
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	14
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	14
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	13
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	13
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	13
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	12
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	12
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	12
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	11
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	11
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	11
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	10
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	10
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	10
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	10
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	10
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	9
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	9
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	9
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	9
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	8
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	8
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	8

$e = 2.71828$

TABLE A. (*Concluded*). — COMMON LOGARITHMS (Log_{10}).

No.	0	1	2	3	4	5	6	7	8	9	Diff.
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	8
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	8
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	8
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	7
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	7
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	7
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	7
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	7
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	7
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	7
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	6
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	6
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	6
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	6
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	6
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	6
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	6
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	6
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	5
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	5
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	5
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	5
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	5
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	5
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	5
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	5
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	5
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	4

Napierian $\log_e = 2.302 \log_{10}$.

TABLE B.—NAPIERIAN LOGARITHMS (log$_e$).

(Also Called Hyperbolic or Natural Logarithms)

No.	0	1	2	3	4	5	6	7	8	9	Avg. Diff.
1.0	0.0000	0100	0198	0296	0392	0488	0583	0677	0770	0862	95
1.1	0953	1044	1133	1222	1310	1398	1484	1570	1655	1740	87
1.2	1823	1906	1989	2070	2151	2231	2311	2390	2469	2546	80
1.3	2624	2700	2776	2852	2927	3001	3075	3148	3221	3293	74
1.4	3365	3436	3507	3577	3646	3716	3784	3853	3920	3988	69
1.5	0.4055	4121	4187	4253	4318	4383	4447	4511	4574	4637	65
1.6	4700	4762	4824	4886	4947	5008	5068	5128	5188	5247	61
1.7	5306	5365	5423	5481	5539	5596	5653	5710	5766	5822	57
1.8	5878	5933	5988	6043	6098	6152	6206	6259	6313	6366	54
1.9	6419	6471	6523	6575	6627	6678	6729	6780	6831	6881	51
2.0	0.6931	6981	7031	7080	7129	7178	7227	7275	7324	7372	49
2.1	7419	7467	7514	7561	7608	7655	7701	7747	7793	7839	47
2.2	7885	7930	7975	8020	8065	8109	8154	8198	8242	8286	44
2.3	8329	8372	8416	8459	8502	8544	8587	8629	8671	8713	43
2.4	8755	8796	8838	8879	8920	8961	9002	9042	9083	9123	41
2.5	0.9163	9203	9243	9282	9322	9361	9400	9439	9478	9517	39
2.6	9555	9594	9632	9670	9708	9746	9783	9821	9858	9895	38
2.7	9933	9969	1.0006	1.0043	1.0080	1.0116	1.0152	1.0188	1.0225	1.0260	36
2.8	1.0296	0332	0367	0403	0438	0473	0508	0543	0578	0613	35
2.9	0647	0682	0716	0750	0784	0818	0852	0886	0919	0953	34
3.0	1.0986	1019	1053	1086	1119	1151	1184	1217	1249	1282	33
3.1	1314	1346	1378	1410	1442	1474	1506	1537	1569	1600	32
3.2	1632	1663	1694	1725	1756	1787	1817	1848	1878	1909	31
3.3	1939	1969	2000	2030	2060	2090	2119	2149	2179	2208	30
3.4	2238	2267	2296	2326	2355	2384	2413	2442	2470	2499	29
3.5	1.2528	2556	2585	2613	2641	2669	2698	2726	2754	2782	28
3.6	2809	2837	2865	2892	2920	2947	2975	3002	3029	3056	27
3.7	3083	3110	3137	3164	3191	3218	3244	3271	3297	3324	27
3.8	3350	3376	3403	3429	3455	3481	3507	3533	3558	3584	26
3.9	3610	3635	3661	3686	3712	3737	3762	3788	3813	3838	25
4.0	1.3863	3888	3913	3938	3962	3987	4012	4036	4061	4085	25
4.1	4110	4134	4159	4183	4207	4231	4255	4279	4303	4327	24
4.2	4351	4375	4398	4422	4446	4469	4493	4516	4540	4563	23
4.3	4586	4609	4633	4656	4679	4702	4725	4748	4770	4793	23
4.4	4816	4839	4861	4884	4907	4929	4951	4974	4996	5019	22
4.5	1.5041	5063	5085	5107	5129	5151	5173	5195	5217	5239	22
4.6	5261	5282	5304	5326	5347	5369	5390	5412	5433	5454	21
4.7	5476	5497	5518	5539	5560	5581	5602	5623	5644	5665	21
4.8	5686	5707	5728	5748	5769	5790	5810	5831	5851	5872	20
4.9	5892	5913	5933	5953	5974	5994	6014	6034	6054	6074	20
5.0	1.6094	6114	6134	6154	6174	6194	6214	6233	6253	6273	20
5.1	6292	6312	6332	6351	6371	6390	6409	6429	6448	6467	19
5.2	6487	6506	6525	6544	6563	6582	6601	6620	6639	6658	19
5.3	6677	6696	6715	6734	6752	6771	6790	6808	6827	6845	18
5.4	6864	6882	6901	6919	6938	6956	6974	6993	7011	7029	18
5.5	1.7047	7066	7084	7102	7120	7138	7156	7174	7192	7210	18
5.6	7228	7246	7263	7281	7299	7317	7334	7352	7370	7387	18
5.7	7405	7422	7440	7457	7475	7492	7509	7527	7544	7561	17
5.8	7579	7596	7613	7630	7647	7664	7681	7699	7716	7733	17
5.9	7750	7766	7783	7800	7817	7834	7851	7867	7884	7901	17

TABLE B. (*Concluded*).—NAPIERIAN LOGARITHMS (Log_e).

No.	0	1	2	3	4	5	6	7	8	9	Avg. Diff.
6.0	1.7918	7934	7951	7967	7984	8001	8017	8034	8050	8066	16
6.1	8083	8099	8116	8132	8148	8165	8181	8197	8213	8229	16
6.2	8245	8262	8278	8294	8310	8326	8342	8358	8374	8390	16
6.3	8405	8421	8437	8453	8469	8485	8500	8516	8532	8547	16
6.4	8563	8579	8594	8610	8625	8641	8656	8672	8687	8703	15
6.5	1.8718	8733	8749	8764	8779	8795	8810	8825	8840	8856	15
6.6	8871	8886	8901	8916	8931	8946	8961	8976	8991	9006	15
6.7	9021	9036	9051	9066	9081	9095	9110	9125	9140	9155	15
6.8	9169	9184	9199	9213	9228	9242	9257	9272	9286	9301	15
6.9	9315	9330	9344	9359	9373	9387	9402	9416	9430	9445	14
7.0	1.9459	9473	9488	9502	9516	9530	9544	9559	9573	9587	14
7.1	9601	9615	9629	9643	9657	9671	9685	9699	9713	9727	14
7.2	9741	9755	9769	9782	9796	9810	9824	9838	9851	9865	14
7.3	9879	9892	9906	9920	9933	9947	9961	9974	9988	2.0001	13
7.4	2.0015	0028	0042	0055	0069	0082	0096	0109	0122	0136	13
7.5	2.0149	0162	0176	0189	0202	0215	0229	0242	0255	0268	13
7.6	0281	0295	0308	0321	0334	0347	0360	0373	0386	0399	13
7.7	0412	0425	0438	0451	0464	0477	0490	0503	0516	0528	13
7.8	0541	0554	0567	0580	0592	0605	0618	0631	0643	0656	13
7.9	0669	0681	0694	0707	0719	0732	0744	0757	0769	0782	12
8.0	2.0794	0807	0819	0832	0844	0857	0869	0882	0894	0906	12
8.1	0919	0931	0943	0956	0968	0980	0992	1005	1017	1029	12
8.2	1041	1054	1066	1078	1090	1102	1114	1126	1138	1150	12
8.3	1163	1175	1187	1199	1211	1223	1235	1247	1258	1270	12
8.4	1282	1294	1306	1318	1330	1342	1353	1365	1377	1389	12
8.5	2.1401	1412	1424	1436	1448	1459	1471	1483	1494	1506	12
8.6	1518	1529	1541	1552	1564	1576	1587	1599	1610	1622	12
8.7	1633	1645	1656	1668	1679	1691	1702	1713	1725	1736	11
8.8	1748	1759	1770	1782	1793	1804	1815	1827	1838	1849	11
8.9	1861	1872	1883	1894	1905	1917	1928	1939	1950	1961	11
9.0	2.1972	1983	1994	2006	2017	2028	2039	2050	2061	2072	11
9.1	2083	2094	2105	2116	2127	2138	2148	2159	2170	2181	11
9.2	2192	2203	2214	2225	2235	2246	2257	2268	2279	2289	11
9.3	2300	2311	2322	2332	2343	2354	2364	2375	2386	2396	11
9.4	2407	2418	2428	2439	2450	2460	2471	2481	2492	2502	11
9.5	2.2513	2523	2534	2544	2555	2565	2576	2586	2597	2607	10
9.6	2618	2628	2638	2649	2659	2670	2680	2690	2701	2711	10
9.7	2721	2732	2742	2752	2762	2773	2783	2793	2803	2814	10
9.8	2824	2834	2844	2854	2865	2875	2885	2895	2905	2915	10
9.9	2925	2935	2946	2956	2966	2976	2986	2996	3006	3016	10
10.0	2.3026	3125	3224	3322	3418	3513	3609	3702	3795	3887	9.5

$\log_{10} a = 0.4343 \log_e a$; $\log_e a = 2.3026 \log_{10} a$; $\log_e 0.1 = 0.6974 - 3$.

Examples: $\log_e 2150 = \log_e 2.15 + 3 \times \log_e 10 = 0.7655 + 3 \times 2.3026$
$\log_e 0.00215 = \log_e 2.15 + 3 \times \log_e 0.1 = 0.7655 + 3(0.6974 - 3)$

TABLE C.—PROPERTIES OF SATURATED STEAM

Abridged and rearranged from Marks and Davis's Steam Tables and Diagrams, 1909, by permission of the publishers, Longmans, Green & Co., with addition of column as indicated by asterisk (*). The first eleven lines constitute a table based on temperatures which are so selected as to give (approximately) intervals of half an inch of Hg in column 1, without using fractional degrees. The rest of the table is based on absolute pressures.

ABS. PRESSURE		TEMPERATURE	HEAT CONTENT, B.T.U. PER LB.		VOLUME, CU. FT. PER LB.		ENTROPY, B.T.U. PER DEG. FAHR.			LATENT HEAT, B.T.U. PER LB.			ABS. PRESS.
In. Hg.	Lb. per Sq. In.	Deg. Fahr.	Sat. Vapor	Sat. Liquid	Sat. Vapor	Sat. Liquid	Sat. Vapor	Sat. Liquid	Vaporization	Total	Internal	External	Lb. per Sq. In.
Saturated Liquid and Vapor													
p''	p	t_{sat}	h_{sat}	h_l	$\overline{V}_{sat}$	$\overline{V}_l$	ϕ_{sat}	ϕ_l	ϕ_v	r	ρ	APu *	p
0.1804	0.0886	32	1073.4	0.0	3294.0	0.01602	2.1832	0.0000	2.1832	1073.4	1019.3	54.1	0.0886
0.50+	0.2472	59	1085.4	27.08	1249.2	0.01603	2.0944	0.0536	2.0408	1058.3	1001.3	57.0	0.2472
1.00−	0.4893	79	1094.3	47.04	657.0	0.01607	2.0358	0.0913	1.9445	1047.3	988.1	59.2	0.4893
1.50+	0.741	92	1100.1	60.0	442.2	0.01610	2.0007	0.1151	1.8856	1040.0	979.4	60.6	0.741
2.00−	0.975	101	1104.0	68.97	341.0	0.01613	1.9770	0.1313	1.8463	1035.1	973.5	61.6	0.975
2.50+	1.235	109	1107.5	76.94	272.9	0.01616	1.9678	0.1454	1.8124	1030.6	968.2	62.4	1.235
3.00−	1.467	115	1110.2	82.92	231.9	0.01618	1.9435	0.1559	1.7876	1027.2	964.2	63.0	1.467
3.50+	1.736	121	1112.8	88.91	197.9	0.01620	1.9296	0.1662	1.7634	1023.9	960.2	63.7	1.736
4.00+	1.992	126	1115.0	93.90	173.9	0.01623	1.9183	0.1747	1.7436	1021.1	956.8	64.3	1.992
4.50+	2.219	130	1116.7	97.89	157.1	0.01625	1.9095	0.1816	1.7279	1018.8	954.1	64.7	2.219
5.00+	2.467	134	1118.4	101.88	142.2	0.01627	1.9008	0.1883	1.7125	1016.5	951.4	65.1	2.467
	1	101.83	1104.4	69.8	333.0	0.01613	1.9754	0.1327	1.8427	1034.6	972.9	61.7	1
	2	126.15	1115.0	94.0	173.5	0.01623	1.9180	0.1749	1.7431	1021.0	956.7	64.3	2
	3	141.52	1121.6	109.4	118.5	0.01630	1.8848	0.2008	1.6840	1012.3	946.4	65.9	3
	4	153.01	1126.5	120.9	90.5	0.01635	1.8614	0.2198	1.6416	1005.7	938.6	67.1	4

5	162.28	1130.5	130.1	73.33	0.01641	1.8432	0.2348	1.6084	1000.3	932.4	67.9	5
6	170.06	1133.7	137.9	61.89	0.01645	1.8285	0.2471	1.5814	995.8	927.0	68.8	6
7	176.85	1136.5	144.7	53.56	0.01648	1.8161	0.2579	1.5582	991.8	922.4	69.4	7
8	182.86	1139.0	150.8	47.27	0.01652	1.8053	0.2673	1.5380	988.2	918.2	70.0	8
9	188.27	1141.1	156.2	42.36	0.01656	1.7958	0.2756	1.5202	985.0	914.4	70.6	9
10	193.22	1143.1	161.1	38.38	0.01660	1.7874	0.2832	1.5042	982.0	910.9	71.1	10
11	197.75	1144.9	165.7	35.10	0.01662	1.7797	0.2902	1.4895	979.2	907.8	71.4	11
12	201.96	1146.5	169.9	32.36	0.01664	1.7727	0.2967	1.4760	976.6	904.8	71.8	12
13	205.87	1148.0	173.8	30.03	0.01667	1.7664	0.3025	1.4639	974.2	902.0	72.2	13
14	209.55	1149.4	177.5	28.02	0.01670	1.7604	0.3081	1.4523	971.9	899.3	72.6	14
14.7	212.00	1150.4	180.0	26.79	0.01672	1.7565	0.3118	1.4447	970.4	897.6	72.8	14.7
15	213.0	1150.7	181.0	26.27	0.01672	1.7549	0.3133	1.4416	969.7	896.8	72.9	15
16	216.3	1152.0	184.4	24.79	0.01674	1.7494	0.3183	1.4311	967.6	894.4	73.2	16
17	219.4	1153.1	187.5	23.38	0.01677	1.7444	0.3229	1.4215	965.6	892.1	73.5	17
18	222.4	1154.2	190.5	22.16	0.01679	1.7400	0.3273	1.4127	963.7	889.9	73.8	18
19	225.2	1155.2	193.4	21.07	0.01681	1.7360	0.3315	1.4045	961.8	887.8	74.0	19
20	228.0	1156.2	196.1	20.08	0.01683	1.7320	0.3355	1.3965	960.0	885.8	74.2	20
21	230.6	1157.1	198.8	19.18	0.01685	1.7280	0.3393	1.3887	958.3	883.9	74.4	21
22	233.1	1158.0	201.3	18.37	0.01687	1.7241	0.3430	1.3811	956.7	882.0	74.7	22
23	235.5	1158.8	203.8	17.62	0.01689	1.7204	0.3465	1.3739	955.1	880.2	74.9	23
24	237.8	1159.6	206.1	16.93	0.01690	1.7169	0.3499	1.3670	953.5	878.5	75.0	24
25	240.1	1160.4	208.4	16.30	0.01692	1.7136	0.3532	1.3604	952.0	876.8	75.2	25
26	242.2	1161.2	210.6	15.72	0.01694	1.7106	0.3564	1.3542	950.6	875.1	75.5	26
27	244.4	1161.9	212.7	15.18	0.01696	1.7077	0.3594	1.3483	949.2	873.5	75.7	27
28	246.4	1162.6	214.8	14.67	0.01697	1.7048	0.3623	1.3425	947.8	872.0	75.8	28
29	248.4	1163.2	216.8	14.19	0.01698	1.7019	0.3652	1.3367	946.4	870.5	75.9	29
30	250.3	1163.9	218.8	13.74	0.01701	1.6991	0.3680	1.3311	945.1	869.0	76.1	30
32	254.1	1165.1	222.6	12.93	0.01702	1.6938	0.3733	1.3205	942.5	866.2	76.3	32
34	257.6	1166.3	226.2	12.22	0.01703	1.6891	0.3784	1.3107	940.1	863.4	76.7	34
36	261.0	1167.3	229.6	11.58	0.01705	1.6846	0.3832	1.3014	937.7	860.8	76.9	36
38	264.2	1168.4	232.9	11.01	0.01708	1.6802	0.3877	1.2925	935.5	858.3	77.2	38

TABLE C.—PROPERTIES OF SATURATED STEAM—(*Continued*)

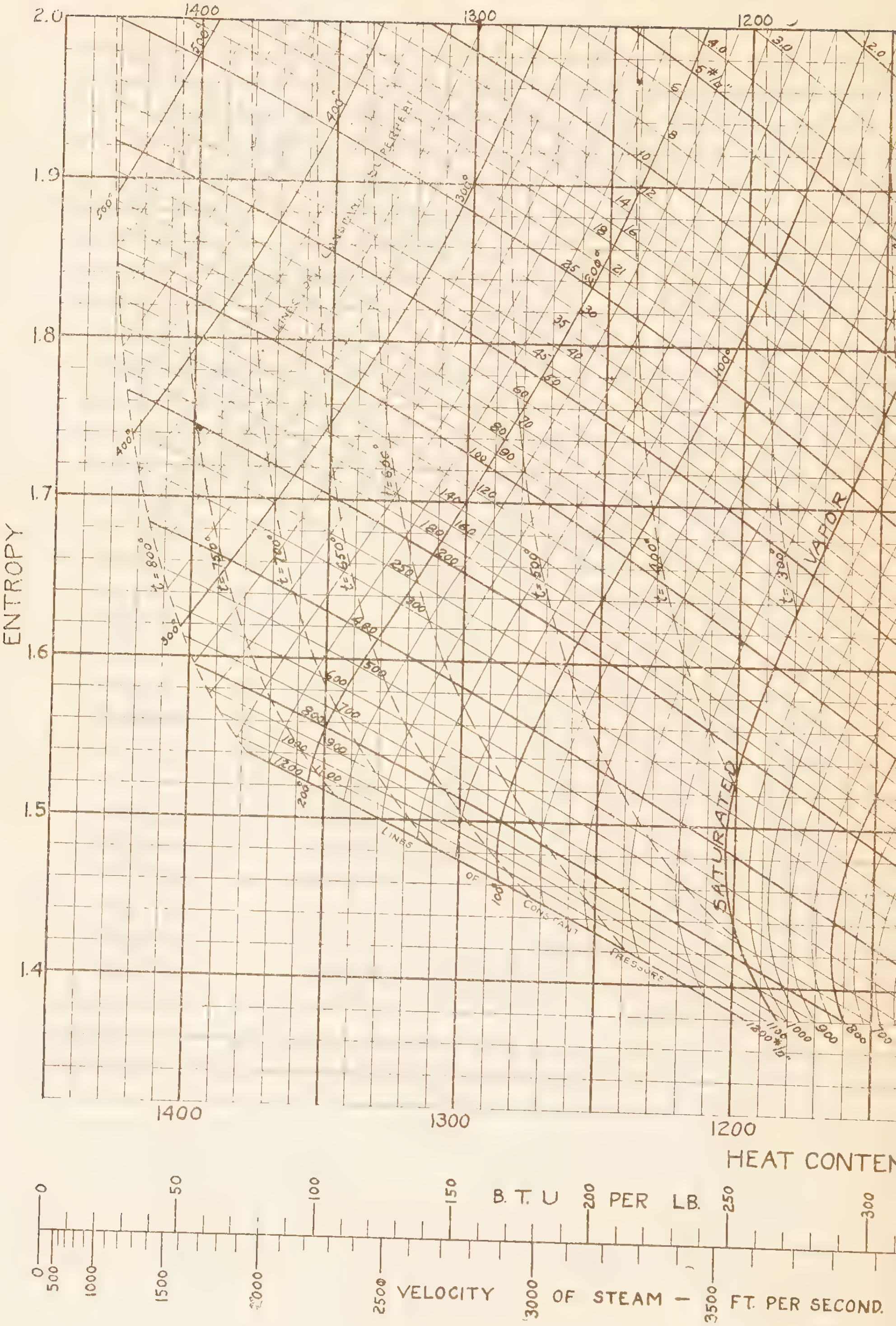
ABS. PRESS. Lb. per Sq. In.	TEMPERATURE Deg. Fahr.	HEAT CONTENT, B.T.U. PER LB.		VOLUME, CU. FT. PER LB.		ENTROPY, B.T.U. PER DEG. FAHR.			LATENT HEAT, B.T.U. PER LB.			ABS. PRESS.
Saturated Liquid and Vapor		Sat. Vapor	Sat. Liquid	Sat. Vapor	Sat. Liquid	Sat. Vapor	Sat. Liquid	Vaporization	Total	Internal	External	Lb. per Sq. In.
p	t_{sat}	h_{sat}	h_l	$\overline{V}_{sat}$	$\overline{V}_l$	ϕ_{sat}	ϕ_l	ϕ_r	r	ρ	APu *	p
40	267.3	1169.4	236.1	10.49	0.01712	1.6761	0.3920	1.2841	933.3	855.9	77.4	40
42	270.2	1170.3	239.1	10.02	0.01716	1.6721	0.3962	1.2759	931.2	853.6	77.6	42
44	273.1	1171.2	242.0	9.59	0.01719	1.6683	0.4002	1.2681	929.2	851.3	77.9	44
46	275.8	1172.0	244.8	9.20	0.01721	1.6647	0.4040	1.2607	927.2	849.2	78.0	46
48	278.5	1172.8	247.5	8.84	0.01723	1.6613	0.4077	1.2536	925.3	847.1	78.2	48
50	281.0	1173.6	250.1	8.51	0.01725	1.6581	0.4113	1.2468	923.5	845.0	78.5	50
52	283.5	1174.3	252.6	8.20	0.01728	1.6549	0.4147	1.2402	921.7	843.1	78.6	52
54	285.9	1175.0	255.1	7.91	0.01730	1.6519	0.4180	1.2339	919.9	841.1	78.8	54
56	288.2	1175.7	257.5	7.65	0.01732	1.6490	0.4212	1.2278	918.2	839.3	78.9	56
58	290.5	1176.4	259.8	7.40	0.01735	1.6460	0.4242	1.2218	916.5	837.4	79.1	58
60	292.7	1177.0	262.1	7.17	0.01738	1.6432	0.4272	1.2160	914.9	835.6	79.3	60
62	294.9	1177.6	264.3	6.95	0.01740	1.6406	0.4302	1.2104	913.3	833.9	79.4	62
64	297.0	1178.2	266.4	6.75	0.01741	1.6380	0.4330	1.2050	911.8	832.2	79.6	64
66	299.0	1178.8	268.5	6.56	0.01743	1.6355	0.4358	1.1998	910.2	830.5	79.7	66
68	301.0	1179.3	270.6	6.38	0.01745	1.6331	0.4385	1.1946	908.7	828.9	79.8	68
70	302.9	1179.8	272.6	6.20	0.01747	1.6307	0.4411	1.1896	907.2	827.3	79.9	70
72	304.8	1180.4	274.5	6.04	0.01749	1.6285	0.4437	1.1848	905.8	825.8	80.0	72
74	306.7	1180.9	276.5	5.89	0.01751	1.6263	0.4462	1.1801	904.4	824.2	80.2	74
76	308.5	1181.4	278.3	5.74	0.01753	1.6242	0.4487	1.1755	903.0	822.7	80.3	76
78	310.3	1181.8	280.2	5.60	0.01754	1.6221	0.4511	1.1710	901.7	821.3	80.4	78

80	312.0	1182.3	282.0	5.47	0.01756	1.6200	0.4535	1.1665	900.3	819.8	80.5	80
82	313.8	1182.8	283.8	5.34	0.01758	1.6180	0.4557	1.1623	899.0	818.4	80.6	82
84	315.4	1183.2	285.5	5.22	0.01760	1.6160	0.4579	1.1581	897.7	817.0	80.7	84
86	317.1	1183.6	287.2	5.10	0.01761	1.6141	0.4601	1.1540	896.4	815.6	80.8	86
88	318.7	1184.0	288.9	5.00	0.01762	1.6123	0.4623	1.1500	895.2	814.3	80.9	88
90	320.3	1184.4	290.5	4.89	0.01765	1.6105	0.4644	1.1461	893.9	813.0	80.9	90
92	321.8	1184.8	292.1	4.79	0.01767	1.6087	0.4664	1.1423	892.7	811.7	81.0	92
94	323.4	1185.2	293.7	4.69	0.01768	1.6069	0.4684	1.1385	891.5	810.4	81.1	94
96	324.9	1185.6	295.3	4.60	0.01770	1.6052	0.4704	1.1348	890.3	809.1	81.2	96
98	326.4	1186.0	296.8	4.51	0.01771	1.6036	0.4724	1.1312	889.2	807.9	81.3	98
100	327.8	1186.3	298.3	4.429	0.01773	1.6020	0.4743	1.1277	888.0	806.6	81.4	100
105	331.4	1187.2	302.0	4.230	0.01778	1.5980	0.4789	1.1191	885.2	803.6	81.6	105
110	334.8	1188.0	305.5	4.047	0.01781	1.5942	0.4834	1.1108	882.5	800.7	81.8	110
115	338.1	1188.8	309.0	3.880	0.01786	1.5907	0.4877	1.1030	879.8	797.9	81.9	115
120	341.3	1189.6	312.3	3.726	0.01790	1.5873	0.4919	1.0954	877.2	795.2	82.0	120
125	344.4	1190.3	315.5	3.583	0.01793	1.5839	0.4959	1.0880	874.7	792.6	82.1	125
130	347.4	1191.0	318.6	3.452	0.01797	1.5807	0.4998	1.0809	872.3	790.0	82.3	130
135	350.3	1191.6	321.7	3.331	0.01800	1.5777	0.5035	1.0742	869.9	787.5	82.4	135
140	353.1	1192.2	324.6	3.219	0.01803	1.5747	0.5072	1.0675	867.6	785.0	82.6	140
145	355.8	1192.8	327.4	3.112	0.01806	1.5719	0.5107	1.0612	865.4	782.7	82.7	145
150	358.5	1193.4	330.2	3.012	0.01809	1.5692	0.5142	1.0550	863.2	780.4	82.8	150
155	361.0	1194.0	332.9	2.920	0.01813	1.5664	0.5175	1.0489	861.0	778.1	82.9	155
160	363.6	1194.5	335.6	2.834	0.01817	1.5639	0.5208	1.0431	858.8	775.8	83.0	160
165	366.0	1195.0	338.2	2.753	0.01820	1.5615	0.5239	1.0376	856.8	773.6	83.2	165
170	368.5	1195.4	340.7	2.675	0.01823	1.5590	0.5269	1.0321	854.7	771.5	83.2	170
175	370.8	1195.9	343.2	2.602	0.01826	1.5567	0.5299	1.0268	852.7	769.4	83.3	175
180	373.1	1196.4	345.6	2.533	0.01829	1.5543	0.5328	1.0215	850.8	767.4	83.4	180
185	375.4	1196.8	348.0	2.468	0.01833	1.5520	0.5356	1.0164	848.8	765.4	83.4	185
190	377.6	1197.3	350.4	2.406	0.01836	1.5498	0.5384	1.0114	846.9	763.4	83.5	190
195	379.8	1197.7	352.7	2.346	0.01839	1.5476	0.5410	1.0066	845.0	761.4	83.6	195

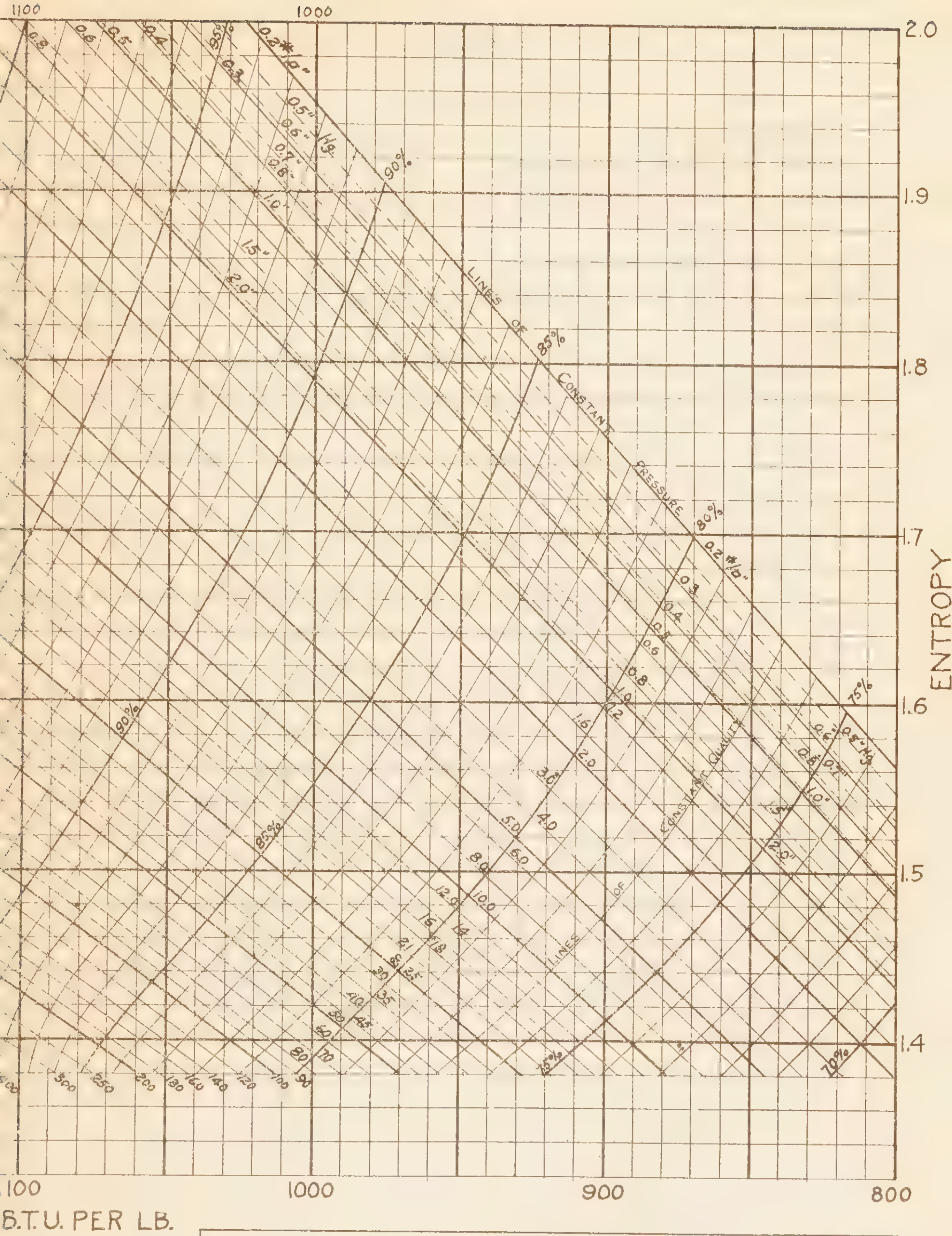
TABLE C.—PROPERTIES OF SATURATED STEAM—(*Continued*)

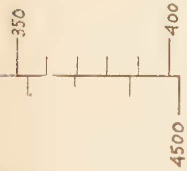
ABS. PRESS.	TEMPERATURE	HEAT CONTENT, B.T.U. PER LB.		VOLUME, CU. FT. PER LB.		ENTROPY, B.T.U. PER DEG. FAHR.			LATENT HEAT, B.T.U. PER LB.			ABS. PRESS.
Lb. per Sq. In.	Deg. Fahr.											
Saturated Liquid and Vapor		Sat. Vapor	Sat. Liquid	Sat. Vapor	Sat. Liquid	Sat. Vapor	Sat. Liquid	Vaporization	Total	Internal	External	Lb. per Sq. In.
p	t_{sat}	h_{sat}	h_l	$\overline{V}_{sat}$	$\overline{V}_l$	ϕ_{sat}	ϕ_l	ϕ_v	r	ρ	APu*	p
200	381.9	1198.1	354.9	2.290	0.01842	1.5456	0.5437	1.0019	843.2	759.5	83.7	200
210	386.0	1198.8	359.2	2.187	0.01848	1.5416	0.5488	0.9928	839.6	755.8	83.8	210
220	389.9	1199.6	363.4	2.091	0.01854	1.5379	0.5538	0.9841	836.2	752.3	83.9	220
230	393.8	1200.2	367.5	2.004	0.01860	1.5344	0.5586	0.9758	832.8	748.8	84.0	230
240	397.4	1200.9	371.4	1.924	0.01866	1.5309	0.5633	0.9676	829.5	745.4	84.1	240
250	401.1	1201.5	375.2	1.850	0.01872	1.5276	0.5676	0.9600	826.3	742.0	84.3	250
260	404.5	1202.1	378.9	1.782	0.01878	1.5244	0.5719	0.9525	823.1	738.9	84.3	260
270	407.9	1202.6	382.5	1.718	0.01884	1.5214	0.5760	0.9454	820.1	735.8	84.3	270
280	411.2	1203.1	386.0	1.658	0.0189	1.5185	0.5800	0.9385	817.1	732.7	84.4	280
290	414.4	1203.6	389.4	1.602	0.0190	1.5156	0.5840	0.9316	814.2	729.7	84.5	290
300	417.5	1204.1	392.7	1.551	0.0191	1.5129	0.5878	0.9251	811.3	726.8	84.5	300
320	423.4	1204.9	399.1	1.456	0.0191	1.5076	0.5951	0.9125	805.8	721.2	84.6	320
340	429.1	1205.7	405.3	1.372	0.0192	1.5026	0.6020	0.9006	800.4	715.9	84.6	340
360	434.6	1206.4	411.2	1.298	0.0193	1.4979	0.6085	0.8894	795.3	710.7	84.6	360
380	439.8	1207.1	416.8	1.231	0.0194	1.4935	0.6147	0.8788	790.3	705.7	84.6	380
400	444.8	1208.0	422.0	1.17	0.0194	1.489	0.621	0.868	786.0	701.0	85.0	400

For pressures higher than 400 see TABLE D.


ENTROPY
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1400
1300
1200
HEAT CONTEN
LINES OF CONSTANT SUPERHEAT
LINES OF CONSTANT PRESSURE
SATURATED
VAPOR
t = 800°
t = 750°
t = 700°
t = 650°
t = 600°
t = 500°
t = 400°
t = 300°
500°
400°
300°
200°
100°
5#/□"
4.0
3.0
2.0
6
8
10
12
14
16
18
21
25
30
35
40
45
50
60
70
80
90
100
120
140
160
180
200
250
300
400
500
600
700
800
900
1000
1100
1200
1200#/□"
B. T. U PER LB.
0
50
100
150
200
250
300
VELOCITY OF STEAM — FT. PER SECOND.
0
500
1000
1500
2000
2500
3000
3500

PL





MOLLIER CHART FOR STEAM

REDRAWN FOR REPRODUCTION TO A SMALLER SCALE FROM THE LARGE CHARTS PUBLISHED IN MECHANICAL ENGINEERING IN FEB. 1926. COPIES OF THE ORIGINAL AND LARGER CHARTS MAY BE OBTAINED FROM THE A.S.M.E.

E I

TABLE D.—SUPERHEATED STEAM

Abridged to the nearest four figures from the charts and tables prepared from the Harvard Throttling Experiments, by J. H. Keenan, in MECHANICAL ENGINEERING, Feb., 1926.

ABS. PRESS.		SUPERHEAT, DEGREES FAHRENHEIT							
Lb. per Sq. In.		0	20	50	100	150	200	300	400
1	t	101.8	121.8	151.8	201.8	251.8	301.8	401.8	501.8
	V	333.0	345.5	363.9	393.9	423.8	453.7	513.4	573.1
	h	1104.0	1113.0	1127.0*	1150.0*	1173.0*	1196.0*	1242.0*	1288.0*
	ϕ	1.975	1.992	2.014*	2.050*	2.084*	2.115*	2.170*	2.222*
5	t	162.3	182.3	212.3	262.3	312.3	362.3	462.3	562.3
	V	73.3	75.9	79.7	85.7	91.8	97.8	109.8	121.8
	h	1131.0	1140.0	1153.0	1176.0	1200.0	1222.0	1269.0*	1315.0*
	ϕ	1.843	1.858	1.879	1.912	1.943	1.972	2.025*	2.074*
10	t	193.2	213.2	243.2	293.2	343.2	393.2	493.2	593.2
	V	38.4	39.7	41.6	44.6	47.6	50.6	56.6	62.5
	h	1143.0	1152.0	1166.0	1190.0	1213.0	1236.0	1283.0	1329.0*
	ϕ	1.787	1.801	1.822	1.854	1.884	1.912	1.964	2.010*
15	t	213.0	233.0	263.0	313.0	363.0	413.0	513.0	613.0
	V	26.29	27.12	28.35	30.40	32.44	34.45	38.46	42.46
	h	1151.0	1160.0	1174.0	1198.0	1222.0	1245.0	1292.0	1340.0
	ϕ	1.754	1.768	1.789	1.820	1.849	1.877	1.929	1.976
20	t	228.0	248.0	278.0	328.0	378.0	428.0	528.0	628.0
	V	20.09	20.73	21.66	23.21	24.74	26.26	29.28	32.28
	h	1156.0	1166.0	1180.0	1204.0	1228.0	1251.0	1299.0	1347.0
	ϕ	1.731	1.745	1.765	1.796	1.826	1.854	1.905	1.951
30	t	250.3	270.3	300.3	350.3	400.3	450.3	550.3	650.3
	V	13.74	14.18	14.81	15.85	16.88	17.90	19.93	21.94
	h	1164.0	1174.0	1188.0	1213.0	1237.0	1261.0	1309.0	1357.0
	ϕ	1.699	1.712	1.732	1.764	1.793	1.820	1.870	1.915
40	t	267.3	287.3	317.3	367.3	417.3	467.3	567.3	667.3
	V	10.49	10.82	11.30	12.10	12.89	13.66	15.18	16.69
	h	1169.0	1180.0	1194.0	1219.0	1244.0	1268.0	1316.0	1365.0
	ϕ	1.676	1.690	1.710	1.741	1.770	1.796	1.846	1.891
60	t	292.7	312.7	342.7	392.7	442.7	492.7	592.7	692.7
	V	7.16	7.39	7.73	8.27	8.81	9.33	10.36	11.38
	h	1177.0	1188.0	1203.0	1229.0	1254.0	1278.0	1327.0	1377.0
	ϕ	1.643	1.658	1.677	1.709	1.736	1.763	1.812	1.857
80	t	312.0	332.0	362.0	412.0	462.0	512.0	612.0	712.0
	V	5.46	5.64	5.90	6.32	6.72	7.11	7.89	8.66
	h	1182.0	1193.0	1210.0	1236.0	1261.0	1286.0	1336.0	1385.0
	ϕ	1.620	1.634	1.654	1.686	1.714	1.740	1.789	1.832
100	t	327.8	347.8	377.8	427.8	477.8	527.8	627.8	727.8
	V	4.429	4.57	4.78	5.11	5.44	5.77	6.38	7.00
	h	1186.0	1198.0	1215.0	1241.0	1267.0	1292.0	1342.0	1392.0
	ϕ	1.602	1.616	1.637	1.668	1.695	1.722	1.770	1.814

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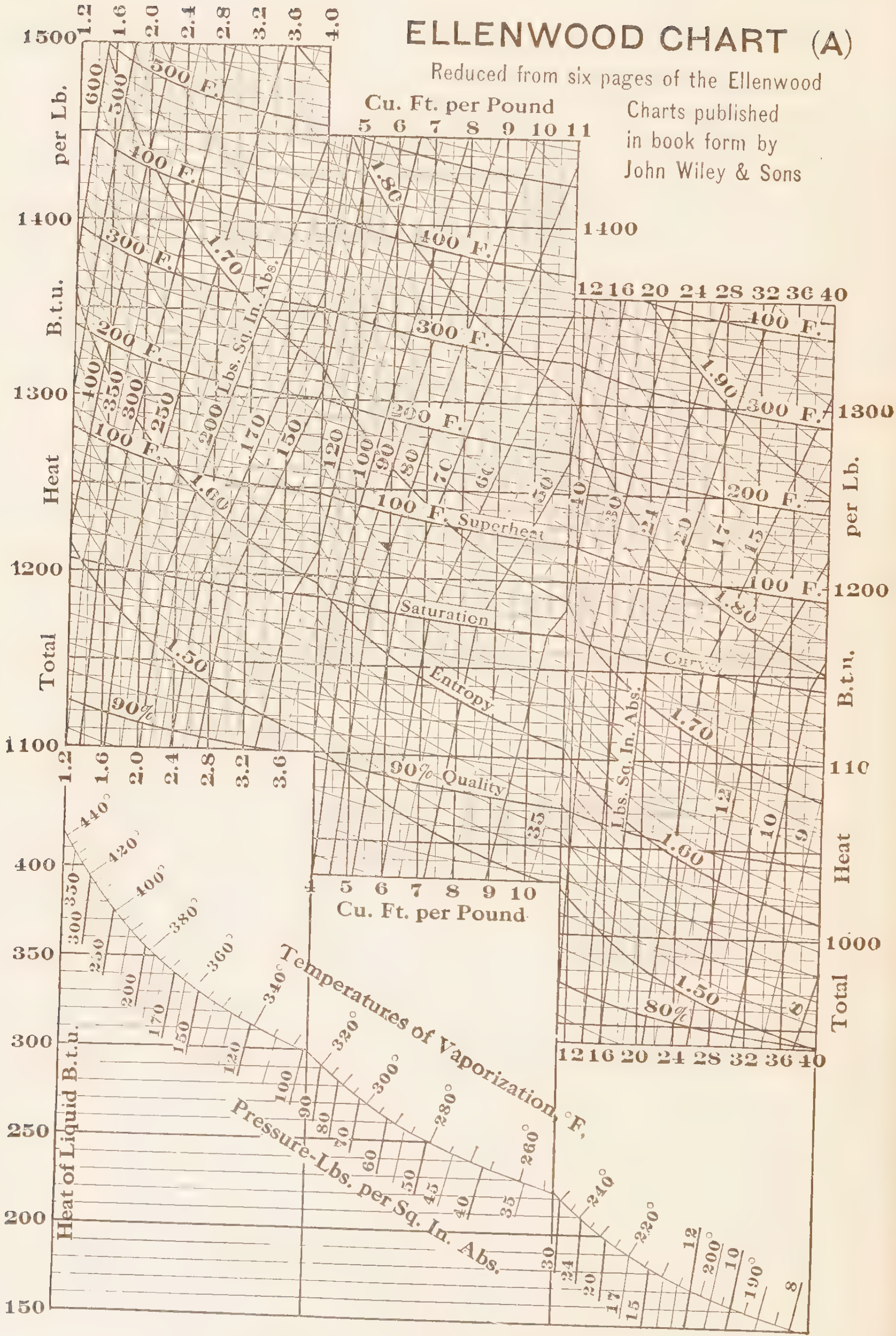
* Value to the nearest four figures, as given in the Steam Tables of Mark and Davis, 1912 Edition

TABLE D.—(*Continued*)

Abs. Press.		Superheat, Degrees Fahrenheit							
Lb. per Sq. In.		0	20	50	100	150	200	300	400
120	t	341.3	361.3	391.3	441.3	491.3	541.3	641.3	741.3
	V	3.724	3.847	4.026	4.315	4.588	4.857	5.38	5.90
	h	1189.0	1202.0	1219.0	1246.0	1272.0	1297.0	1348.0	1398.0
	ϕ	1.587	1.602	1.622	1.653	1.681	1.707	1.755	1.799
140	t	353.1	373.1	403.1	453.1	503.1	553.1	653.1	753.1
	V	3.216	3.326	3.486	3.731	3.969	4.205	4.65	5.10
	h	1192.0	1205.0	1222.0	1250.0	1276.0	1302.0	1352.0	1403.0
	ϕ	1.574	1.589	1.610	1.641	1.669	1.695	1.743	1.786
160	t	363.6	383.6	413.6	463.6	513.6	563.6	663.6	763.6
	V	2.833	2.929	3.069	3.281	3.498	3.706	4.10	4.50
	h	1194.0	1207.0	1225.0	1253.0	1279.0	1305.0	1357.0	1408.0
	ϕ	1.564	1.579	1.600	1.631	1.659	1.685	1.732	1.776
180	t	373.1	393.1	423.1	473.1	523.1	573.1	673.1	773.1
	V	2.531	2.614	2.744	2.940	3.13	3.313	3.67	4.03
	h	1196.0	1209.0	1228.0	1256.0	1282.0	1309.0	1360.0	1412.0
	ϕ	1.554	1.569	1.590	1.621	1.649	1.675	1.723	1.766
200	t	381.9	401.9	431.9	481.9	531.9	581.9	681.9	781.9
	V	2.287	2.360	2.480	2.667	2.839	2.999	3.32	3.63
	h	1198.0	1211.0	1230.0	1258.0	1285.0	1312.0	1364.0	1415.0
	ϕ	1.545	1.560	1.582	1.613	1.641	1.666	1.719	1.758
250	t	401.1	421.1	451.1	501.1	551.1	601.1	701.1	801.1
	V	1.843	1.909	2.003	2.154	2.295	2.430	2.69	
	h	1200.0	1214.0	1234.0	1263.0	1291.0	1318.0	1371.0	
	ϕ	1.526	1.541	1.563	1.595	1.623	1.649	1.697	
300	t	417.5	437.5	467.5	517.5	567.5	617.5	717.5	817.5
	V	1.544	1.601	1.684	1.812	1.933	2.047	2.266	
	h	1202.0	1217.0	1237.0	1267.0	1296.0	1323.0	1377.0	
	ϕ	1.510	1.526	1.548	1.580	1.609	1.635	1.682	
350	t	431.9	451.9	481.9	531.9	581.9	631.9	731.9	831.9
	V	1.326	1.378	1.452	1.562	1.668	1.769	1.958	
	h	1203.0	1218.0	1239.0	1271.0	1300.0	1328.0	1382.0	
	ϕ	1.497	1.513	1.535	1.568	1.596	1.622	1.670	
400	t	444.8	464.8	494.8	544.8	594.8	644.8	744.8	844.8
	V	1.162	1.208	1.275	1.374	1.466	1.558	1.725	
	h	1203.0	1220.0	1241.0	1273.0	1303.0	1332.0	1386.0	
	ϕ	1.483	1.501	1.524	1.557	1.585	1.612	1.660	
450	t	456.5	476.5	506.5	556.5	606.5	656.5	756.5	856.5
	V	1.033	1.075	1.136	1.227	1.314	1.392	1.543	
	h	1203.0	1220.0	1242.0	1276.0	1306.0	1335.0	1390.0	
	ϕ	1.472	1.491	1.514	1.547	1.576	1.603	1.650	

TABLE D—(*Continued*)

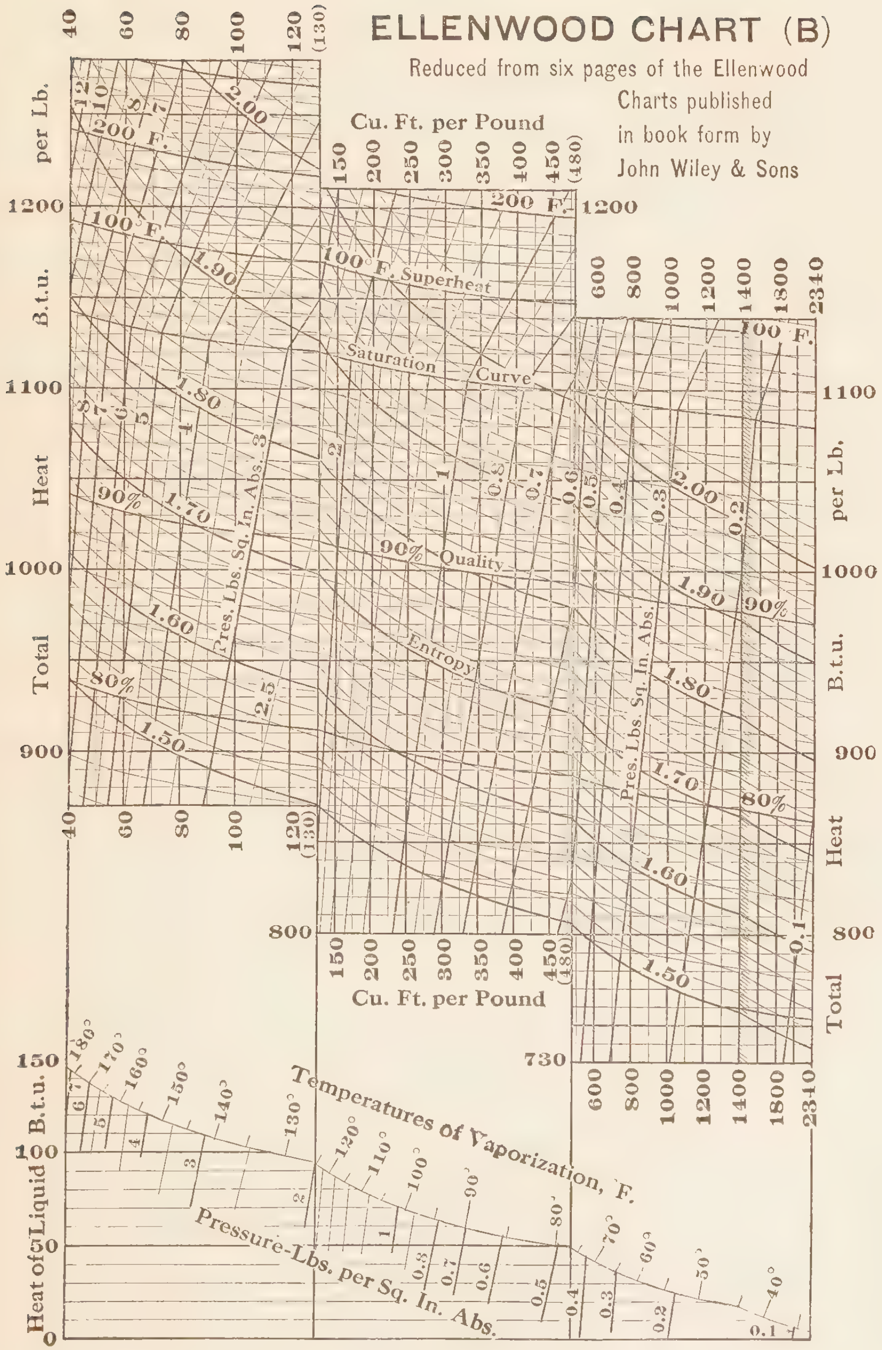
ABS. PRESS. Lb. per Sq. In.		SUPERHEAT, DEGREE FAHRENHEIT							
		0	20	50	100	150	200	300	400
500	t	467.2	487.2	517.2	567.2	617.2	667.2	767.2	867.2
	$\overline{V}$	0.928	0.967	1.021	1.107	1.186	1.256	1.394	
	h	1202.0	1221.0	1243.0	1278.0	1308.0	1338.0	1394.0	
	ϕ	1.462	1.481	1.504	1.538	1.567	1.594	1.642	
550	t	477.2	497.2	527.2	577.2	627.2	677.2	777.2	877.2
	$\overline{V}$	0.842	0.874	0.928	1.009	1.080	1.146	1.272	
	h	1202.0	1221.0	1244.0	1279.0	1311.0	1340.0	1397.0	
	ϕ	1.452	1.472	1.496	1.530	1.560	1.587	1.635	
600	t	486.5	506.5	536.5	586.5	636.5	686.5	786.5	886.5
	$\overline{V}$	0.769	0.804	0.851	0.927	0.994	1.055	1.171	
	h	1201.0	1221.0	1245.0	1281.0	1312.0	1343.0	1400.0	
	ϕ	1.444	1.464	1.489	1.523	1.553	1.580	1.628	
650	t	495.2	515.2	545.2	595.2	645.2	695.2	795.2	895.2
	$\overline{V}$	0.707	0.739	0.786	0.857	0.919	0.978	1.086	
	h	1200.0	1221.0	1246.0	1282.0	1314.0	1345.0	1402.0	
	ϕ	1.434	1.456	1.481	1.517	1.547	1.574	1.621	
700	t	503.4	523.4	553.4	603.4	653.4	703.4	803.4	903.4
	$\overline{V}$	0.655	0.685	0.730	0.794	0.855	0.911	1.013	
	h	1198.0	1221.0	1246.0	1283.0	1316.0	1346.0		
	ϕ	1.427	1.450	1.474	1.510	1.541	1.568		
750	t	511.1	531.1	561.1	611.1	661.1	711.1	811.1	911.1
	$\overline{V}$	0.609	0.638	0.680	0.741	0.798	0.852	0.950	
	h	1197.0	1220.0	1246.0	1284.0	1317.0	1348.0		
	ϕ	1.420	1.443	1.469	1.504	1.535	1.563		
800	t	518.5	538.5	568.5	618.5	668.5	718.5	818.5	918.5
	$\overline{V}$	0.568	0.596	0.636	0.694	0.749	0.800	0.896	
	h	1195.0	1219.0	1246.0	1284.0	1318.0	1350.0		
	ϕ	1.412	1.436	1.463	1.498	1.529	1.556		
900	t	532.3	552.3	582.3	632.3	682.3	732.3	832.3	932.3
	$\overline{V}$	0.500	0.525	0.562	0.617	0.667	0.713		
	h	1191.0	1217.0	1246.0	1285.0	1320.0	1352.0		
	ϕ	1.398	1.424	1.452	1.488	1.520	1.546		
1000	t	544.9	564.9	594.9	644.9	694.9	744.9	844.9	944.9
	$\overline{V}$	0.444	0.469	0.503	0.556	0.601	0.643		
	h	1186.0	1215.0	1246.0	1285.0	1321.0	1354.0		
	ϕ	1.385	1.413	1.442	1.479	1.510	1.538		
1100	t	556.6	576.6	606.6	656.6	706.6	756.6	856.6	956.6
	$\overline{V}$	0.400	0.423	0.455	0.504	0.547	0.588		
	h	1181.0	1212.0	1245.0	1285.0	1322.0	1355.0		
	ϕ	1.373	1.402	1.433	1.470	1.501	1.530		
1200	t	567.7	587.7	617.3	667.7	717.3	767.7	867.7	967.7
	$\overline{V}$	0.361	0.384	0.414	0.460	0.502	0.541		
	h	1177.0	1210.0	1243.0	1285.0	1322.0	1357.0		
	ϕ	1.362	1.394	1.422	1.462	1.494	1.522		



ELLENWOOD CHART (A)

Reduced from six pages of the Ellenwood Charts published in book form by John Wiley & Sons

Plate IIA. The Ellenwood Chart.



ELLENWOOD CHART (B)

Reduced from six pages of the Ellenwood Charts published in book form by John Wiley & Sons

Plate IIB. The Ellenwood Chart (Cont.)

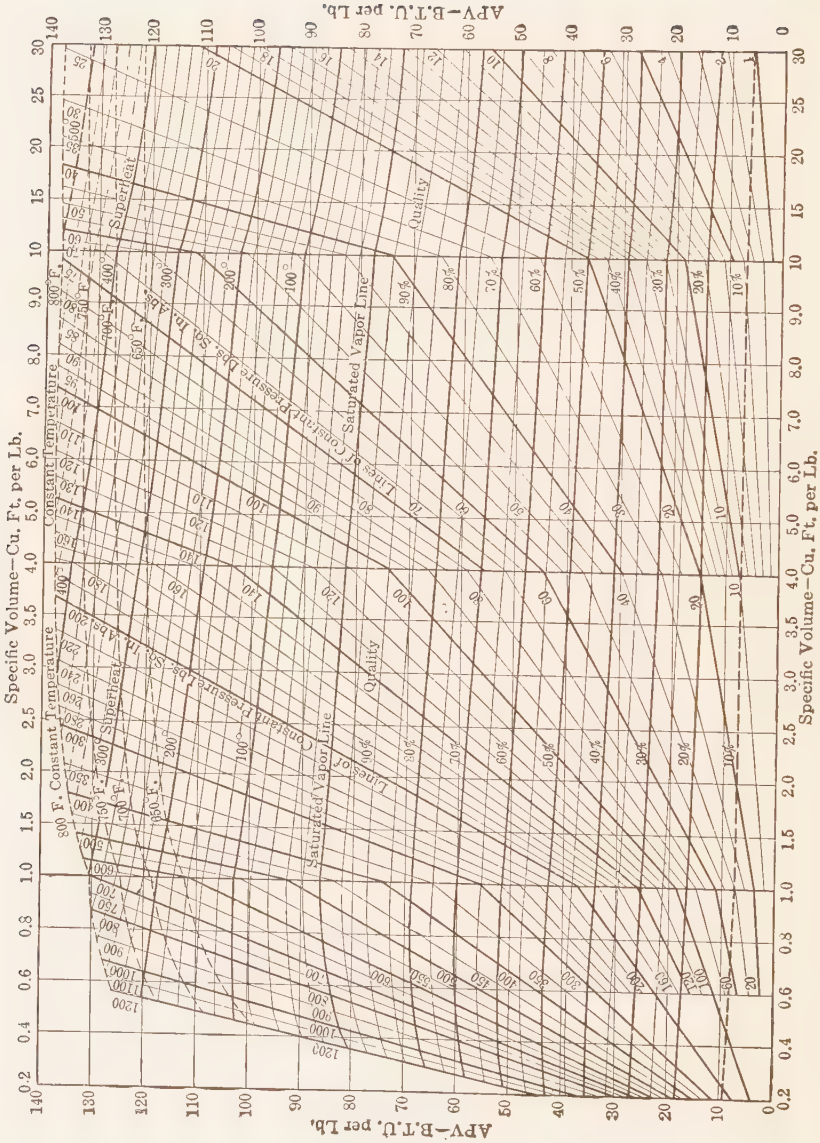
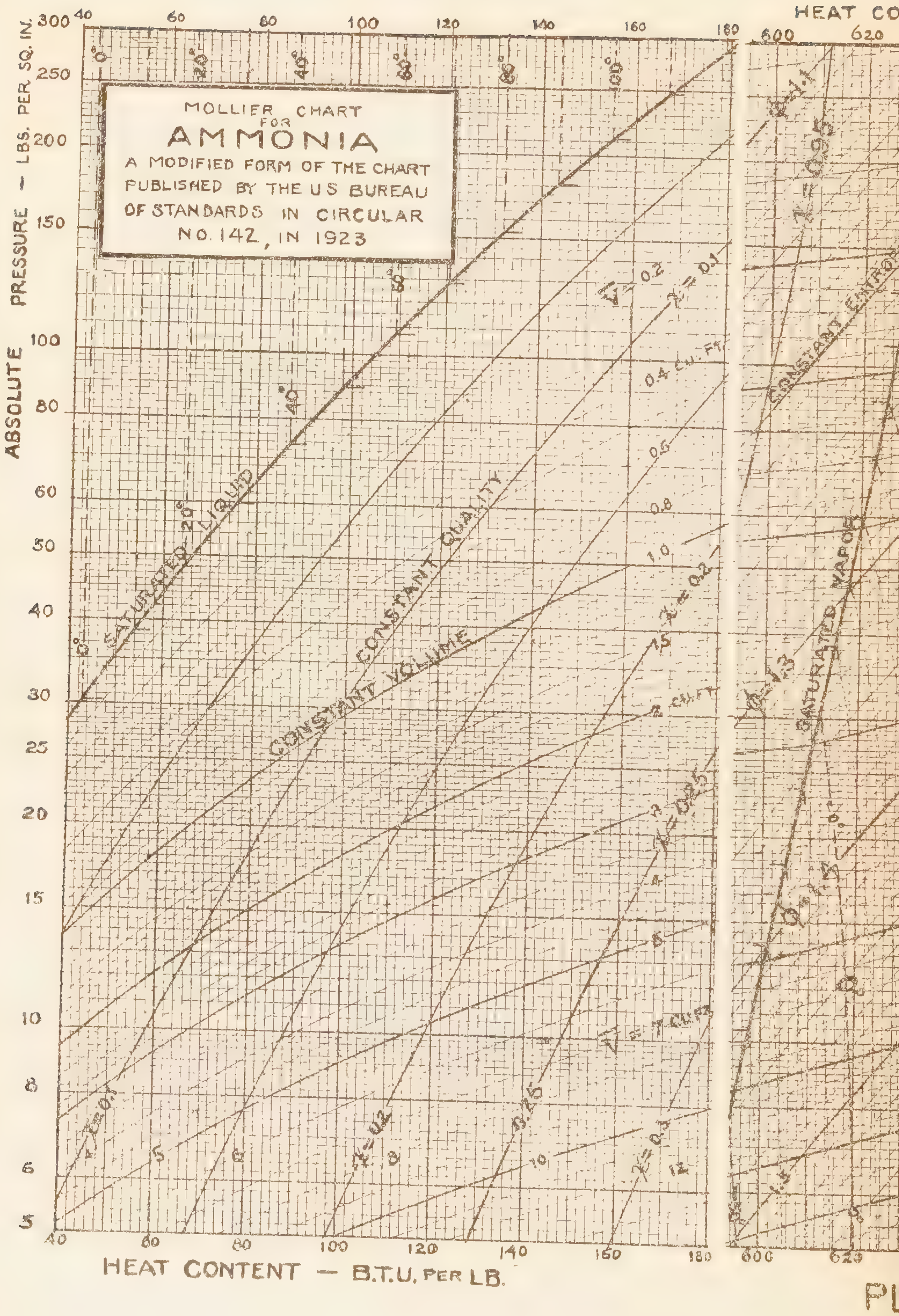
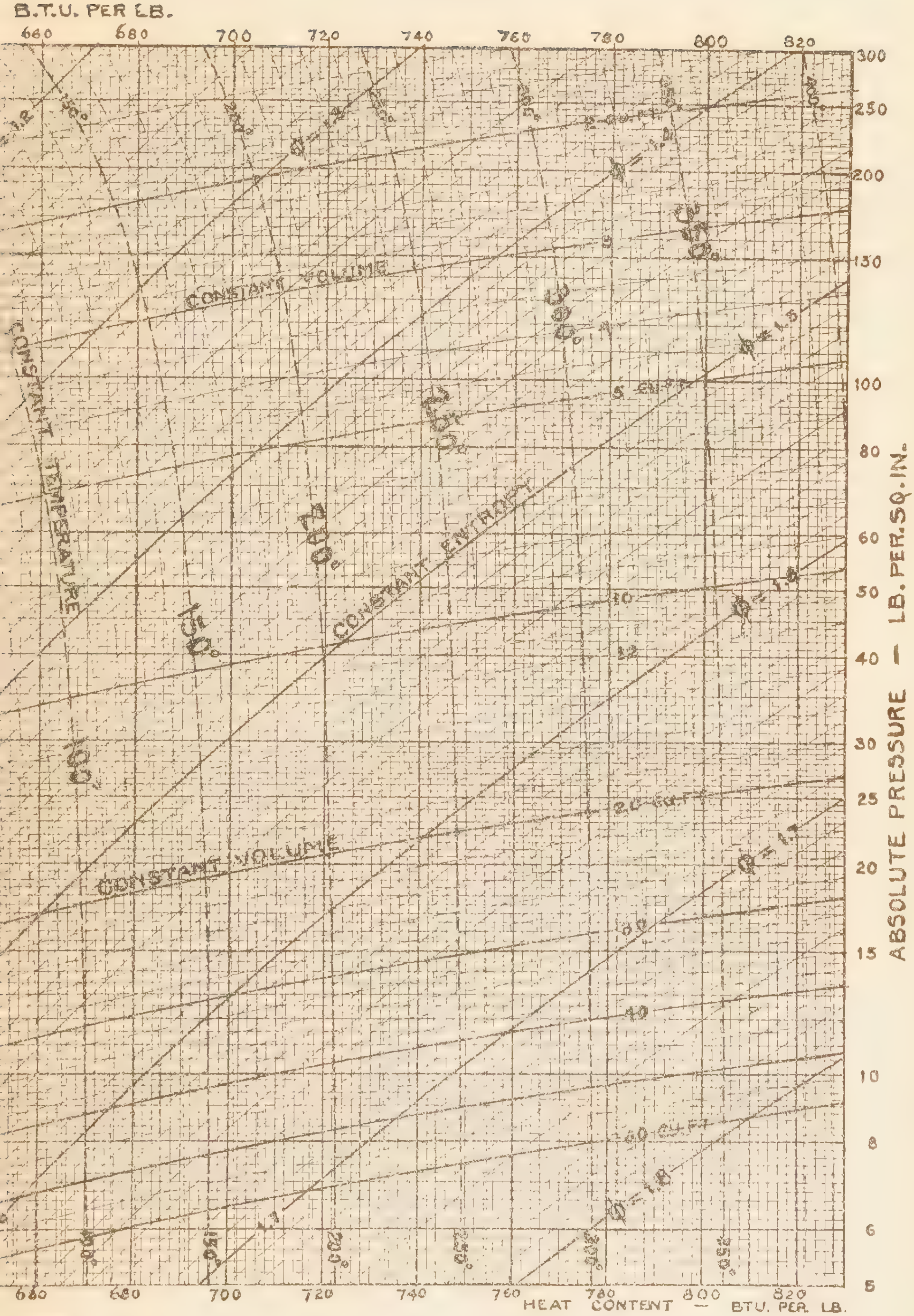


PLATE III. ELLENWOOD APV-V CHART.

MOLLIER CHART FOR AMMONIA
A MODIFIED FORM OF THE CHART PUBLISHED BY THE U S BUREAU OF STANDARDS IN CIRCULAR No. 142, IN 1923
ABSOLUTE PRESSURE – LBS. PER SQ. IN.
300
250
200
150
100
80
60
50
40
30
25
20
15
10
8
6
5
HEAT CONTENT – B.T.U. PER LB.
40
60
80
100
120
140
160
180
600
620
HEAT CO
0°
20°
40°
60°
80°
100°
SATURATED LIQUID
CONSTANT QUALITY
CONSTANT VOLUME
x = 0.1
x = 0.2
x = 0.25
x = 0.3
v = 0.2
0.4 CU. FT.
0.6
0.8
1.0
1.5
2 CU. FT.
3
4
5
6
7 CU. FT.
8
10
12
CONSTANT ENTROPY
SATURATED VAPOR
x = 0.95
1.1
1.3
1.5

PL


B.T.U. PER LB.
660
680
700
720
740
760
780
800
820
300
250
200
150
100
80
60
50
40
30
25
20
15
10
8
6
5
ABSOLUTE PRESSURE — LB. PER SQ. IN.
HEAT CONTENT — BTU. PER. LB.
CONSTANT VOLUME
CONSTANT VOLUME
CONSTANT ENTROPY
CONSTANT TEMPERATURE
100°
150°
200°
250°
300°
350°
400°
5 CU.FT.
10
12
2.0 CU.FT.
30
40
60 CU.FT.
Φ=1.5
Φ=1.6
Φ=1.7
Φ=1.8

IV

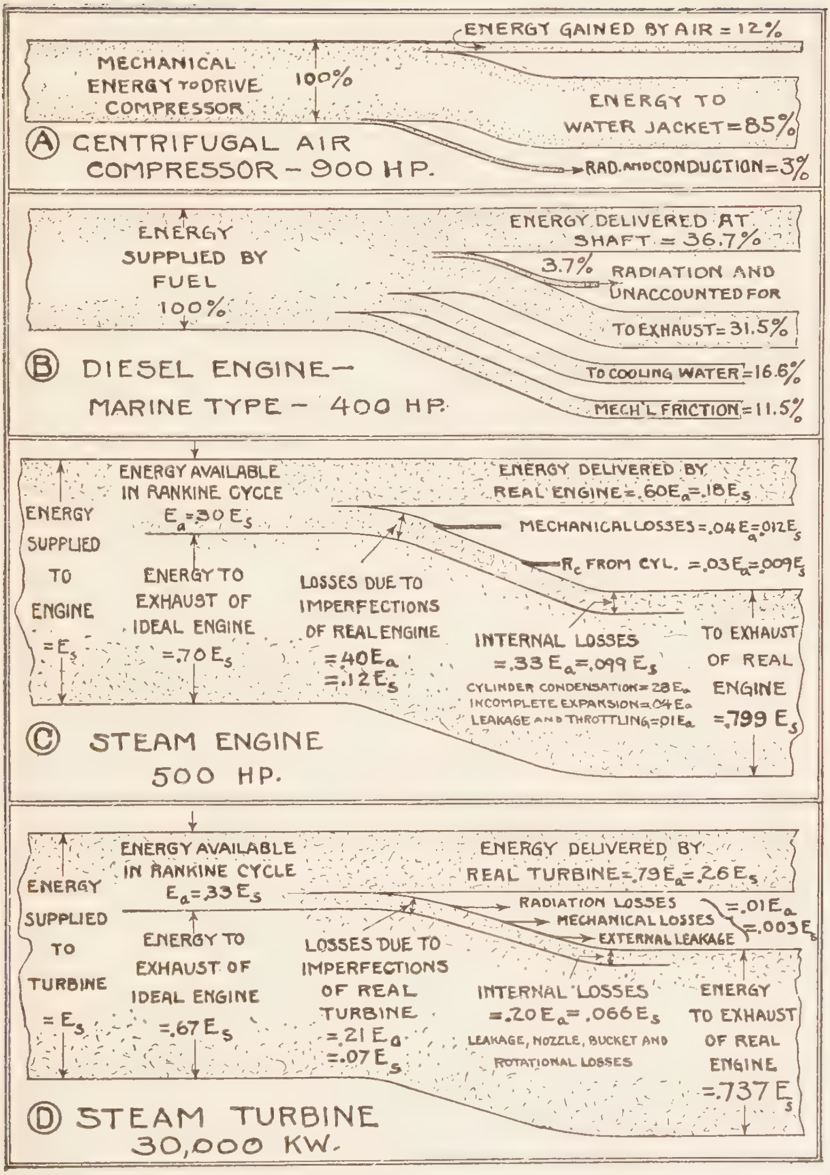

ENERGY GAINED BY AIR = 12%
MECHANICAL ENERGY TO DRIVE COMPRESSOR
100%
ENERGY TO WATER JACKET = 85%
RAD. AND CONDUCTION = 3%
Ⓐ CENTRIFUGAL AIR COMPRESSOR – 900 HP.
ENERGY SUPPLIED BY FUEL 100%
ENERGY DELIVERED AT SHAFT = 36.7%
3.7% RADIATION AND UNACCOUNTED FOR
TO EXHAUST = 31.5%
TO COOLING WATER = 16.6%
MECH'L FRICTION = 11.5%
Ⓑ DIESEL ENGINE — MARINE TYPE – 400 HP.
ENERGY AVAILABLE IN RANKINE CYCLE $E_a = .30 E_s$
ENERGY DELIVERED BY REAL ENGINE = $.60 E_a = .18 E_s$
MECHANICAL LOSSES = $.04 E_a = .012 E_s$
R_c FROM CYL. = $.03 E_a = .009 E_s$
ENERGY SUPPLIED TO ENGINE = E_s
ENERGY TO EXHAUST OF IDEAL ENGINE = $.70 E_s$
LOSSES DUE TO IMPERFECTIONS OF REAL ENGINE = $.40 E_a$ = $.12 E_s$
INTERNAL LOSSES = $.33 E_a = .099 E_s$
CYLINDER CONDENSATION = $.28 E_a$
INCOMPLETE EXPANSION = $.04 E_a$
LEAKAGE AND THROTTLING = $.01 E_a$
TO EXHAUST OF REAL ENGINE = $.799 E_s$
Ⓒ STEAM ENGINE 500 HP.
ENERGY AVAILABLE IN RANKINE CYCLE $E_a = .33 E_s$
ENERGY DELIVERED BY REAL TURBINE = $.79 E_a = .26 E_s$
RADIATION LOSSES
MECHANICAL LOSSES
EXTERNAL LEAKAGE
= $.01 E_a$ = $.003 E_s$
ENERGY SUPPLIED TO TURBINE = E_s
ENERGY TO EXHAUST OF IDEAL ENGINE = $.67 E_s$
LOSSES DUE TO IMPERFECTIONS OF REAL TURBINE = $.21 E_a$ = $.07 E_s$
INTERNAL LOSSES = $.20 E_a = .066 E_s$
LEAKAGE, NOZZLE, BUCKET AND ROTATIONAL LOSSES
ENERGY TO EXHAUST OF REAL ENGINE = $.737 E_s$
Ⓓ STEAM TURBINE 30,000 KW.

Plate V. Energy Streams Typical of Large Modern Machines.

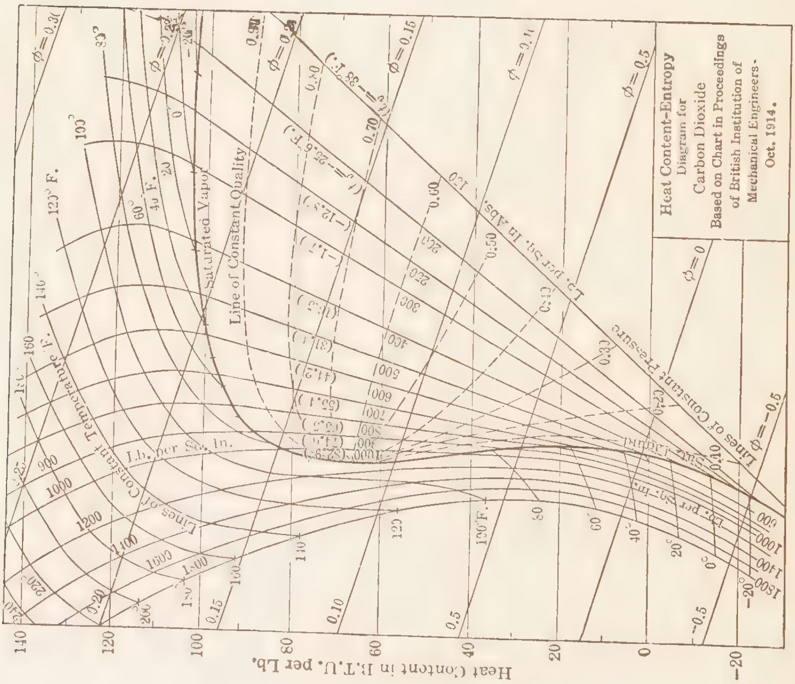
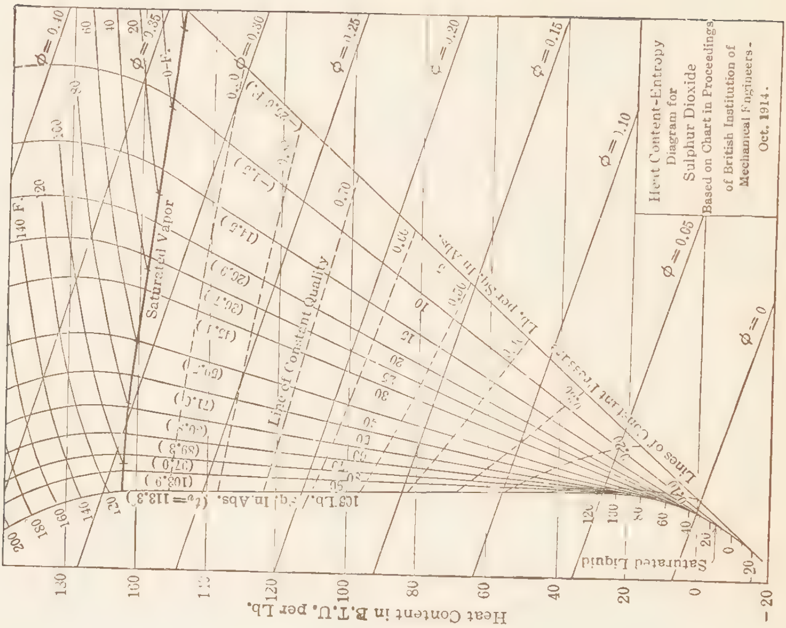


PLATE VI.



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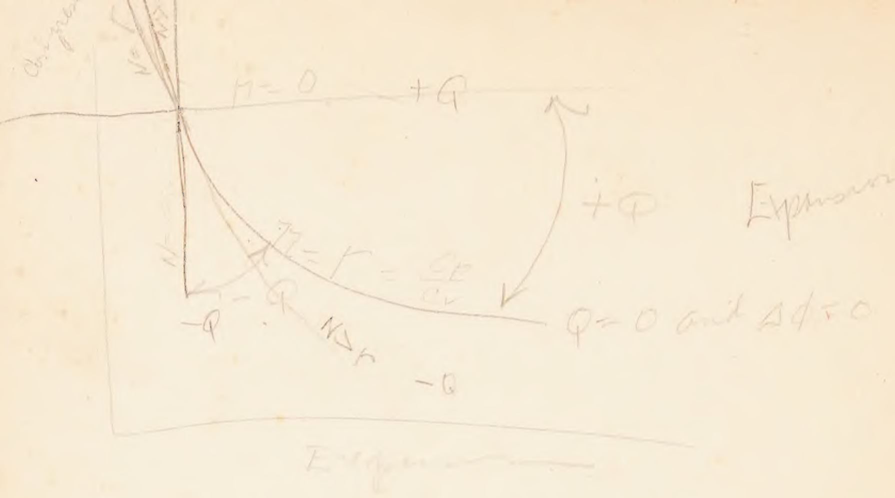
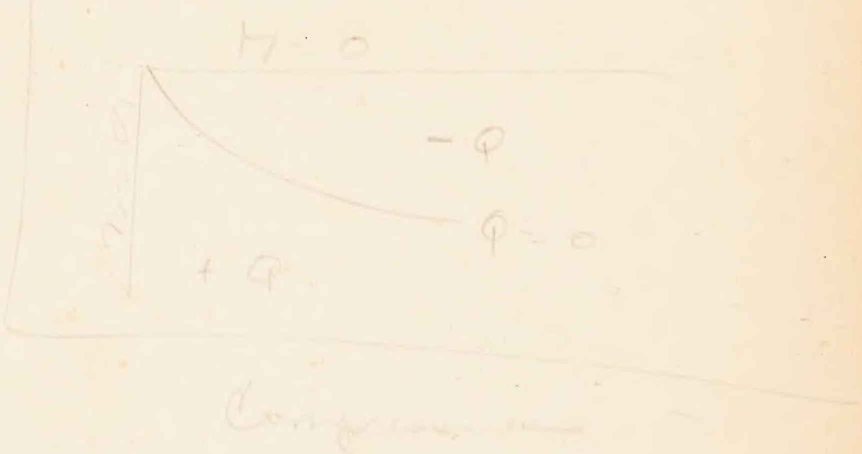
$h_2 = h_1 - A\bar{V}_f (P_1 - P_2)$


Compression
N=0
+Q
N=γ = Cp/Cv
N=
-Q
-Q
N>γ
+Q
Expansion
Q=0 and Δt=0
-Q
Expansion

$PV^N = C$


N=0
-Q
Q=0
+Q
Compression

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